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Mixed frequency models with MA components

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Abstract

Temporal aggregation in general introduces a moving average (MA) component in the aggregated model. A similar feature emerges when not all but only a few variables are aggregated, which generates a mixed frequency model. The MA component is generally neglected, likely to preserve the possibility of OLS estimation, but the consequences have never been properly studied in the mixed frequency context. In this paper, we show, analytically, in Monte Carlo simulations and in a forecasting application on U.S. macroeconomic variables, the relevance of considering the MA component in mixed-frequency MIDAS and Unrestricted-MIDAS models (MIDAS-ARMA and UMIDAS-ARMA). Specifically, the simulation results indicate that the short-term forecasting performance of MIDAS-ARMA and UMIDAS-ARMA is better than that of, respectively, MIDAS and UMIDAS. The empirical applications on nowcasting U.S. GDP growth, investment growth and GDP deflator inflation confirm this ranking. Moreover, in both simulation and empirical results, MIDAS-ARMA is better than UMIDAS-ARMA.

Keywords: Temporal aggregation, MIDAS models, ARMA models.

JEL Classification Code: E37, C53.

Non-technical summary

Temporal aggregation generally introduces a moving average (MA) component in the model for the aggregate variable. A similar feature should be present in the mixed frequency models, and indeed we show formally that this is in general the case. The effects of neglecting the MA component have been rarely explicitly considered in a single frequency context. For mixed frequency models, there are no results available.

We close this gap and analyze the relevance of the inclusion of an MA component in mixed-data sampling (MIDAS) and unrestricted mixed-data sampling (UMIDAS) models, with the resulting specifications labeled, respectively, MIDAS-ARMA and UMIDAS-ARMA. We first compare the forecasting performance of the mixed frequency models with and without the MA component in a set of Monte Carlo experiments, using a variety of Data Generating Processes (DGPs). Next, we carry out an empirical investigation, where we predict several quarterly macroeconomic variables using timely monthly indicators. In particular, we forecast three relevant quarterly U.S. macroeconomic variables: real GDP growth, real private non residential fixed investment (PNFI) growth and GDP deflator inflation.

In the Monte Carlo simulations, the short-term forecasting performance is better when including the MA component, and the gains are higher the more persistent is the series. Moreover, in general the MIDAS-ARMA specifications are slightly better than the UMIDAS-ARMA specifications. This pattern suggests that adding the MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure. In the empirical exercise, the inclusion of an MA component generally improves the forecasting performance substantially. For all variables, and in line with the simulation results, MIDAS-ARMA is better than UMIDAS-ARMA.

1 Introduction

The use of mixed-frequency models has become increasingly popular among academics and practitioners. It is in fact by now well recognized that a good nowcast or short-term forecast for a low frequency variable, such as GDP growth and its components, requires to exploit the timely information contained in higher frequency macroeconomic or financial indicators, such as surveys or spreads. A growing literature has flourished proposing different methods to deal with the mixed-frequency feature. In particular, models cast in state-space form, such as vector autoregressions (VAR) and factor models, can deal with mixed-frequency data, taking advantage of the Kalman filter to interpolate the missing observations of the series only available at low frequency (see, among many others, Mariano and Murasawa (2010) and Giannone et al. (2008) in a classical context, and Eraker et al. (2015) and Schorfheide and Song (2015) in a Bayesian context). A second approach has been proposed by Ghysels (2016). He introduces a different class of mixed-frequency VAR models, in which the vector of endogenous variables includes both high and low frequency variables, with the former stacked according to the timing of the data releases. A third approach is the mixed-data sampling (MIDAS) regression, introduced by Ghysels et al. (2006), and its unrestricted version (UMIDAS) by Foroni et al. (2015). MIDAS models are tightly parameterized, parsimonious models, which allow for the inclusion of many lags of the explanatory variables. Given their non-linear form, MIDAS models are typically estimated by non-linear least squares (NLS)[1]. UMIDAS models are the unrestricted counterpart of MIDAS models, which can be estimated by simple ordinary least squares (OLS), but work well only when the frequency mismatch is small.[2]

In this paper, we start from the observation that temporal aggregation generally introduces a moving average (MA) component in the model for the aggregate variable (see, e.g., Marcellino (1999) and the references therein). A similar feature should be present in the mixed frequency models, and indeed we show formally that this is in general the case[3]. The MA component is often neglected, both in same frequency and in mixed frequency models, likely to preserve the

[1] In a recent paper Ghysels and Qian (2018) propose to use OLS estimation of the MIDAS regression slope and intercept combined with profiling the polynomial weighting scheme parameters.

[2] The literature on mixed-frequency approaches is vast. The papers cited in the text are a non-exhaustive list of key contributions to the field. For a review of the mixed-frequency literature, see Bai et al. (2013) and Foroni and Marcellino (2013) among many others.

[3] An analysis of identifiability on ARMA processes with mixed-frequency observations is provided by Anderson et al. (2016), on VARMA processes by Deistler et al. (2017).

possibility of OLS estimation and on the grounds that it can be approximated by a sufficiently long autoregressive (AR) component.

The effects of neglecting the MA component have been rarely explicitly considered. In a single frequency context, Lutkepohl (2006) showed that VARMA models are especially appropriate in forecasting, since they can capture the dynamic relations between time series with a small number of parameters. Further, Dufour and Stevanovic (2013) showed that a VARMA instead of VAR model for the factors provides better forecasts for several key macroeconomic aggregates relative to standard factor models, as well as producing a more precise representation of the effects and transmission of monetary policy. Leroux et al. (2017) found that ARMA(1,1) models predict well the inflation change and outperform many data-rich models, confirming the evidence on forecasting inflation by Stock and Watson (2007), Faust and Wright (2013) and Marcellino et al. (2006). Finally, VARMA models are often the correct reduced form representation of DSGE models (see, for example, Ravenna (2007)). For mixed frequency models, there are no results available.

We close this gap and analyze the relevance of the inclusion of an MA component in MIDAS and UMIDAS models, with the resulting specifications labeled, respectively, MIDAS-ARMA and UMIDAS-ARMA. We first compare the forecasting performance of the mixed frequency models with and without the MA component in a set of Monte Carlo experiments, using a variety of Data Generating Processes (DGPs). It turns out that the short-term forecasting performance is better when including the MA component, and the gains are higher the more persistent is the series. Moreover, in general the MIDAS-ARMA specifications are slightly better than the UMIDAS-ARMA specifications, though the differences are minor. This pattern is in contrast with the findings in Foroni et al. (2015), and suggests that adding the MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure.

Next, we carry out an empirical investigation, where we predict several quarterly macroeconomic variables using timely monthly indicators. In particular, we forecast three relevant quarterly U.S. macroeconomic variables: real GDP growth, real private non residential fixed investment (PNFI) growth and GDP deflator inflation. The latter variable is particularly relevant, as Stock and Watson (2007) show that the MA component for US inflation is important, especially after 1984. In fact, while during the 1970s the inflation process could be very well approximated by a low order AR, after the 1980s this has become less accurate and the inclusion of an MA component more relevant. Evidence on the importance of the MA component for the U.S. inflation is also found by Ng and Perron (2001) and Perron and Ng (1996). As monthly explanatory variables, we consider industrial production and employment for the real GDP growth and the PNFI growth, CPI inflation and personal consumption expenditures (PCE) inflation for the GDP deflator. The inclusion of an MA component generally improves the forecasting performance substantially. In particular, adding the MA part to forecast GDP growth one-year ahead ameliorates the MSE up to 10%, while for PNFI we obtain even bigger gains, up to 30% one-year ahead. Also in the case of GDP deflator we obtain robust improvements, which go up to 15%. For all variables, and in line with the simulation results, MIDAS-ARMA is better that UMIDAS-ARMA. Lastly, full sample estimates of MA coefficients are significant and important in most of MIDAS-ARMA and UMIDAS-ARMA specifications.

The remainder of the paper proceeds as follows. In Section 2 we show that temporal aggregation generally creates an MA component also in mixed frequency models. In Section 3 we describe parameter estimators for the MIDAS-ARMA and UMIDAS-ARMA models. In Section 4 we present the design and results of the simulation exercises. In Section 5 we develop the empirical applications on forecasting U.S. quarterly variables with monthly indicators. In Section 6 we summarize main results and conclude. Robustness analysis on the Monte Carlo simulations and the empirical applications are reported in Appendix.

2 The rationale for an MA component in mixed frequency models

The UMIDAS regression approach can be derived by aggregation of a general dynamic linear model in high frequency, as shown by Foroni et al. (2015), while the MIDAS model imposes specific restrictions on the UMIDAS coefficients in order to reduce their number, which is particularly relevant when the frequency mismatch is large (for example, with daily and quarterly series). In Section 2.1, we briefly review the derivation of the UMIDAS model, highlighting that, in general, there should be an MA component, even though it is generally disregarded. In Section 2.2, we provide two simple analytical examples in which, starting from a high-frequency model without MA term, we end up with a mixed frequency model in which the MA component is present. We discuss estimation of mixed frequency models with an MA component in a separate section.

2.1 UMIDAS regressions and dynamic linear models

Let us assume that the Data Generating Process (DGP) for the variable y and the N variables x is an $ARDL(p, q)$ process, as in Foroni et al. (2015):

$$a(L)y_{t_m} = b_1(L)x_{1t_m} + ... + b_N(L)x_{Nt_m} + e_{yt_m} \tag{1}$$

where $a(L) = 1 - a_1L - ... - a_pL$, $b_j(L) = b_{j1}L + ... + b_{jq}L^q$, $j = 1, ..., N$, and the error e_{yt_m} is white noise. We assume, for simplicity, that $p = q$ and the starting values $y_{-p}, ..., y_0$ and $x_{-p}, ..., x_0$ are all fixed and equal to zero.

We then assume that x can be observed for each period t_m, while y can be only observed every m periods. We define $t = 1, ..., T$ as the low frequency (LF) time unit and $t_m = 1, ..., T_m$ as the high frequency (HF) time unit. The HF time unit is observed m times in the LF time unit. As an example, if we are working with quarterly (LF) and monthly (HF) data, it is $m = 3$ (i.e., three months in a quarter). Moreover, L indicates the lag operator at t_m frequency, while L^m is the lag operator at t frequency.

We also introduce the aggregation operator

$$\omega(L) = \omega_0 + \omega_1L + ... + \omega_{m-1}L^{m-1}, \tag{2}$$

which characterizes the temporal aggregation scheme. For example, $\omega(L) = 1 + L + ... + L^{m-1}$ indicates the sum of the high-frequency observations over the low-frequency period, typically used in the case of flow variables, while $\omega(L) = 1$ corresponds to point-in-time sampling and is typically used for stock variables. As we will see, different aggregation schemes will play a role in generating MA components.

To derive the generating mechanism for y at mixed frequency (MF), we introduce a polynomial in the lag operator, $\beta(L)$, whose degree in L is at most equal to $pm - p$ and which is such that the product $h(L) = \beta(L)a(L)$ only contains powers of L^m. This means that $h(L)$ is a polynomial of the form $h_0L^0 + h_1L^m + h_2L^{2m} + ... + h_{pm-p}L^{pm-p}$. It can be shown that such a polynomial always exists, and its coefficients depend on those of $a(L)$, see Marcellino (1999) for details.

In order to determine the AR component of the MF process, we then multiply both sides of (1) by $\omega(L)$ and $\beta(L)$ to get

$$h(L)\omega(L)y_{t_m} = \beta(L)b_1(L)\omega(L)x_{1t_m} + ... + \beta(L)b_N(L)\omega(L)x_{Nt_m} + \beta(L)\omega(L)e_{yt_m}. \quad (3)$$

Hence, the autoregressive component only depends on LF values of y. Let us consider now the x variables, which are observable at high frequency t_m. Each HF x_{it_m} influences the LF variable y via a polynomial $\beta(L)b_j(L)\omega(L) = b_j(L)\beta(L)\omega(L)$, $j = 1, ..., N$. We see that it is a particular combination of high-frequency values of x_j, equal to $\beta(L)\omega(L)x_{jt_m}$, that affects the low-frequency values of y.

Only under certain, rather strict conditions, it is possible to recover the polynomials $a(L)$ and $b_j(L)$ that appear in the HF model for y from the MF model, and in these cases also $\beta(L)$ can be identified. Therefore, when $\beta(L)$ cannot be identified, we can estimate a model as

$$\begin{aligned} c(L^m)\omega(L)y_{t_m} &= \delta_1(L)x_{1t_m-1} + ... + \delta_N(L)x_{Nt_m-1} + \epsilon_{t_m}, \\ t_m &= m, 2m, 3m, ... \end{aligned} \quad (4)$$

where $c(L^m) = (1 - c_1L^m - ... - c_cL^{mc})$, $\delta_j(L) = (\delta_{j,0} + \delta_{j,1}L + ... + \delta_{j,v}L^v)$, $j = 1, ..., N$.

We can focus now on the error term of equation (3). In general, there is an MA component in the MF model, $q(L^m)u_{yt_m}$, with $q(L^m) = (1+q_1L^m+...+q_qL^{mq})$. The order of $q(L^m)$, q, coincides with the highest multiple of m non zero lag in the autocovariance function of $\beta(L)\omega(L)e_{yt_m}$. The coefficients of the MA component have to be such that the implied autocovariances of $q(L^m)u_{yt_m}$ coincide with those of $\beta(L)\omega(L)e_{yt_m}$ evaluated at all multiples of m. Consequently, also the error term ϵ_{t_m} in the approximate mixed frequency model (4), which is the UMIDAS model, in general has an MA structure. It can be shown that the maximum order of the MA structure is p for average sampling and p-1 for point-in-time sampling, where p is the order of the AR component in the high frequency model for y_{t_m} (see, e.g., Marcellino (1999) for a derivation of this results)[4].

[4]The fact that an aggregated model is misspecified and leads to biased and inconsistent estimators has been highlighted by Andreou et al. (2010). However, contrary to us, they do not focus on the presence of an MA component in the aggregation.

2.2 Two analytical examples

In this section, we consider two simple DGPs and show that, even in these basic cases, an MA component appears in the mixed frequency model. In the first example, we consider an ARDL(1,1) with average sampling, in the second one an ARDL(2,2) with point-in-time sampling. In both cases, we work with monthly and quarterly variables, therefore $m = 3$, as in the empirical applications that will be presented later on. The examples could be easily generalized to consider higher order models and different frequency mismatches m.

ARDL(1,1) with average sampling

Let us assume an ARDL(1,1) as HF DGP:

$$y_{t_m} = ay_{t_m-1} + bx_{t_m-1} + e_{yt_m}, \tag{5}$$

where y_{t_m} is a variable unobservable at HF, x_{t_m} is the high-frequency variable, e_{yt_m} is white noise, and t_m is the high-frequency time index. Although we do not observe y_{t_m}, we observe the quarterly aggregated values of the series.

In order to obtain the model for the quarterly aggregated series, let us write (5) as

$$(1 - aL)\, y_{t_m} = bLx_{t_m} + e_{yt_m}. \tag{6}$$

We consider average sampling, and therefore we define the aggregation operator $\omega(L) = 1 + L + L^2$. Then, we first introduce a polynomial in the lag operator, $\beta(L)$, which is such that the product $h(L) = \beta(L)(1 - aL)$ only contains powers of L^3. This polynomial exists and it is equal to $\left(1 + aL + a^2L^2\right)$. We then multiply both sides of equation (6) by $\omega(L)$ and $\beta(L)$ and we obtain:

$$\begin{aligned}\left(1 + aL + a^2L^2\right)(1 - aL)\left(1 + L + L^2\right) y_{t_m} &= \left(1 + aL + a^2L^2\right) bL \left(1 + L + L^2\right) x_{t_m} + \\ &\quad \left(1 + aL + a^2L^2\right)\left(1 + L + L^2\right) e_{yt_m},\end{aligned} \tag{7}$$

or equivalently:

$$\begin{aligned}\left(1 - a^3L^3\right) \widetilde{y}_{t_m} &= \left(1 + aL + a^2L^2\right) bL \left(1 + L + L^2\right) x_{t_m} + \\ &\quad \left(1 + (a+1)L + \left(a^2 + a + 1\right)L^2 + \left(a^2 + a\right)L^3 + a^2L^4\right) e_{yt_m},\end{aligned} \tag{8}$$

where $\widetilde{y}_{t_m} = \left(1 + L + L^2\right) y_{t_m}$ and $t_m = 3, 6, 9, ...$.

As we saw it in Section 2.1, the order of the MA component coincides with the highest multiple of 3 non zero lag in the autocovariance function of the error term in equation (8), and it is bounded above by the AR order of the model for y_{t_m}.

Eq. (8) is then estimated at quarterly frequency, but making use of all the information available in the HF variable x_{t_m}, and including the MA component, which is of order 1 in this case (being the relevant lag for the quarterly model L^3). The model in eq. (8) is therefore a UMIDAS-AR with an MA(1) component.

ARDL(2,2) with point-in-time sampling

Let us now assume an ARDL(2,2) as HF DGP:

$$y_{t_m} = a_1 y_{t_m-1} + a_2 y_{t_m-2} + b_1 x_{t_m-1} + b_2 x_{t_m-2} + e_{yt_m}, \tag{9}$$

or, equivalently,

$$\left(1 - a_1 L - a_2 L^2\right) y_{t_m} = \left(b_1 L + b_2 L^2\right) x_{t_m} + e_{yt_m}, \tag{10}$$

where y_{t_m}, x_{t_m}, e_{yt_m} and t_m are defined as in the previous example.

We consider point-in-time sampling, and therefore $\omega(L) = 1$. Next, we need to multiply both sides of equation (9) by $\omega(L)$ and find a polynomial $\beta(L)$ such that the product $h(L) = \beta(L)\left(1 - a_1 L - a_2 L^2\right)$ only contains powers of L^3. In can be easily shown that $\beta(L)$ exists and it is equal to

$$\left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right).$$

The resulting mixed frequency model for the low-frequency variable is:

$$\begin{aligned} \left(1 - \left(a_1^3 + 3a_2 a_1\right) L^3 - a_2^3 L^6\right) y_{t_m} \quad = \quad & \left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right)\left(b_1 L + b_2 L^2\right) x_{t_m} + \\ & \left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right) e_{yt_m}, \end{aligned} \tag{11}$$

with $t_m = 3, 6, 9, ...$ Hence, also in this case there is an MA component in the mixed frequency model for y. Its order coincides with the highest multiple of 3 non zero lag in the autocovariance function of $\left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right) e_{yt_m}$, and it is bounded above by the AR order of the model for y_{t_m} minus one, which is 1 in this example. Following the same line of reasoning as in the previous example, the MA component is of order 1.

3 UMIDAS-ARMA and MIDAS-ARMA: forecasting specifications and estimation

We describe now in more detail the model specifications we consider for forecasting, and the estimation details. We first recall the main features of the standard MIDAS regression, introduced by Ghysels et al. (2006), and its unrestricted version, as in Foroni et al. (2015). Then, we discuss their extensions to allow for an MA component and we discuss the estimation of the models.

The starting point for our MF models is equation (4). In order to simplify the notation, we assume $\omega(L) = 1$ and one explanatory variable x_{t_m}[5]. Further, we allow for incorporating leads of the high frequency variable in the projections, which captures asynchronous releases.

The equation we are going to estimate to generate an h_m-step ahead forecast is the following:

$$y_{t_m} = \tilde{c}(L^m) y_{t_m - h_m} + \delta(L) x_{t_m - h_m + w} + \epsilon_{t_m}, \tag{12}$$

where $\tilde{c}(L^m)$ is a modified lag structure of equation (4) to obtain a direct forecast and w is the number of months with which x is leading y.

If ϵ_{t_m} is serially uncorrelated, equation (12) represents the UMIDAS-AR model. Given that the model is linear, the UMIDAS-AR regression can be estimated by simple OLS. Empirically, the lag length of the high frequency variable x is often selected by means of an information criterion, such as the BIC.

Adding an MA component to the UMIDAS-AR yields the UMIDAS-ARMA model:

$$y_{t_m} = \tilde{c}(L^m) y_{t_m - h_m} + \delta(L) x_{t_m - h_m + w} + u_{t_m} + q(L^m) u_{t_m - h_m}, \tag{13}$$

where u_{t_m} is a (weak) white noise with $\mathrm{E}(u_{t_m}) = 0$ and $\mathrm{E}(u_{t_m} u'_{t_m}) = \sigma_u^2 < \infty$, and all the remaining terms stay the same as in equation (12). Given that MIDAS models are direct forecasting tools, we decided to follow a direct approach also when modelling the MA component. Note that if equation (4) coincides with the DGP, then the errors in equation (12) will be serially correlated. This provides an additional justification for the use of MA errors.

[5] This is an innocuous simplification, as with a generic aggregation scheme $\omega(L) \neq 1$ we could just work with the redefined variable $\tilde{y}_{t_m} = \omega(L) y_{t_m}$.

OLS estimation of the UMIDAS-ARMA model is no longer possible, because of the MA component in the residuals. We then estimate the model as in the standard ARMA literature, by maximum likelihood or, as we will actually do to be coherent with the MIDAS literature, by non-linear least squares (NLS).

The MIDAS-AR specification is a restricted version of the UMIDAS-AR. The MIDAS-AR model as in Ghysels et al. (2006), specified for forecasting h_m periods ahead, can be written as follows:

$$y_{t_m} = \tilde{c}(L^m) y_{t_m - h_m} + \beta B(L, \theta) x_{t_m - h_m + w} + \epsilon_{t_m}, \tag{14}$$

where

$$\begin{aligned} B(L, \theta) &= \sum_{j=0}^{K} b(j, \theta) L^j, \\ b(j, \theta) &= \frac{exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=0}^{K} exp(\theta_1 j + \theta_2 j^2)}, \end{aligned}$$

and K is the maximum number of lags included of the explanatory variable.

As it is clear by comparing equation (12) and equation (14), the MIDAS model is nested into the UMIDAS model.

The MIDAS-AR model in equation (14) is estimated by NLS. Given that it is h_m-dependent, as in the UMIDAS case it has to be re-estimated for each forecast horizon.

Exactly as for the UMIDAS, we extend the MIDAS-AR in equation (14) to incorporate an MA component:

$$y_{t_m} = \tilde{c}(L^m) y_{t_m - h_m} + \beta B(L, \theta) x_{t_m - h_m + w} + u_{t_m} + q(L^m) u_{t_m - h_m}, \tag{15}$$

where the error term is defined as in (13). Given the nonlinearity of the model, we estimate its parameters by NLS. Appendix A provides additional details on the NLS estimation procedures.

To conclude, it is worth briefly comparing the use for forecasting of UMIDAS-ARMA versus the Kalman filter. The latter is clearly optimal in the presence of mixed frequency data and linear models. However, UMIDAS-ARMA is equivalent if it is theoretically derived from a known high frequency linear dynamic model, as UMIDAS-ARMA coincides with the mixed frequency data generating process. The ranking of the two approaches is unclear if the high frequency model is mis-specified. Moreover, the Kalman filter can incur into computational problems when the frequency mismatch is large. Something similar happens to UMIDAS-ARMA, due to parameter proliferation, and in this case the parsimony of MIDAS-ARMA can be particularly helpful. Bai et al. (2013) propose a more detailed comparison of the Kalman filter and the MIDAS approach.

4 Monte Carlo evaluation

We now assess the forecasting relevance of including an MA component in MIDAS and UMIDAS models by means of simulation experiments. We use two designs, closely related to the two analytical examples described in Section 2.2. We present first the Monte Carlo designs and then the results. Finally, we summarize the results obtained from other variations of the Monte Carlo design, aimed at making the evidence more robust. The detailed robustness checks are presented in Appendix B.

4.1 Monte Carlo design

In the first design, the DGP is the HF ARDL(1,1):

$$y_{t_m} = \rho y_{t_m-1} + \delta_l x_{t_m-1} + e_{y,t_m}, \tag{16}$$

where y_{t_m} is unobservable at HF, but available at LF, while x_{t_m} is the HF variable, t_m is the HF time index, the aggregation frequency is $m = 3$ (as in the case of quarterly and monthly frequencies), and t is the LF time index, with $t = 3t_m$. We assume that $\omega(L) = 1 + L + L^2$, corresponding to average sampling.

The shocks e_{y,t_m} are independent and sampled from a normal distribution. The variance of x and that of the error e_{y,t_m} are set in such a way that the R^2 in the model for y is equal to 0.9 in each simulation. We consider different combinations of ρ and δ_l, representing different degrees of persistence and correlation between the HF and the LF variables. In detail, we evaluate the following parameter sets:

$$(\rho, \delta_l) = \{(0.1, 0.1), (0.5, 0.1), (0.9, 1), (0.94, 1)\}, \tag{17}$$

and we would expect theoretically the relative importance of the MA component to increase with the value of ρ.

Finally, x_{t_m} is generated as an AR(1) with coefficient ρ. We highlight that there is no need to play on the persistence of x_{t_m} to obtain high or low R^2, but we can obtain the same results by changing the variance of the $e_{x_{t_m}}$, and consequently of x_{t_m}.

In the second design, the DGP is the HF ARDL(2,2):

$$y_{t_m} = \rho_1 y_{t_m-1} + \rho_2 y_{t_m-2} + \delta_{l1} x_{t_m-1} + \delta_{l2} x_{t_m-2} + e_{y,t_m}. \tag{18}$$

We still assume $m = 3$ but now $\omega(L) = 1$, so that the LF variable is skip-sampled every $m = 3$ observations.

In this second DGP, we consider the following parameter combinations:

$$(\rho_1, \rho_2, \delta_{l1}, \delta_{l2}) = \{(0.05, 0.1, 0.5, 1), (0.125, 0.5, 0.125, 0.5), (0.25, 0.5, 0.5, 1)\}. \tag{19}$$

All the other design features are as in the first DGP.

We focus on typical sample sizes for the estimation sample, with $T = 50, 100$. The size of the evaluation sample is set to 50, and the estimation sample is recursively expanded as we progress in the recursive forecasting exercise. The number of replications is 500.

The competing forecasting models are the following:

1. A MIDAS-AR model, with 12 lags in the exogenous HF variable and 1 lag in the AR component;
2. A MIDAS-ARMA model, as in the previous point but with the addition of an MA component;
3. A MIDAS-ARMA model, with only 3 lags in the exogenous HF variable and 1 AR lag;
4. A UMIDAS-AR model, with lag length selected according to the BIC criterion, where the maximum lag length is set equal to 12;
5. A UMIDAS-ARMA model, as in the previous point, with the addition of an MA component;

6. A UMIDAS-ARMA, fixing at 3 the number of lags of the HF exogenous variable.

In all ARMA models there is an MA(1) component, in line with the theoretical results, but an higher order can be allowed. Further, a higher number of lags in the autoregressive component can also be included.[6]

We evaluate the competing one-step ahead forecasts on the basis of their associated mean square prediction error (MSE), assuming that information on the first two months of the quarter is available (as it is common in nowcasting exercises).

4.2 Results

In Tables 1 to 4 we report the mean relative MSE across simulations, and numbers smaller than one indicate that the model is better than the benchmark (model 1, the MIDAS-AR). We also report the $10^{th}, 25^{th}, 50^{th}, 75^{th}$ and 90^{th} percentiles, to provide a measure of the dispersion in the results.

Tables 1 and 2 present the results for the first DGP (the ARDL(1,1) with average sampling), using $T = 100$ in Table 1 and $T = 50$ in Table 2. The corresponding Tables 3 and 4 are based on the second DGP (the ARDL(2,2) with point-in-time sampling).

A few key findings emerge. First, adding an MA component to the MIDAS model generally helps. The gains are not very large but they are visible at all percentiles, with a few exceptions for the second DGP. The gains are larger either with substantial persistence ($\rho = 0.9$ or $\rho = 0.94$ in the first DGP and $\rho_1 = 0.25$, $\rho_2 = 0.5$ in the second DGP) or with low persistence in the first DGP ($\rho = 0.1$), but in the latter case the result is mainly due to a deterioration in the absolute performance of the standard MIDAS model. The more parsimonious specification with 3 lags only of the HF variable is generally better, except when $\rho = 0.5$.

Second, adding an MA component to the UMIDAS model is also generally helpful, though the gains remain small.

Third, in general the MIDAS-ARMA specifications are slightly better than the UMIDAS-ARMA specifications, though the differences are minor. This pattern is in contrast with the

[6] We computed results with 3 lags in the MIDAS-AR and UMIDAS-AR, to check whether inserting more lags in the AR part serves as a proxy for the MA component. However, the relevance of the MA component is confirmed. Results are available upon request.

findings in Foroni et al. (2015), and suggests that adding the MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure.

Finally, results are consistent across sample sizes, and the models do not seem sensitive to short sample sizes.

4.3 Robustness checks

We run a battery of robustness checks on our Monte Carlo experiments, with the scope to strengthen the evidence emerging from our main results. For the sake of conciseness, here we report a summary of the checks and the main conclusions. The detailed results are presented in Appendix B.

As a first robustness exercise, we modify the set up by reducing the explanatory power of the x variable in such a way that the total R^2 in our DGP for y is equal to 0.3, 0.5 or 0.7. The MSE are obviously bigger in absolute value, but the MA component is still generally improving the forecasting performance of the (U)MIDAS models. Second, we compute the results for a longer estimation sample, with $T_q = 200$, which corresponds to 50 years of quarterly observations, in line for example with empirical applications using long macroeconomic time series for the US. The results show broadly the same features as when $T = 100$.

Finally, we consider not only nowcasting, but also 2- and 4-quarter ahead forecasts. Results are overall robust at the 2-quarter ahead horizon, while at the 4-quarter ahead results are much weaker. Theoretically, the relevance of the MA component should decline at longer horizons. Indeed, the Monte Carlo results show smaller gains from the use of an MA component for 2-quarter ahead forecasting, and no gains at 4-quarter horizon.

5 Empirical applications

Applications of MIDAS regressions to forecasting macroeconomic variables is fairly standard by now, starting from the paper by Clements and Galvo (2008) [7]. Our focus is on the inclusion of an MA component. Therefore, in this section, we look at the performance of our MA augmented

[7] Other studies which consider MIDAS applications are, among many others, Schumacher (2016) and Kim and Swanson (2017). For a comprehensive review, see Foroni and Marcellino (2013).

mixed frequency models in forecasting exercises with actual data. The analysis focuses on forecasting quarterly U.S. variables.

In particular, we consider three relevant quarterly U.S. macroeconomic variables: real GDP growth, real private non residential fixed investment (PNFI) growth and GDP deflator inflation. As monthly explanatory variables, we consider industrial production and employment for the real GDP growth and the PNFI growth, while we consider CPI inflation and personal consumption inflation for the GDP deflator. A complete description of data sources and transformations is available in Table 5.

The total sample spans over 50 years of data, from the first quarter of 1960 to the end of 2015. The forecasts are computed in pseudo real time, with progressively expanding samples. The evaluation period goes from 1980Q1 to the end of the sample, covering roughly 35 years. At each point in time, we compute forecasts from 1- up to 4-quarter ahead. The forecasting target is the annualized growth rate. Although the information contained in the monthly variables updates every month, we focus on the case in which the first two months of the quarter are already available.

We consider the models (1) to (7) as described in Section 4.1, plus a simple low frequency AR(1) model as a further benchmark for the usefulness of the mixed-frequency data[8]. In particular, we consider the direct forecast resulting from the model:

$$y_t = c + \rho y_{t-h} + e_t. \tag{20}$$

We evaluate the forecasts both in terms of mean squared errors (MSE) and in terms of mean absolute errors (MAE). We then compare the forecasting performance relative to a standard MIDAS model with an autoregressive component and 12 lags of the explanatory variable (as the model (1) in Section 4).

In Tables 6 to 8 we report the results for, respectively, the real GDP growth, the real PNFI growth and the GDP deflator inflation rate. Each table is organized in the same way: it reports the value of MSE and MAE for each model, the ratio of those criteria for each model relative to the MIDAS-AR, our benchmark model, and the p-value of the Diebold-Mariano test, to check

[8] Despite for a variable like inflation more accurate benchmarks could be chosen, we consider an AR process, which is nested in all the models under comparison, and we keep it the same for all the variables under analysis to test the usefulness of mixed frequency data.

the statistical significance of the differences in forecast measures with respect to the benchmark (see Diebold and Mariano (1995)).

The tables are broadly supportive of the inclusion of the MA component in the mixed frequency models, as the MSE and MAE ratios are often smaller than one for the MIDAS-ARMA and UMIDAS-ARMA models when compared with their versions without MA.[9] More in detail: for forecasting GDP growth, adding the MA component does not provide substantial improvements with respect to standard mixed frequency models for $h = 1$, with industrial production being the best indicator. When $h = 2$, employment becomes better than industrial production, and adding an MA term matters, with gains of 8% for the MIDAS-ARMA model. A similar results holds for $h = 3$, with gains increasing to 20%. Four-quarter ahead, industrial production returns best, and MIDAS-ARMA leads to a decrease of 10% in the MSE. For PNFI growth, MIDAS-ARMA is best at all horizons, with employment preferred to industrial production except for $h = 1$. The gains are small for $h = 1, 2, 3$, in the range 1%-8%, but increase to 30% for $h = 4$. For GDP deflator, PCE inflation is systematically better than CPI, and MIDAS-ARMA yields gains for $h = 1$ and 2 of, respectively, 15% and 10%. It is also worth mentioning that MSE and MAE lead to the same rankings, and that the gains from adding the MA parts are generally statistically significant. Finally, the models perform well with respect to the AR benchmark. Confirming the widespread evidence in the literature, the mixed frequency models perform the best at short horizons. However, we get satisfactory results also up to $h = 4$.

The empirical gains resulting from the use of the MA term in the mixed frequency models are somewhat larger than those in the Monte Carlo experiments. A possible reason is model mis-specification, and in particular some form of parameter instability. In that case, the use of an MA component can be helpful to put the forecasts "back on track", see for example Clements and Hendry (1998) for details.

We now decompose the MSE in bias and variance, as:

$$MSE = \underbrace{(E(e))^2}_{\text{Bias}} + \underbrace{Var(e)}_{\text{Variance}} \tag{21}$$

with $e = y - \hat{y}$. We find that the MA part helps especially in reducing the bias, suggesting that the MA part is important to well approximate the conditional mean of y (the optimal forecast under

[9]The models which include an MA component are indicated in bold in the tables, while the lowest MSE and MAE values are underlined.

the quadratic loss). When the models with the MA component are not performing well, this is due especially to the variance term, instead. Detailed results on the bias/variance decomposition are presented in Table 9. In particular, in the table we report the ratio of the bias and of the variance of each model relative to the bias and variance of the MIDAS-AR model, which is taken as a benchmark.

The MSE and MAE are computed over the entire evaluation sample. To check whether the performance of our models remains good across the entire sample, in Figure 1 we report the one-quarter ahead forecasts of the benchmark MIDAS-AR model and of one of the MA augmented models, together with the realized series. In Figure 2, instead, we report the 4-quarter ahead forecasts.[10] It turns out that, on average, MIDAS models perform well throughout the sample, both with and without an MA component.

Tables 10 and 11 report the full-sample estimates of MA coefficients in MIDAS-ARMA and UMIDAS-ARMA models that have been used in the forecasting exercise. The corresponding t-statistics are shown in parentheses. We observe that many MA coefficients are significant. For instance, MA(1) coefficient in MIDAS-ARMA-3lags model on GDP growth equation with employment growth is precisely estimated at horizons $h = 1, 2, 3$. This MIDAS-ARMA model was also the best in out-of-sample forecasting exercise, see table 6. In case of PNFI, when forecasting one quarter ahead with industrial production as high frequency predictor, MA coefficient is significant in all models. Same result holds at longer horizons with employment growth. When it comes to GDP deflator prediction, an interesting finding is that the MA(2) component is highly strong and significant for most of the horizons and models.

In Appendix C, we expand the empirical exercise along several dimensions. First, we analyse a shorter sample ending in 2007Q3, to assess the effects of the recent crisis. Second, we report results for the cases in which only one of the months of the quarters are available and when instead all three months are already available. Third, we use a real-time dataset with the different vintages.[11]

All in all, the robustness exercises confirm the evidence we find in this section. Excluding the crisis, results do not change substantially, and remain broadly supportive of the inclusion of the

[10] Figures 1 and 2 focus only on a small portion of results that we have available. The same figures for other models, other forecast horizons and other explanatory variables are available upon request.

[11] As in the Monte Carlo, we could extend the number of lags in the AR component. However, we keep our benchmark specification of one AR lag, consistent with most of the empirical studies involving MIDAS. Results with 4 lags are available upon request.

MA component in the mixed-frequency models. In most of the cases, the best performing model up to 2007 remains the best in the full sample. The magnitude of improvements is also very comparable. For the other nowcasting horizons (that is, including only one month of monthly information or, instead, all the three months), results confirm that in most of the cases the best performance is obtained when the MA component is added. Finally, when using real-time data for the macroeconomic series considered, we see the same patterns as with pseudo real-time data.

6 Conclusions

In this paper, we start from the observation that temporal aggregation in general introduces a moving average component in the aggregated model. We show that a similar feature also emerges when not all but only a few variables are aggregated, which generates a mixed frequency model. Hence, an MA component should be added to mixed frequency models, while this is generally neglected in the literature.

We illustrate in a set of Monte Carlo simulations that indeed adding an MA component to MIDAS and UMIDAS models further improves their nowcasting and forecasting abilities, though in general the gains are limited and particularly evident in the presence of persistence. Interestingly, the relative performance of MIDAS versus UMIDAS further improves when adding an MA component, with the latter attenuating the effects of imposing a particular polynomial structure in the dynamic response of the low frequency to the high frequency variable.

A similar pattern emerges in an empirical exercise based on actual data. Specifically, we find that the inclusion of an MA component can substantially improve the forecasting performance of quarterly macroeconomic U.S. variables, as GDP growth, PNFI growth and GDP deflator inflation. MIDAS-ARMA models perform particularly well, suggesting that the addition of an MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure. Finally, full sample estimates of MA coefficients are significant and important in most of MIDAS-ARMA and UMIDAS-ARMA specifications.

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Table 1: Monte Carlo simulations results: MSE(model) relative to MSE(MIDAS) - DGP: ARDL(1,1) with average sampling, $T = 100$.

	PANEL (A): $\rho = 0.94, \delta_l = 1$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.974	0.986	0.981	0.968	0.970	0.967
MIDAS-ARMA-3 (3)	0.966	0.981	0.969	0.962	0.959	0.966
UMIDAS-AR (4)	0.997	0.997	0.986	1.005	0.994	0.990
UMIDAS-ARMA (5)	0.969	0.983	0.974	0.970	0.961	0.973
UMIDAS-ARMA-3 (6)	0.971	0.977	0.974	0.971	0.964	0.975
	PANEL (B): $\rho = 0.9, \delta_l = 1$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.976	0.989	0.984	0.979	0.973	0.976
MIDAS-ARMA-3 (3)	0.975	0.983	0.988	0.969	0.978	0.971
UMIDAS-AR (4)	1.030	1.024	1.023	1.029	1.038	1.040
UMIDAS-ARMA (5)	1.019	1.018	1.023	1.012	1.024	1.028
UMIDAS-ARMA-3 (6)	0.976	0.979	0.984	0.977	0.977	0.981
	PANEL (C): $\rho = 0.5, \delta_l = 0.1$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.986	0.990	0.994	0.984	0.983	0.978
MIDAS-ARMA-3 (3)	1.184	1.197	1.178	1.174	1.202	1.176
UMIDAS-AR (4)	1.005	1.000	1.003	1.006	1.013	0.995
UMIDAS-ARMA (5)	1.000	1.005	0.992	0.998	0.993	0.992
UMIDAS-ARMA-3 (6)	1.182	1.212	1.185	1.175	1.198	1.179
	PANEL (D): $\rho = 0.1, \delta_l = 0.1$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.981	0.985	0.989	0.983	0.980	0.972
MIDAS-ARMA-3 (3)	0.833	0.848	0.846	0.834	0.827	0.828
UMIDAS-AR (4)	0.825	0.837	0.834	0.823	0.824	0.819
UMIDAS-ARMA (5)	0.832	0.841	0.844	0.833	0.831	0.836
UMIDAS-ARMA-3 (6)	0.833	0.846	0.846	0.834	0.829	0.829

Note: The four panels report the results for four different DGPs for 1-quarter ahead horizon (with the information of the first two months of the quarter available). The numbers (2) to (6) refer to the corresponding models described in Section 4. The results reported are the average, median and the $10^{th}, 25^{th}, 75^{th}, 90^{th}$ percentiles of the MSE of the indicated model relative to the average, median and the $10^{th}, 25^{th}, 75^{th}, 90^{th}$ percentiles of the MSE of the benchmark MIDAS (model (1) in Section 4) computed over 500 replications.

Table 2: Monte Carlo simulations results: MSE(model) relative to MSE(MIDAS) - DGP: ARDL(1,1) with average sampling, $T = 50$

	PANEL (A): $\rho = 0.94, \delta_l = 1$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.985	0.975	0.996	0.983	0.990	0.968
MIDAS-ARMA-3 (3)	0.957	0.949	0.982	0.947	0.957	0.944
UMIDAS-AR (4)	0.982	0.984	0.998	0.968	0.986	0.979
UMIDAS-ARMA (5)	0.957	0.950	0.984	0.954	0.965	0.939
UMIDAS-ARMA-3 (6)	0.968	0.950	1.003	0.962	0.975	0.955
	PANEL (B): $\rho = 0.9, \delta_l = 1$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.997	1.001	1.012	0.994	0.986	0.982
MIDAS-ARMA-3 (3)	0.973	0.978	1.006	0.964	0.961	0.977
UMIDAS-AR (4)	1.033	1.074	1.041	1.025	1.034	1.013
UMIDAS-ARMA (5)	1.020	1.041	1.040	1.018	1.019	1.016
UMIDAS-ARMA-3 (6)	0.981	0.982	1.023	0.968	0.971	0.983
	PANEL (C): $\rho = 0.5, \delta_l = 0.1$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.013	1.007	1.010	1.022	0.999	1.014
MIDAS-ARMA-3 (3)	1.188	1.182	1.172	1.179	1.168	1.249
UMIDAS-AR (4)	1.038	1.056	1.054	1.026	1.046	1.064
UMIDAS-ARMA (5)	1.061	1.089	1.059	1.049	1.049	1.062
UMIDAS-ARMA-3 (6)	1.197	1.186	1.181	1.181	1.173	1.241
	PANEL (D): $\rho = 0.1, \delta_l = 0.1$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.984	0.987	0.989	0.983	0.989	0.973
MIDAS-ARMA-3 (3)	0.824	0.809	0.807	0.825	0.830	0.846
UMIDAS-AR (4)	0.810	0.791	0.814	0.819	0.814	0.820
UMIDAS-ARMA (5)	0.834	0.824	0.826	0.831	0.834	0.853
UMIDAS-ARMA-3 (6)	0.830	0.826	0.816	0.827	0.832	0.859

Note: See Table 2.

Table 3: Monte Carlo simulations results: MSE(model) relative to MSE(MIDAS) - DGP: ARDL(2,2) with point-in-time sampling, $T = 100$

	mean	10 prct	25 prct	median	75 prct	90 prct
			PANEL (A): $\rho_1 = 0.05, \rho_2 = 0.1, \delta_{l1} = 0.5, \delta_{l2} = 1$			
MIDAS-ARMA-12 (2)	1.007	1.010	1.003	1.007	1.010	1.025
MIDAS-ARMA-3 (3)	1.006	1.006	0.997	1.003	1.014	1.018
UMIDAS-AR (4)	1.014	1.007	1.016	1.005	1.006	1.030
UMIDAS-ARMA (5)	1.015	1.000	1.014	1.007	1.014	1.026
UMIDAS-ARMA-3 (6)	1.007	0.998	1.004	1.006	1.016	1.023
			PANEL (B): $\rho_1 = 0.125, \rho_2 = 0.5, \delta_{l1} = 0.125, \delta_{l2} = 0.5$			
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.956	0.955	0.960	0.963	0.949	0.959
MIDAS-ARMA-3 (3)	0.940	0.932	0.950	0.950	0.931	0.943
UMIDAS-AR (4)	0.938	0.921	0.938	0.945	0.929	0.946
UMIDAS-ARMA (5)	0.921	0.927	0.922	0.926	0.908	0.939
UMIDAS-ARMA-3 (6)	0.943	0.921	0.950	0.947	0.932	0.948
			PANEL (C): $\rho_1 = 0.25, \rho_2 = 0.5, \delta_{l1} = 0.5, \delta_{l2} = 1$			
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.984	0.968	0.981	0.985	0.991	0.998
MIDAS-ARMA-3 (3)	0.980	0.981	0.982	0.968	0.981	0.999
UMIDAS-AR (4)	1.021	1.032	1.020	1.006	1.032	1.036
UMIDAS-ARMA (5)	0.992	0.987	0.986	0.988	1.001	1.004
UMIDAS-ARMA-3 (6)	0.983	0.978	0.979	0.980	0.985	0.998

Note: The four panels report the results for three different DGPs. The numbers (2) to (6) refer to the corresponding models described in Section 4. The results reported are the average, median and the $10^{th}, 25^{th}, 75^{th}, 90^{th}$ percentiles of the MSE of the indicated model relative to the average, median and the $10^{th}, 25^{th}, 75^{th}, 90^{th}$ percentiles of the MSE of the benchmark MIDAS (model (1) in Section 4) computed over 500 replications.

Table 4: Monte Carlo simulations results: MSE(model) relative to MSE(MIDAS) - DGP: ARDL(2,2) with point-in-time sampling, $T = 50$

	mean	10 prct	25 prct	median	75 prct	90 prct
			PANEL (A): $\rho_1 = 0.05, \rho_2 = 0.1, \delta_{l1} = 0.5, \delta_{l2} = 1$			
MIDAS-ARMA-12 (2)	1.020	1.003	1.024	1.021	1.009	1.017
MIDAS-ARMA-3 (3)	1.003	0.990	1.015	1.014	0.986	0.994
UMIDAS-AR (4)	1.006	0.982	1.036	1.019	1.011	0.988
UMIDAS-ARMA (5)	1.018	0.955	1.033	1.037	1.023	1.030
UMIDAS-ARMA-3 (6)	1.018	1.000	1.024	1.028	1.021	1.010
			PANEL (B): $\rho_1 = 0.125, \rho_2 = 0.5, \delta_{l1} = 0.125, \delta_{l2} = 0.5$			
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.017	0.980	1.006	1.004	1.019	1.042
MIDAS-ARMA-3 (3)	0.967	0.934	0.970	0.995	0.953	0.991
UMIDAS-AR (4)	0.971	0.973	0.979	0.979	0.961	0.997
UMIDAS-ARMA (5)	0.983	0.983	0.977	1.000	0.958	0.980
UMIDAS-ARMA-3 (6)	1.009	0.970	1.002	1.016	1.000	1.023
			PANEL (C): $\rho_1 = 0.25, \rho_2 = 0.5, \delta_{l1} = 0.5, \delta_{l2} = 1$			
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.016	1.003	0.991	1.005	1.012	1.010
MIDAS-ARMA-3 (3)	0.990	0.993	0.965	0.984	0.988	0.979
UMIDAS-AR (4)	1.046	1.024	1.016	1.047	1.059	1.039
UMIDAS-ARMA (5)	1.041	1.051	1.035	1.018	1.045	1.038
UMIDAS-ARMA-3 (6)	1.016	1.008	1.014	0.991	1.024	1.018

Note: see Table 3.

Table 5: Data description

Series	Source	Source Code	Transformation	Frequency
US data				
GDP Deflator	FRED	GDPDEF	Log-difference	Quarterly
Real GDP	FRED	GDP	Log-difference	Quarterly
Private Nonresidential Fixed Investment	FRED	PNFI	Level	Quarterly
Nonresidential (implicit price deflator)	FRED	A008RD3Q086SBEA	Level	Quarterly
Real Private Nonresidential Fixed Investment		PNFI / A008RD3Q086SBEA	Log-difference	Quarterly
Consumer Price Index (CPI)	FRED	CPIAUCSL	Log-difference	Monthly
Personal Consumption Expenditures: Price Index (PCE)	FRED	PCEPI	Log-difference	Monthly
Employment	FRED	PAYEMS	Log-difference	Monthly
Industrial Production	FRED	INDPRO	Log-difference	Monthly

Table 6: Forecasting U.S. GDP growth

	Explanatory variable: Industrial production growth						**Explanatory variable: Employment growth**					
	h=1						h=1					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	4.06	1.00	NaN	1.58	1.00	NaN	4.84	1.00	NaN	1.73	1.00	NaN
MIDAS-ARMA-12lags	4.05	1.00	0.41	1.59	1.01	0.13	4.73	0.98	0.33	1.72	0.99	0.37
MIDAS-ARMA-3	4.27	1.05	0.07	1.60	1.01	0.22	4.76	0.98	0.39	1.72	0.99	0.42
UMIDAS-biclags	4.21	1.04	0.15	1.58	1.00	0.41	4.70	0.97	0.31	1.73	1.00	0.49
UMIDAS-ARMA-biclags	4.18	1.03	0.19	1.59	1.01	0.26	4.37	0.90	0.05	1.67	0.96	0.17
UMIDAS-ARMA-3	4.27	1.05	0.07	1.60	1.01	0.22	4.76	0.98	0.39	1.72	0.99	0.42
AR	7.60	1.87	0.01	1.97	1.25	0.01	7.60	1.57	0.02	1.97	1.14	0.05
	h=2						h=2					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	6.68	1.00	NaN	1.86	1.00	NaN	6.07	1.00	NaN	1.79	1.00	NaN
MIDAS-ARMA-12lags	6.35	0.95	0.03	1.84	0.99	0.29	5.58	0.92	0.03	1.76	0.98	0.22
MIDAS-ARMA-3	6.59	0.99	0.31	1.90	1.02	0.16	5.78	0.95	0.12	1.78	1.00	0.41
UMIDAS-biclags	7.05	1.05	0.00	1.94	1.04	0.00	6.07	1.00	0.06	1.79	1.00	0.05
UMIDAS-ARMA-biclags	6.89	1.03	0.12	1.92	1.03	0.02	5.81	0.96	0.15	1.78	1.00	0.45
UMIDAS-ARMA-3	7.04	1.05	0.09	1.89	1.01	0.19	6.10	1.00	0.46	1.82	1.01	0.20
AR	7.77	1.16	0.10	1.98	1.06	0.06	7.77	1.28	0.02	1.98	1.10	0.01
	h=3						h=3					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	8.14	1.00	NaN	2.01	1.00	NaN	8.85	1.00	NaN	2.10	1.00	NaN
MIDAS-ARMA-12lags	8.22	1.01	0.34	2.04	1.01	0.28	9.39	1.06	0.21	2.31	1.10	0.03
MIDAS-ARMA-3	7.44	0.91	0.00	1.89	0.94	0.00	7.06	0.80	0.00	1.90	0.90	0.00
UMIDAS-biclags	8.12	1.00	0.45	1.99	0.99	0.19	7.62	0.86	0.00	1.94	0.92	0.00
UMIDAS-ARMA-biclags	12.00	1.47	0.01	2.46	1.22	0.00	15.23	1.72	0.00	3.11	1.48	0.00
UMIDAS-ARMA-3	8.24	1.01	0.42	1.97	0.98	0.16	7.93	0.90	0.07	1.99	0.95	0.07
AR	8.77	1.08	0.11	2.05	1.02	0.19	8.77	0.99	0.43	2.05	0.98	0.19
	h=4						h=4					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	9.14	1.00	NaN	2.11	1.00	NaN	11.11	1.00	NaN	2.31	1.00	NaN
MIDAS-ARMA-12lags	8.56	0.94	0.09	2.07	0.98	0.24	11.20	1.01	0.46	2.43	1.05	0.13
MIDAS-ARMA-3	8.27	0.90	0.03	2.02	0.96	0.05	10.14	0.91	0.02	2.16	0.94	0.01
UMIDAS-biclags	8.77	0.96	0.19	2.05	0.97	0.10	10.37	0.93	0.05	2.17	0.94	0.01
UMIDAS-ARMA-biclags	8.91	0.98	0.40	2.10	0.99	0.45	11.13	1.00	0.49	2.38	1.03	0.30
UMIDAS-ARMA-3	10.01	1.09	0.30	2.08	0.99	0.40	9.52	0.86	0.00	2.10	0.91	0.00
AR	8.69	0.95	0.11	2.05	0.97	0.10	8.69	0.78	0.00	2.05	0.89	0.00

Note: The table reports the results on the forecasting performance of the different models. In the columns "value" we report the MSE and the MAE respectively. In the columns "ratio" we report the MSE and MAE of each model relative to the MIDAS-AR benchmark. In the columns "DM" we report the p-value of the Diebold-Mariano test. The forecasts are evaluated over the sample 1980Q1-2015Q4. The lowest values for each variable are underlined.

Table 7: Forecasting U.S. Real Private Nonresidential Fixed Investment growth

	Explanatory variable: Industrial production growth						**Explanatory variable: Employment growth**					
	h=1						h=1					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	30.70	1.00	NaN	4.40	1.00	NaN	33.01	1.00	NaN	4.56	1.00	NaN
MIDAS-ARMA-12lags	29.32	0.96	0.06	4.25	0.97	0.02	33.38	1.01	0.18	4.57	1.00	0.33
MIDAS-ARMA-3	28.13	0.92	0.01	4.19	0.95	0.01	33.15	1.00	0.40	4.57	1.00	0.37
UMIDAS-biclags	31.91	1.04	0.22	4.45	1.01	0.24	33.31	1.01	0.18	4.58	1.01	0.12
UMIDAS-ARMA-biclags	29.27	0.95	0.13	4.27	0.97	0.06	34.11	1.03	0.20	4.64	1.02	0.11
UMIDAS-ARMA-3	28.15	0.92	0.01	4.19	0.95	0.01	32.98	1.00	0.48	4.57	1.00	0.37
AR	44.91	1.46	0.01	5.11	1.16	0.01	44.91	1.36	0.02	5.11	1.12	0.05
	h=2						h=2					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	38.90	1.00	NaN	4.72	1.00	NaN	36.33	1.00	NaN	4.71	1.00	NaN
MIDAS-ARMA-12	38.59	0.99	0.34	4.74	1.00	0.37	36.13	0.99	0.40	4.74	1.01	0.32
MIDAS-ARMA-3	41.46	1.07	0.07	4.86	1.03	0.11	36.70	1.01	0.29	4.76	1.01	0.21
UMIDAS-biclags	41.92	1.08	0.00	4.87	1.03	0.00	37.84	1.04	0.03	4.78	1.02	0.08
UMIDAS-ARMA-biclags	44.76	1.15	0.00	5.11	1.08	0.00	38.40	1.06	0.03	4.85	1.03	0.07
UMIDAS-ARMA-3	43.93	1.13	0.03	4.92	1.04	0.06	36.69	1.01	0.36	4.80	1.02	0.11
AR	47.14	1.21	0.08	5.18	1.10	0.05	47.14	1.30	0.04	5.18	1.10	0.05
	h=3						h=3					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	43.65	1.00	NaN	5.15	1.00	NaN	43.77	1.00	NaN	5.20	1.00	NaN
MIDAS-ARMA-12	41.97	0.96	0.07	4.99	0.97	0.05	40.70	0.93	0.02	5.00	0.96	0.01
MIDAS-ARMA-3	45.90	1.05	0.13	5.21	1.01	0.25	41.31	0.94	0.05	5.06	0.97	0.06
UMIDAS-biclags	52.95	1.21	0.02	5.51	1.07	0.00	45.51	1.04	0.05	5.25	1.01	0.17
UMIDAS-ARMA-biclags	47.52	1.09	0.07	5.26	1.02	0.17	42.62	0.97	0.16	5.06	0.97	0.06
UMIDAS-ARMA-3	45.86	1.05	0.14	5.21	1.01	0.28	41.36	0.95	0.07	5.08	0.98	0.11
AR	55.91	1.28	0.03	5.60	1.09	0.04	55.91	1.28	0.04	5.60	1.08	0.08
	h=4						h=4					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	52.70	1.00	NaN	5.72	1.00	NaN	69.36	1.00	NaN	6.18	1.00	NaN
MIDAS-ARMA-12	50.68	0.96	0.21	5.48	0.96	0.04	59.35	0.86	0.02	5.78	0.94	0.01
MIDAS-ARMA-3	50.04	0.95	0.17	5.44	0.95	0.02	48.38	0.70	0.00	5.29	0.86	0.00
UMIDAS-biclags	53.47	1.01	0.41	5.59	0.98	0.16	57.78	0.83	0.01	5.73	0.93	0.01
UMIDAS-ARMA-biclags	54.95	1.04	0.26	5.60	0.98	0.18	56.52	0.81	0.03	5.70	0.92	0.05
UMIDAS-ARMA-3	52.92	1.00	0.48	5.48	0.96	0.12	53.55	0.77	0.01	5.55	0.90	0.01
AR	63.44	1.20	0.04	5.91	1.03	0.17	63.44	0.91	0.01	5.91	0.96	0.03

Note: See Table 6.

Table 8: Forecasting U.S. GDP Deflator

	Explanatory variable: CPI inflation						**Explanatory variable: PCE inflation**					
	h=1						h=1					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	0.65	1.00	NaN	0.61	1.00	NaN	0.51	1.00	NaN	0.52	1.00	NaN
MIDAS-ARMA-12	0.65	0.99	0.42	0.61	1.00	0.45	0.50	0.98	0.37	0.51	0.99	0.28
MIDAS-ARMA-3	0.61	0.94	0.09	0.59	0.97	0.17	0.43	0.85	0.07	0.49	0.95	0.20
UMIDAS-biclags	0.65	0.99	0.42	0.60	0.98	0.22	0.53	1.04	0.24	0.52	1.00	0.47
UMIDAS-ARMA-biclags	0.61	0.94	0.09	0.59	0.97	0.17	0.51	1.01	0.44	0.52	1.00	0.47
UMIDAS-ARMA-3	0.61	0.94	0.09	0.59	0.97	0.17	0.49	0.96	0.11	0.51	0.98	0.14
AR	0.79	1.20	0.11	0.68	1.11	0.07	0.79	1.55	0.00	0.68	1.32	0.00
	h=2						h=2					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	0.79	1.00	NaN	0.67	1.00	NaN	0.55	1.00	NaN	0.55	1.00	NaN
MIDAS-ARMA-12	0.68	0.86	0.08	0.64	0.95	0.09	0.56	1.01	0.38	0.55	1.01	0.36
MIDAS-ARMA-3	0.68	0.86	0.13	0.64	0.96	0.16	0.50	0.90	0.14	0.53	0.96	0.24
UMIDAS-biclags	0.74	0.94	0.31	0.66	0.99	0.38	0.55	1.00	0.49	0.56	1.01	0.44
UMIDAS-ARMA-biclags	0.70	0.89	0.15	0.67	1.00	0.46	0.52	0.95	0.30	0.55	1.00	0.47
UMIDAS-ARMA-3	0.68	0.87	0.13	0.64	0.96	0.16	0.50	0.90	0.14	0.53	0.96	0.24
AR	0.82	1.04	0.43	0.71	1.05	0.27	0.82	1.48	0.00	0.71	1.28	0.00
	h=3						h=3					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	0.80	1.00	NaN	0.69	1.00	NaN	0.57	1.00	NaN	0.57	1.00	NaN
MIDAS-ARMA-12	0.87	1.08	0.29	0.73	1.07	0.15	0.64	1.13	0.04	0.61	1.06	0.07
MIDAS-ARMA-3	0.73	0.91	0.23	0.68	0.99	0.42	0.60	1.05	0.28	0.58	1.01	0.46
UMIDAS-biclags	0.84	1.05	0.31	0.72	1.05	0.17	0.71	1.24	0.05	0.63	1.10	0.03
UMIDAS-ARMA-biclags	0.84	1.05	0.36	0.74	1.07	0.15	0.76	1.33	0.00	0.66	1.15	0.01
UMIDAS-ARMA-3	0.75	0.93	0.25	0.68	0.99	0.43	0.60	1.05	0.27	0.58	1.01	0.45
AR	0.85	1.05	0.39	0.73	1.05	0.26	0.85	1.49	0.00	0.73	1.27	0.00
	h=4						h=4					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	0.86	1.00	NaN	0.74	1.00	NaN	0.63	1.00	NaN	0.61	1.00	NaN
MIDAS-ARMA-12	1.08	1.25	0.00	0.84	1.13	0.00	0.81	1.27	0.00	0.70	1.16	0.00
MIDAS-ARMA-3	0.85	0.98	0.42	0.73	0.99	0.36	0.69	1.09	0.09	0.63	1.04	0.14
UMIDAS-biclags	1.00	1.16	0.00	0.80	1.07	0.01	0.74	1.17	0.01	0.65	1.08	0.02
UMIDAS-ARMA-biclags	1.06	1.23	0.01	0.81	1.09	0.02	0.83	1.31	0.00	0.70	1.15	0.00
UMIDAS-ARMA-3	0.98	1.13	0.06	0.77	1.03	0.20	0.73	1.15	0.02	0.65	1.07	0.05
AR	0.89	1.03	0.41	0.76	1.01	0.41	0.89	1.41	0.00	0.76	1.25	0.00

Note: See Table 6.

Table 9: Bias/Variance decomposition of MSE

		Bias				Variance			
		h=1	h=2	h=3	h=4	h=1	h=2	h=3	h=4
GDP with Industrial Production	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.72	0.68	0.59	0.89	1.00	0.97	1.04	0.94
	MIDAS-ARMA-3lags	0.85	0.97	1.01	0.90	1.06	0.99	0.91	0.90
	UMIDAS-biclags	1.09	1.04	1.08	0.87	1.04	1.06	0.99	0.97
	UMIDAS-ARMA-biclags	0.96	1.03	1.13	1.06	1.03	1.03	1.50	0.97
	UMIDAS-ARMA-3lags	0.85	1.12	1.23	0.98	1.06	1.05	1.00	1.10
	AR	3.06	1.02	1.23	1.01	1.84	1.17	1.07	0.95
GDP with Employment	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.26	0.70	2.22	1.69	0.98	0.92	0.95	0.93
	MIDAS-ARMA-3lags	0.18	1.44	0.51	0.65	0.98	0.94	0.82	0.94
	UMIDAS-biclags	5.60	1.00	0.51	0.74	0.97	1.00	0.89	0.95
	UMIDAS-ARMA-biclags	19.59	1.52	1.88	1.30	0.90	0.95	1.71	0.97
	UMIDAS-ARMA-3lags	0.18	1.40	0.56	0.61	0.98	1.00	0.93	0.88
	AR	348.15	4.11	0.87	0.56	1.51	1.24	1.00	0.81
PNFI with Industrial Production	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	5.39	0.94	0.86	0.86	0.95	0.99	0.96	0.96
	MIDAS-ARMA-3lags	10.55	1.13	1.54	1.12	0.91	1.07	1.05	0.95
	UMIDAS-biclags	0.09	1.38	1.31	1.01	1.04	1.08	1.21	1.01
	UMIDAS-ARMA-biclags	1.52	1.50	1.65	1.20	0.95	1.15	1.08	1.04
	UMIDAS-ARMA-3lags	10.59	1.54	1.54	0.92	0.91	1.13	1.05	1.01
	AR	33.09	2.46	2.77	2.10	1.45	1.20	1.27	1.19
PNFI with Employment	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.99	1.15	0.00	0.73	1.01	0.99	0.93	0.86
	MIDAS-ARMA-3lags	0.78	0.42	4.78	0.17	1.01	1.01	0.94	0.72
	UMIDAS-biclags	0.91	0.85	0.35	0.24	1.01	1.04	1.04	0.86
	UMIDAS-ARMA-biclags	0.81	0.32	7.20	0.23	1.04	1.06	0.97	0.84
	UMIDAS-ARMA-3lags	0.78	0.44	4.22	0.23	1.01	1.01	0.94	0.80
	AR	0.25	2.53	230.03	0.58	1.40	1.29	1.25	0.93
GDP Deflator with CPI inflation	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	1.08	0.79	0.83	0.95	0.98	0.88	1.15	1.35
	MIDAS-ARMA-3lags	0.87	0.61	0.51	0.72	0.94	0.92	1.03	1.07
	UMIDAS-biclags	0.63	0.61	0.63	0.98	1.02	1.01	1.18	1.21
	UMIDAS-ARMA-biclags	0.87	0.64	0.59	0.76	0.94	0.95	1.19	1.37
	UMIDAS-ARMA-3lags	0.87	0.61	0.53	0.69	0.94	0.92	1.06	1.27
	AR	0.79	0.62	0.67	0.89	1.24	1.13	1.17	1.07
GDP Deflator with PCE inflation	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	1.23	1.00	0.88	1.04	0.97	1.01	1.19	1.33
	MIDAS-ARMA-3lags	0.45	0.58	0.62	0.77	0.87	0.95	1.15	1.17
	UMIDAS-biclags	0.70	0.58	0.60	0.87	1.06	1.07	1.39	1.24
	UMIDAS-ARMA-biclags	0.96	0.70	0.49	0.81	1.01	0.99	1.51	1.43
	UMIDAS-ARMA-3lags	0.90	0.58	0.62	0.73	0.96	0.95	1.15	1.26
	AR	2.09	1.16	1.24	1.45	1.52	1.53	1.54	1.40

Note: The table the decomposition of the MSE of the different models as presented in Section 4 into bias and variance, for different forecasting horizons. The forecasts are evaluated over the sample 1980Q1-2015Q4. The numbers reported are the ratio of the bias and of the variance of each model relative to the bias and variance of the MIDAS-AR model.

Table 10: Full-sample estimated MA coefficients

Forecasting U.S. GDP growth

	Explanatory variable: Industrial Production					Explanatory variable: Employment			
	h=1	h=2		h=3	h=4	h=1	h=2	h=3	h=4
	MA(1)	MA(1)	MA(2)	MA(1)	MA(1)	MA(1)	MA(1)	MA(1)	MA(1)
MIDAS-ARMA-12lags	0.06	0.03	0.11	0.29	0.26	0.39	0.35	0.24	0.12
	(0.45)	(0.46)	(0.72)	(1.33)	(0.78)	(3.59)	(2.26)	(1.18)	(0.42)
MIDAS-ARMA-3lags	-0.10	0.05	0.17	0.31	0.32	0.41	0.44	0.37	0.26
	(-0.89)	(0.73)	(1.13)	(1.71)	(1.20)	(4.00)	(2.85)	(1.95)	(1.01)
UMIDAS-ARMA-biclags	0.01	0.06	0.19	0.11	0.25	0.05	0.42	0.25	0.28
	(0.08)	(0.84)	(1.14)	(0.43)	(1.22)	(0.18)	(2.98)	(1.47)	(1.15)
UMIDAS-ARMA-3lags	-0.10	0.05	0.17	0.31	0.16	0.41	0.45	0.33	0.27
	(-0.89)	(0.73)	(1.13)	(1.80)	(0.60)	(4.00)	(3.03)	(1.70)	(1.07)

Forecasting PNFI growth

	Explanatory variable: Industrial Production				Explanatory variable: Employment			
	h=1	h=2	h=3	h=4	h=1	h=2	h=3	h=4
	MA(1)	MA(1)	MA(1)	MA(1)	MA(1)	MA(1)	MA(1)	MA(1)
MIDAS-ARMA-12lags	-0.33	-0.14	0.13	-0.03	0.13	0.29	0.35	0.46
	(-3.82)	(-1.15)	(0.65)	(-0.13)	(1.27)	(2.22)	(2.37)	(2.67)
MIDAS-ARMA-3lags	-0.39	-0.14	0.15	-0.07	0.11	0.27	0.37	0.48
	(-5.83)	(-1.54)	(1.14)	(-0.43)	(1.23)	(2.86)	(2.65)	(2.94)
UMIDAS-ARMA-biclags	-0.31	0.05	0.17	-0.09	-0.04	0.26	0.35	0.48
	(-3.70)	(0.50)	(1.33)	(-0.51)	(-0.42)	(2.78)	(2.41)	(2.46)
UMIDAS-ARMA-3lags	-0.39	-0.12	0.15	-0.07	0.08	0.29	0.37	0.48
	(-5.83)	(-1.24)	(1.14)	(-0.43)	(0.83)	(3.00)	(2.65)	(2.98)

Note: The table reports the estimated values of MA coefficients using the full sample 1960-2015. The values in parentheses are t-statistics calculated with Newey-West standard errors to take into account the serial autocorrelation of order $h - 1$, possibly induced by direct forecasting.

Table 11: Full-sample estimated MA coefficients

Forecasting U.S. GDP deflator

Explanatory variable: Consumer Price Index

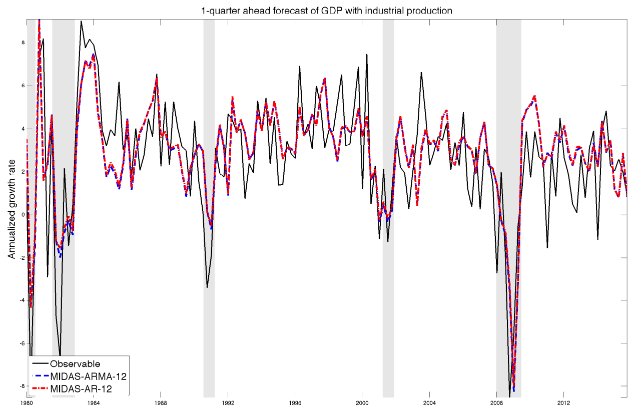
	h=1		h=2		h=3		h=4	
	MA(1)	MA(2)	MA(1)	MA(2)	MA(1)	MA(2)	MA(1)	MA(2)
MIDAS-ARMA-12lags	.-0.09	-0.31	0.18	-0.30	0.29	-0.08	0.19	0.14
	(-1.03)	(-2.43)	(1.89)	(-2.78)	(2.78)	(-0.67)	(1.70)	(1.46)
MIDAS-ARMA-3lags	-0.09	-0.36	0.20	-0.34	0.28	-0.10	0.18	0.11
	(-1.13)	(-4.65)	(2.09)	(-4.48)	(2.99)	(-0.93)	(1.76)	(1.08)
UMIDAS-ARMA-biclags	-0.09	-0.36	0.18	-0.35	0.25	-0.12	0.16	0.09
	(-1.13)	(-4.65)	(1.78)	(-4.58)	(2.62)	(-1.09)	(1.58)	(0.91)
UMIDAS-ARMA-3lags	-0.09	-0.36	0.20	-0.34	0.28	-0.10	0.18	0.11
	(-1.13)	(-4.65)	(2.06)	(-4.44)	(2.99)	(-0.93)	(1.76)	(1.08)

Explanatory variable: PCE Price Index

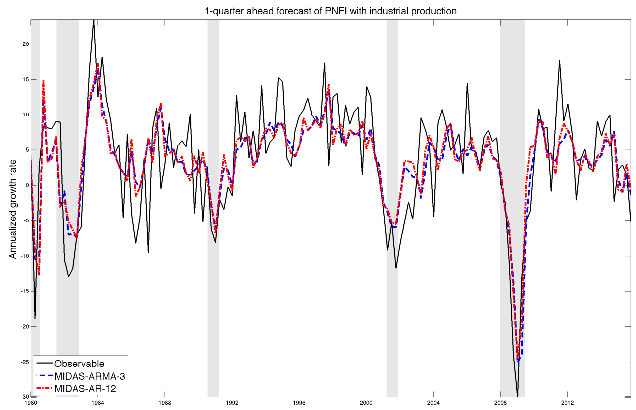
	h=1		h=2		h=3		h=4	
	MA(1)	MA(2)	MA(1)	MA(2)	MA(1)	MA(2)	MA(1)	MA(2)
MIDAS-ARMA-12lags	-0.06	0.09	0.13	-0.22	0.25	-0.03	0.19	0.19
	(-0.68)	(0.55)	(1.50)	(-1.62)	(2.48)	(-0.30)	(1.84)	(2.07)
MIDAS-ARMA-3lags	0.15	0.32	0.14	-0.30	0.24	-0.05	0.18	0.16
	(1.97)	(3.25)	(1.56)	(-3.86)	(2.95)	(-0.53)	(1.91)	(1.53)
UMIDAS-ARMA-biclags	-0.13	-0.29	0.14	-0.30	0.22	-0.06	0.17	0.16
	(-1.84)	(-3.60)	(1.56)	(-3.86)	(2.63)	(-0.63)	(1.82)	(1.51)
UMIDAS-ARMA-3lags	-0.13	-0.29	0.14	-0.30	0.24	-0.05	0.18	0.16
	(-1.84)	(-3.60)	(1.56)	(-3.86)	(2.95)	(-0.53)	(1.91)	(1.53)

Note: The table reports the estimated values of MA coefficients using the full sample 1960-2015. The values in parentheses are t-statistics calculated with Newey-West standard errors to take into account the serial autocorrelation of order $h-1$, possibly induced by direct forecasting.

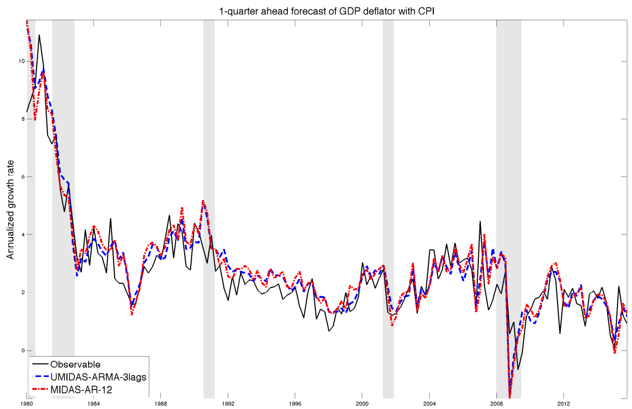
Figure 1: Out-of-sample performance: one-quarter ahead



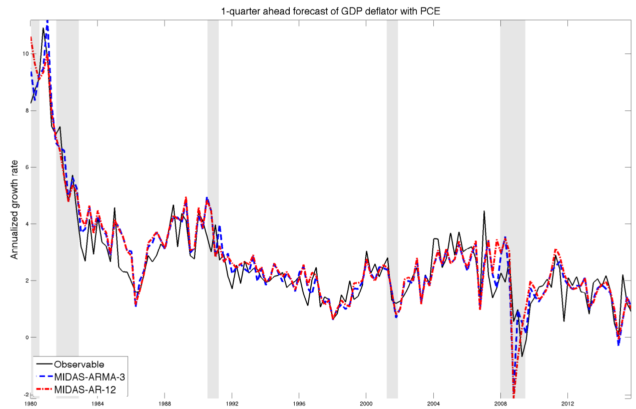
(a) GDP with monthly industrial production



(b) PNFI with monthly industrial production

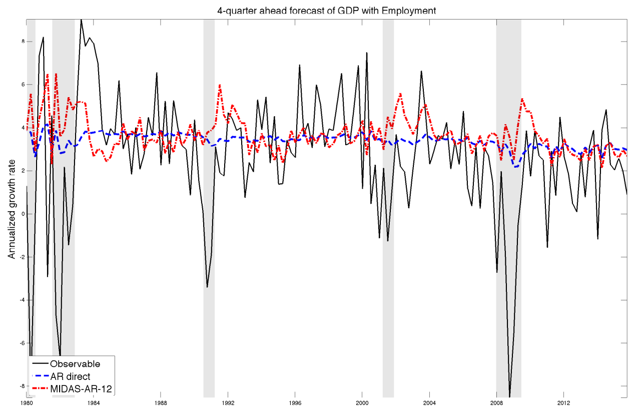


(c) GDP deflator with monthly CPI

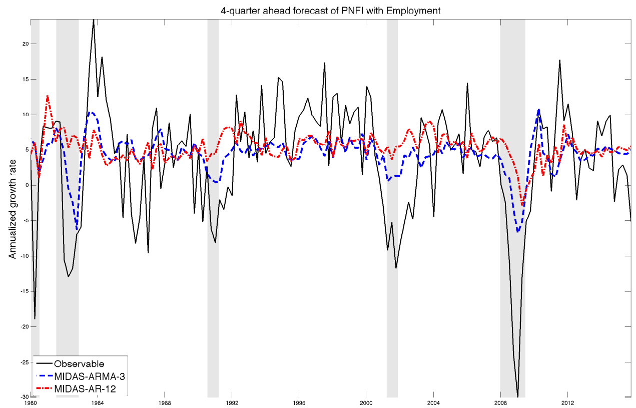


(d) GDP deflator with monthly PCE

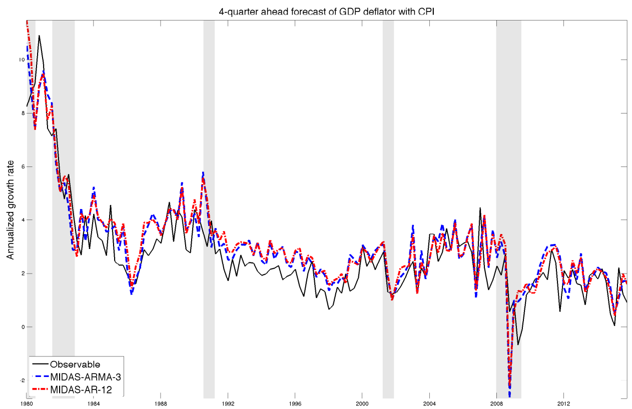
Figure 2: Out-of-sample performance: four-quarters ahead



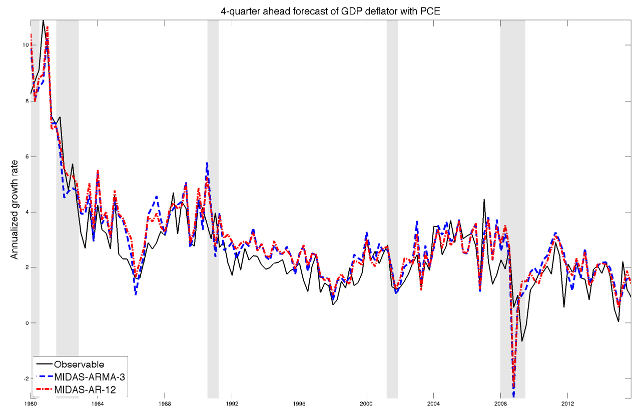
(a) GDP with monthly employment



(b) PNFI with monthly employment



(c) GDP deflator with monthly CPI



(d) GDP deflator with monthly PCE

Appendix for “Mixed frequency models with MA components”

A Estimation of (U)MIDAS-ARMA models

In this section we detail the algorithm used to estimate the (U)MIDAS-ARMA models with non-linear least squares (NLS).

A.1 NLS estimation of MIDAS-ARMA

The MIDAS-ARMA specification is as in (15):

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \beta B(L,\theta)x_{t_m-h_m+w} + u_{t_m} + q(L^m)u_{t_m-h_m}.$$

The estimation procedure, for given orders of the lag polynomials, consists of the following steps:

Step 1 Get initial values for all the parameters but $q(L^m)$ following Foroni et al. (2015) (the starting values are chosen with a grid search over a set of values for θ, which minimize the residual sum of squares). Set the initial values for $q(L^m)$. We initialize the MA coefficients by a draw from the uniform U(0.1, 0.5) distribution. In principle, it is possible to include the selection of initial value of $q(L^m)$ into the grid search.

Step 2 Estimate all the parameters, including the weights in the Almon polynomial, simultaneously by NLS, numerically minimizing the residual sum of squares, starting from the initial values obtained in Step 1.

A.2 NLS estimation of UMIDAS-ARMA

The UMIDAS-ARMA specification is as in (13):

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \delta(L)x_{t_m-h_m+w} + u_{t_m} + q(L^m)u_{t_m-h_m},$$

with p, d and r being the lag orders of $\tilde{c}(L^m)$, $\delta(L)$ and $q(L^m)$ respectively. The estimation procedure consists of the following steps:

Step 1 Get the initial values for all the parameters but $q(L^m)$ by projecting y_{t_m} on $\sum_{j=0}^{p} y_{t_m-h_m-j}$ and $\sum_{j=0}^{d} x_{t_m-h_m+w-j}$. Set the initial value for $q(L^m)$. We initialize the MA coefficients by a draw from the uniform U(0.1, 0.5) distribution. In principle, it is possible to perform a grid search and find a set of starting values of $q(L^m)$ which minimize the residual sum of squares.

Step 2 Estimate all the parameters simultaneously by NLS, numerically minimizing the residual sum of squares, starting from the initial values obtained in Step 1.

B Monte Carlo experiments: robustness checks

We run a battery of robustness checks on our Monte Carlo experiments.

1. We modify the set up by reducing the explanatory power of the x variable in such a way that the total R^2 of the DGP for y is equal to 0.3, 0.5 and 0.7. We obtain that by changing the value of the variance of $e_{x_{tm}}$ in the DGP for x. As examples, we consider the cases with a less persistent AR component, $\rho = 0.5$, where it is more likely to have a smaller R^2. The rest of the simulation is set up according to the DGP1 described in Section 4.1. The results are shown in Table B.1, and are similar to those in the main analysis. In general, adding an MA component (especially to the MIDAS) still improves the performance of the models.

2. We take again DGP1, and keep everything the same, except the sample size. We consider a longer estimation sample, with $T_q = 200$, which corresponds to 50 years of quarterly observations. The results are shown in Table B.2. The long sample does not affect substantially the findings.

3. We evaluate the results for DGP1 also at 2- and 4-quarter ahead horizons. The results are shown in Tables B.3 and B.4. At 2-quarter ahead horizon, adding an MA component is generally still useful, while at 4-quarter ahead horizon the MA is not useful anymore.

Table B.1: Robustness check: x explains a smaller share of y

DGP: average sampling, $\rho = 0.5$, $\delta_l = 0.1$, $T = 100$, $h = 1$						
			PANEL (A): $R^2 = 0.3$			
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	1.020	1.034	0.989	1.017	1.013	1.072
MIDAS-ARMA-3lags	1.009	1.027	1.000	1.032	1.010	0.998
UMIDAS-biclags	1.023	1.030	1.031	1.047	1.019	0.986
UMIDAS-ARMA-biclags	1.040	1.065	1.012	1.048	1.037	1.035
UMIDAS-ARMA-3lags	1.027	1.048	1.012	1.044	1.024	1.008
			PANEL (B): $R^2 = 0.5$			
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	0.996	1.018	1.082	0.996	1.112	0.933
MIDAS-ARMA-3lags	1.007	1.019	1.124	1.050	1.012	0.942
UMIDAS-biclags	0.991	1.052	1.133	1.013	0.956	0.917
UMIDAS-ARMA-biclags	1.015	1.150	1.130	1.138	0.980	0.916
UMIDAS-ARMA-3lags	1.022	1.088	1.049	1.110	1.000	0.922
			PANEL (C): $R^2 = 0.7$			
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	0.996	1.008	0.987	1.002	0.985	0.981
MIDAS-ARMA-3lags	1.041	1.073	1.011	1.054	1.038	1.051
UMIDAS-biclags	1.017	1.055	1.009	0.999	1.007	1.025
UMIDAS-ARMA-biclags	1.021	1.036	1.004	1.012	1.006	1.024
UMIDAS-ARMA-3lags	1.042	1.068	1.032	1.063	1.040	1.035

Note: The table is organized as Table 1 in Section 4.

Table B.2: Robustness check: longer sample $T = 200$

DGP: average sampling, $T = 200$, $h = 1$						
	PANEL (A): $\rho = 0.94$, $\delta_l = 1$					
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	0.973	0.978	0.940	0.983	0.975	0.956
MIDAS-ARMA-3lags	0.964	0.964	0.931	0.978	0.966	0.943
UMIDAS-biclags	1.011	1.019	1.003	1.012	1.003	1.014
UMIDAS-ARMA-biclags	0.979	0.995	0.940	0.990	1.000	0.949
UMIDAS-ARMA-3lags	0.972	0.963	0.958	0.985	0.977	0.952
	PANEL (B): $\rho = 0.9$, $\delta_l = 1$					
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	0.969	0.971	0.946	0.992	0.970	0.943
MIDAS-ARMA-3lags	0.972	0.965	0.954	0.988	0.985	0.950
UMIDAS-biclags	1.020	1.018	1.046	1.005	1.019	1.028
UMIDAS-ARMA-biclags	1.014	1.014	1.031	1.034	1.049	0.986
UMIDAS-ARMA-3lags	0.975	0.965	0.972	0.989	0.980	0.962
	PANEL (C): $\rho = 0.5$, $\delta_l = 0.1$					
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	0.980	0.984	0.971	0.983	0.978	0.969
MIDAS-ARMA-3lags	1.200	1.217	1.220	1.202	1.155	1.210
UMIDAS-biclags	0.980	0.959	1.019	0.971	0.999	0.962
UMIDAS-ARMA-biclags	0.966	0.955	1.031	0.959	0.986	0.950
UMIDAS-ARMA-3lags	1.193	1.208	1.223	1.202	1.146	1.208
	PANEL (D): $\rho = 0.1$, $\delta_l = 0.1$					
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	0.983	0.979	0.943	0.955	0.992	0.997
MIDAS-ARMA-3lags	0.781	0.819	0.802	0.778	0.754	0.748
UMIDAS-biclags	0.775	0.786	0.802	0.781	0.757	0.755
UMIDAS-ARMA-biclags	0.777	0.786	0.806	0.780	0.760	0.756
UMIDAS-ARMA-3lags	0.782	0.813	0.817	0.778	0.757	0.747

Note: The table is organized as Table 1 in Section 4.

Table B.3: Robustness check: forecasting 2 quarters ahead

DGP: average sampling, T=100, h=2						
			PANEL (A): $\rho = 0.94,\ \delta_l = 1$			
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	1.029	0.924	0.977	1.060	1.041	0.969
MIDAS-ARMA-3lags	0.999	0.905	0.979	0.977	1.067	0.964
UMIDAS-biclags	0.961	0.913	0.988	0.943	0.973	0.943
UMIDAS-ARMA-biclags	0.990	0.884	0.984	0.958	1.051	0.978
UMIDAS-ARMA-3lags	1.001	0.900	0.976	0.996	1.061	0.960
			PANEL (B): $\rho = 0.9,\ \delta_l = 1$			
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	1.026	0.999	0.962	1.080	1.056	1.017
MIDAS-ARMA-3lags	1.019	1.008	0.981	1.059	1.036	1.003
UMIDAS-biclags	1.023	1.030	1.000	1.043	0.982	1.016
UMIDAS-ARMA-biclags	1.064	1.048	1.010	1.094	1.096	1.062
UMIDAS-ARMA-3lags	1.036	1.041	0.982	1.104	1.000	1.031
			PANEL (C): $\rho = 0.5,\ \delta_l = 0.1$			
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	0.979	1.004	1.054	0.980	0.938	0.931
MIDAS-ARMA-3lags	0.972	1.006	1.011	0.968	0.989	0.900
UMIDAS-biclags	1.179	1.100	1.189	1.310	1.147	1.180
UMIDAS-ARMA-biclags	1.172	1.107	1.230	1.267	1.132	1.124
UMIDAS-ARMA-3lags	1.025	1.108	1.078	1.082	0.963	1.001
			PANEL (D): $\rho = 0.1,\ \delta_l = 0.1$			
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	1.018	0.989	1.007	0.987	1.041	0.981
MIDAS-ARMA-3lags	0.993	1.012	1.002	0.997	0.968	0.969
UMIDAS-biclags	0.951	1.003	1.014	0.964	0.959	0.849
UMIDAS-ARMA-biclags	1.031	1.075	1.087	1.052	1.072	0.878
UMIDAS-ARMA-3lags	1.105	1.125	1.201	1.146	1.126	0.985

Note: The table is organized as Table 1 in Section 4. The forecasting horizon is 2 quarters ahead.

Table B.4: Robustness check: forecasting 4 quarters ahead

DGP: average sampling, T=100, h=4						
	PANEL (A): $\rho = 0.94$, $\delta_l = 1$					
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	1.130	1.005	1.050	1.154	1.217	1.110
MIDAS-ARMA-3lags	1.076	0.951	1.154	1.127	1.078	1.039
UMIDAS-biclags	0.976	0.921	1.021	1.036	1.015	0.946
UMIDAS-ARMA-biclags	1.057	0.937	1.172	1.069	1.069	1.042
UMIDAS-ARMA-3lags	1.078	0.962	1.255	1.115	1.056	1.038
	PANEL (B): $\rho = 0.9$, $\delta_l = 1$					
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	1.147	1.208	1.259	1.033	1.109	1.085
MIDAS-ARMA-3lags	1.048	1.128	1.146	1.035	0.929	1.007
UMIDAS-biclags	1.019	1.005	1.030	1.033	0.939	1.023
UMIDAS-ARMA-biclags	1.080	1.130	1.154	1.094	0.975	1.023
UMIDAS-ARMA-3lags	1.077	1.099	1.202	1.063	0.966	1.011
	PANEL (C): $\rho = 0.5$, $\delta_l = 0.1$					
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	1.029	1.071	0.998	0.997	1.042	1.026
MIDAS-ARMA-3lags	1.003	1.016	1.061	1.036	0.998	0.947
UMIDAS-biclags	0.992	1.050	1.028	1.058	0.935	0.913
UMIDAS-ARMA-biclags	1.015	1.157	1.040	1.082	0.945	0.918
UMIDAS-ARMA-3lags	1.091	1.126	1.076	1.122	1.058	1.024
	PANEL (D): $\rho = 0.1$, $\delta_l = 0.1$					
	mse	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12lags	1.023	1.020	1.030	1.013	1.017	1.062
MIDAS-ARMA-3lags	1.013	0.991	0.987	1.022	1.036	1.019
UMIDAS-biclags	0.957	1.000	0.894	0.978	1.045	0.936
UMIDAS-ARMA-biclags	1.044	1.078	1.070	1.001	1.095	1.069
UMIDAS-ARMA-3lags	1.030	1.138	1.016	1.000	1.021	1.059

Note: The table is organized as Table 1 in Section 4. The forecasting horizon is 4 quarters ahead.

C Empirical applications: robustness checks

To see whether our results are robust, we run several additional exercises.

1. We recompute the forecast evaluation stopping our evaluation sample in 2007Q3, to exclude the possibility that they are driven by the Great Recession. Tables C.1 to C.3 are the equivalent of Tables 6 to 8 for the full sample. Results do not change substantially, and remain broadly supportive of the inclusion of the MA component in the mixed-frequency models. In most of the cases, the best performing model up to 2007 stays the best in the full sample also. Also the magnitude of improvements is very comparable.

2. While in Section 5 we report only the results when two months of monthly information are available, here we complete the set of possible results, considering the cases when only one month of the quarter is available or when all three months are available. Results are reported in Table C.4. Inclusion of the MA component helps at all horizons and, as expected, the absolute performance generally improves when adding additional information.

3. We reestimate our models with real-time data. In particular:

    - For forecasting GDP deflator inflation: GDP deflator is from Philadelphia Fed (GDPDEF). CPI and PCEPI are from Alfred. CPI vintages are available only from 1994M02, so the out-of-sample period in the exercise with GDPDEF / CPI starts on 1994Q2. PCEPI vintages are available only from 2007M07, so the out-of-sample period in the exercise for GDPDEF / PCEPI starts on 2000Q3.
    - For forecasting GDP growth: GDP, industrial production and employment are all available from Philadelphia Fed real-time data base. All vintages are available so the out-of-sample period is the same as with historical data: 1980Q1-2015Q4.
    - For forecasting Investment growth: we use Real Gross Private Domestic Investment: nonresidential (RINVBF) available from Philadelphia Fed (we were not able to construct Real PNFI in real-time.). Industrial production and employment are the same as in GDP exercise. The out-of-sample period is 1980Q1 - 2015Q4.

    In evaluating the forecasts we consider as realized values those in the vintage of 2016Q1. Results are available in Tables C.5 to C.7. The main findings Inclusion of the MA component helps at all horizons and, as expected, the absolute performance generally improves when adding additional information.

Table C.1: Forecasting U.S. GDP growth: evaluation sample 1980Q1- 2007Q3

	Explanatory variable: Industrial production growth						**Explanatory variable: Employment growth**					
	h=1						h=1					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	3.81	1.00	NaN	1.51	1.00	NaN	4.82	1.00	NaN	1.71	1.00	NaN
MIDAS-ARMA-12	3.83	1.01	0.36	1.53	1.01	0.04	4.81	1.00	0.48	1.70	1.00	0.44
MIDAS-ARMA-3	3.89	1.02	0.26	1.51	1.00	0.47	4.86	1.01	0.45	1.71	1.00	0.49
UMIDAS-biclags	3.88	1.02	0.30	1.51	1.00	0.44	4.74	0.98	0.39	1.72	1.01	0.43
UMIDAS-ARMA-biclags	3.86	1.01	0.35	1.52	1.01	0.36	4.43	0.92	0.10	1.66	0.97	0.24
UMIDAS-ARMA-3	3.89	1.02	0.26	1.51	1.00	0.47	4.86	1.01	0.45	1.71	1.00	0.49
AR	7.37	1.93	0.01	1.94	1.29	0.01	7.37	1.53	0.04	1.94	1.14	0.09
	h=2						h=2					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	6.59	1.00	NaN	1.85	1.00	NaN	6.15	1.00	NaN	1.80	1.00	NaN
MIDAS-ARMA-12	6.26	0.95	0.05	1.84	0.99	0.40	5.63	0.92	0.06	1.76	0.98	0.20
MIDAS-ARMA-3	6.53	0.99	0.39	1.91	1.03	0.10	5.76	0.94	0.11	1.77	0.98	0.26
UMIDAS-biclags	7.04	1.07	0.00	1.96	1.06	0.00	6.15	1.00	0.12	1.80	1.00	0.09
UMIDAS-ARMA-biclags	6.74	1.02	0.22	1.92	1.04	0.03	5.76	0.94	0.12	1.77	0.98	0.26
UMIDAS-ARMA-3	6.71	1.02	0.21	1.86	1.00	0.42	6.19	1.01	0.45	1.82	1.01	0.30
AR	7.45	1.13	0.19	1.94	1.05	0.16	7.45	1.21	0.06	1.94	1.08	0.04
	h=3						h=3					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	7.76	1.00	NaN	2.00	1.00	NaN	8.79	1.00	NaN	2.12	1.00	NaN
MIDAS-ARMA-12	7.91	1.02	0.27	2.02	1.01	0.28	9.38	1.07	0.23	2.35	1.11	0.04
MIDAS-ARMA-3	6.91	0.89	0.00	1.85	0.93	0.00	6.88	0.78	0.00	1.89	0.89	0.00
UMIDAS-biclags	7.67	0.99	0.32	1.96	0.98	0.12	7.36	0.84	0.01	1.93	0.91	0.01
UMIDAS-ARMA-biclags	11.40	1.47	0.02	2.44	1.22	0.02	16.27	1.85	0.00	3.35	1.58	0.00
UMIDAS-ARMA-3	7.33	0.94	0.12	1.90	0.95	0.04	7.90	0.90	0.13	1.99	0.94	0.09
AR	8.22	1.06	0.23	2.00	1.00	0.49	8.22	0.94	0.03	2.00	0.94	0.03
	h=4						h=4					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	8.26	1.00	NaN	2.03	1.00	NaN	10.56	1.00	NaN	2.26	1.00	NaN
MIDAS-ARMA-12	8.08	0.98	0.28	2.04	1.00	0.44	10.94	1.04	0.36	2.43	1.08	0.09
MIDAS-ARMA-3	7.73	0.94	0.06	1.97	0.97	0.14	9.85	0.93	0.09	2.12	0.94	0.03
UMIDAS-biclags	8.26	1.00	0.50	2.00	0.98	0.27	9.89	0.94	0.10	2.11	0.94	0.02
UMIDAS-ARMA-biclags	7.04	0.85	0.04	1.96	0.96	0.26	11.10	1.05	0.31	2.39	1.06	0.18
UMIDAS-ARMA-3	9.45	1.14	0.29	2.01	0.99	0.43	9.26	0.88	0.01	2.06	0.91	0.01
AR	8.09	0.98	0.30	1.99	0.98	0.23	8.09	0.77	0.01	1.99	0.88	0.00

Note: The table reports the results on the forecasting performance of the different models. In the columns "value" we report the MSE and the MAE respectively. In the columns "ratio" we report the MSE and MAE of each model relative to the MIDAS-AR benchmark. In the columns "DM" we report the p-value of the Diebold-Mariano test. The forecasts are evaluated over the sample 1980Q1-2007Q3. The lowest values for each variable are underlined.

Table C.2: Forecasting U.S. Real Private Nonresidential Fixed Investment growth growth: evaluation sample 1980Q1- 2007Q3

	Explanatory variable: Industrial production growth						**Explanatory variable: Employment growth**					
	h=1						h=1					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	33.02	1.00	NaN	4.56	1.00	NaN	33.56	1.00	NaN	4.55	1.00	NaN
MIDAS-ARMA-12	30.91	0.94	0.01	4.41	0.97	0.02	34.30	1.02	0.06	4.60	1.01	0.07
MIDAS-ARMA-3	29.68	0.90	0.00	4.37	0.96	0.03	33.90	1.01	0.30	4.60	1.01	0.16
UMIDAS-biclags	34.77	1.05	0.19	4.65	1.02	0.19	33.97	1.01	0.15	4.60	1.01	0.01
UMIDAS-ARMA-biclags	30.99	0.94	0.09	4.44	0.97	0.10	35.23	1.05	0.16	4.69	1.03	0.03
UMIDAS-ARMA-3	29.70	0.90	0.00	4.37	0.96	0.03	33.58	1.00	0.49	4.59	1.01	0.26
AR	43.71	1.32	0.01	5.13	1.12	0.03	43.71	1.30	0.03	5.13	1.13	0.06
	h=2						h=2					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	39.79	1.00	NaN	4.87	1.00	NaN	36.79	1.00	NaN	4.75	1.00	NaN
MIDAS-ARMA-12	39.82	1.00	0.48	4.89	1.00	0.42	37.40	1.02	0.21	4.82	1.02	0.13
MIDAS-ARMA-3	42.43	1.07	0.09	5.04	1.03	0.13	37.83	1.03	0.08	4.85	1.02	0.08
UMIDAS-biclags	42.57	1.07	0.00	5.04	1.03	0.00	38.74	1.05	0.03	4.84	1.02	0.09
UMIDAS-ARMA-biclags	46.29	1.16	0.00	5.34	1.10	0.00	39.91	1.08	0.01	4.96	1.05	0.03
UMIDAS-ARMA-3	45.18	1.14	0.04	5.11	1.05	0.07	37.75	1.03	0.22	4.89	1.03	0.04
AR	44.72	1.12	0.22	5.20	1.07	0.15	44.72	1.22	0.07	5.20	1.09	0.06
	h=3						h=3					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	43.65	1.00	NaN	5.24	1.00	NaN	42.71	1.00	NaN	5.18	1.00	NaN
MIDAS-ARMA-12	42.16	0.97	0.13	5.11	0.97	0.11	40.70	0.95	0.07	5.06	0.98	0.06
MIDAS-ARMA-3	44.05	1.01	0.36	5.25	1.00	0.47	41.21	0.96	0.19	5.11	0.99	0.22
UMIDAS-biclags	48.10	1.10	0.00	5.46	1.04	0.04	44.94	1.05	0.05	5.25	1.01	0.17
UMIDAS-ARMA-biclags	45.37	1.04	0.10	5.30	1.01	0.28	42.65	1.00	0.48	5.10	0.98	0.19
UMIDAS-ARMA-3	44.00	1.01	0.37	5.24	1.00	0.49	41.27	0.97	0.21	5.14	0.99	0.33
AR	50.82	1.16	0.07	5.52	1.05	0.12	50.82	1.19	0.05	5.52	1.07	0.10
	h=4						h=4					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	50.48	1.00	NaN	5.71	1.00	NaN	63.84	1.00	NaN	6.13	1.00	NaN
MIDAS-ARMA-12	47.04	0.93	0.12	5.39	0.94	0.02	54.59	0.86	0.05	5.74	0.94	0.04
MIDAS-ARMA-3	46.28	0.92	0.10	5.34	0.93	0.01	45.45	0.71	0.01	5.21	0.85	0.00
UMIDAS-biclags	47.34	0.94	0.06	5.46	0.96	0.02	54.11	0.85	0.01	5.63	0.92	0.01
UMIDAS-ARMA-biclags	50.38	1.00	0.49	5.50	0.96	0.08	52.81	0.83	0.06	5.67	0.93	0.10
UMIDAS-ARMA-3	48.15	0.95	0.30	5.35	0.94	0.07	52.06	0.82	0.03	5.54	0.91	0.03
AR	56.22	1.11	0.08	5.76	1.01	0.40	56.22	0.88	0.01	5.76	0.94	0.01

Note: See Table C.1.

Table C.3: Forecasting U.S. GDP Deflator: evaluation sample 1980Q1- 2007Q3

	Explanatory variable: CPI inflation						**Explanatory variable: PCE inflation**					
	h=1						h=1					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	0.63	1.00	NaN	0.60	1.00	NaN	0.42	1.00	NaN	0.48	1.00	NaN
MIDAS-ARMA-12	0.63	1.00	0.49	0.61	1.02	0.29	0.43	1.03	0.29	0.49	1.02	0.23
MIDAS-ARMA-3	0.58	0.92	0.09	0.58	0.96	0.15	0.39	0.95	0.32	0.48	1.02	0.36
UMIDAS-biclags	0.59	0.93	0.19	0.58	0.97	0.19	0.43	1.04	0.29	0.49	1.02	0.29
UMIDAS-ARMA-biclags	0.58	0.92	0.09	0.58	0.96	0.15	0.45	1.08	0.18	0.49	1.03	0.19
UMIDAS-ARMA-3	0.58	0.92	0.09	0.58	0.96	0.15	0.41	1.00	0.47	0.48	1.01	0.39
AR	0.77	1.23	0.15	0.68	1.14	0.06	0.77	1.86	0.00	0.68	1.43	0.00
	h=2						h=2					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	0.83	1.00	NaN	0.68	1.00	NaN	0.51	1.00	NaN	0.53	1.00	NaN
MIDAS-ARMA-12	0.69	0.83	0.08	0.65	0.95	0.14	0.54	1.05	0.11	0.54	1.01	0.15
MIDAS-ARMA-3	0.64	0.77	0.04	0.64	0.93	0.09	0.43	0.84	0.08	0.52	0.98	0.35
UMIDAS-biclags	0.71	0.85	0.14	0.66	0.97	0.27	0.48	0.93	0.30	0.54	1.02	0.40
UMIDAS-ARMA-biclags	0.66	0.80	0.04	0.64	0.94	0.12	0.44	0.85	0.08	0.52	0.98	0.40
UMIDAS-ARMA-3	0.64	0.77	0.04	0.64	0.93	0.09	0.43	0.84	0.08	0.52	0.98	0.35
AR	0.81	0.97	0.46	0.71	1.04	0.33	0.81	1.57	0.01	0.71	1.35	0.00
	h=3						h=3					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	0.83	1.00	NaN	0.70	1.00	NaN	0.51	1.00	NaN	0.54	1.00	NaN
MIDAS-ARMA-12	0.92	1.10	0.28	0.75	1.08	0.14	0.60	1.18	0.03	0.58	1.07	0.08
MIDAS-ARMA-3	0.72	0.87	0.18	0.69	0.99	0.42	0.53	1.04	0.38	0.55	1.02	0.40
UMIDAS-biclags	0.83	0.99	0.48	0.72	1.03	0.32	0.56	1.11	0.18	0.58	1.07	0.14
UMIDAS-ARMA-biclags	0.85	1.01	0.46	0.74	1.06	0.25	0.64	1.26	0.03	0.61	1.13	0.06
UMIDAS-ARMA-3	0.74	0.89	0.19	0.69	0.99	0.43	0.53	1.04	0.37	0.55	1.02	0.40
AR	0.84	1.00	0.49	0.73	1.05	0.29	0.84	1.65	0.00	0.73	1.35	0.00
	h=4						h=4					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12	0.85	1.00	NaN	0.75	1.00	NaN	0.55	1.00	NaN	0.59	1.00	NaN
MIDAS-ARMA-12	1.11	1.31	0.00	0.86	1.14	0.00	0.74	1.34	0.00	0.68	1.16	0.00
MIDAS-ARMA-3	0.80	0.95	0.28	0.73	0.97	0.28	0.60	1.08	0.18	0.60	1.03	0.27
UMIDAS-biclags	0.97	1.14	0.03	0.79	1.05	0.05	0.62	1.13	0.08	0.61	1.05	0.14
UMIDAS-ARMA-biclags	1.02	1.21	0.04	0.80	1.06	0.11	0.68	1.23	0.01	0.65	1.11	0.01
UMIDAS-ARMA-3	0.97	1.14	0.10	0.78	1.03	0.23	0.65	1.18	0.05	0.62	1.06	0.10
AR	0.89	1.04	0.39	0.76	1.01	0.43	0.89	1.61	0.00	0.76	1.30	0.00

Note: See Table C.1.

Table C.4: Nowcasting when one month of information is available ($h_m = 2$) or all months of information are available ($h_m = 0$)

	$h_m = 2$						$h_m = 0$					
		MSE			MAE			MSE			MAE	
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
Forecasting GDP deflator with CPI inflation												
MIDAS-AR-12	0.76	1.00	NaN	0.67	1.00	NaN	0.71	1.00	NaN	0.63	1.00	NaN
MIDAS-ARMA-12	0.72	0.95	0.08	0.65	0.98	0.12	0.66	0.92	0.24	0.62	0.99	0.41
MIDAR-ARMA-3	0.77	1.01	0.46	0.65	0.98	0.39	0.69	0.97	0.40	0.64	1.02	0.36
UMIDAS-biclags	0.70	0.92	0.19	0.66	0.98	0.37	0.66	0.92	0.13	0.60	0.96	0.12
UMIDAS-ARMA-biclags	0.67	0.87	0.03	0.64	0.96	0.15	0.62	0.87	0.02	0.60	0.95	0.08
UMIDAS-ARMA-3	0.73	0.95	0.12	0.66	0.99	0.40	0.65	0.91	0.11	0.61	0.96	0.17
AR	0.79	1.03	0.41	0.68	1.03	0.35	0.79	1.10	0.26	0.68	1.08	0.14
Forecasting GDP deflator with PCE inflation												
MIDAS-AR-12	0.68	1.00	NaN	0.63	1.00	NaN	0.54	1.00	NaN	0.52	1.00	NaN
MIDAS-ARMA-12	0.65	0.95	0.20	0.61	0.96	0.08	0.61	1.14	0.16	0.61	1.18	0.00
MIDAR-ARMA-3	0.67	0.98	0.42	0.61	0.97	0.29	0.66	1.23	0.04	0.62	1.20	0.00
UMIDAS-biclags	0.66	0.97	0.39	0.63	1.00	0.49	0.53	0.99	0.43	0.52	1.00	0.48
UMIDAS-ARMA-biclags	0.66	0.96	0.35	0.63	1.00	0.47	0.51	0.95	0.26	0.51	0.99	0.36
UMIDAS-ARMA-3	0.65	0.96	0.25	0.62	0.99	0.31	0.51	0.94	0.21	0.51	0.99	0.45
AR	0.79	1.15	0.12	0.68	1.08	0.11	0.79	1.47	0.02	0.68	1.32	0.00
Forecasting GDP with industrial production												
MIDAS-AR-12	4.24	1.00	NaN	1.60	1.00	NaN	3.97	1.00	NaN	1.56	1.00	NaN
MIDAS-ARMA-12	4.19	0.99	0.15	1.59	0.99	0.21	3.96	1.00	0.47	1.57	1.01	0.12
MIDAR-ARMA-3	4.28	1.01	0.28	1.61	1.00	0.38	4.68	1.18	0.13	1.62	1.04	0.20
UMIDAS-biclags	4.43	1.04	0.12	1.62	1.01	0.23	4.06	1.02	0.21	1.56	1.00	0.49
UMIDAS-ARMA-biclags	4.44	1.05	0.10	1.63	1.01	0.17	4.02	1.01	0.35	1.56	1.00	0.43
UMIDAS-ARMA-3	4.26	1.01	0.38	1.60	1.00	0.48	4.41	1.11	0.13	1.64	1.05	0.11
AR	7.60	1.79	0.01	1.97	1.23	0.01	7.60	1.92	0.01	1.97	1.27	0.00
Forecasting GDP with employment												
MIDAS-AR-12	5.14	1.00	NaN	1.76	1.00	NaN	4.75	1.00	NaN	1.71	1.00	NaN
MIDAS-ARMA-12	4.99	0.97	0.18	1.73	0.98	0.17	4.64	0.98	0.35	1.70	1.00	0.44
MIDAR-ARMA-3	4.97	0.97	0.18	1.72	0.98	0.13	4.64	0.98	0.35	1.70	1.00	0.44
UMIDAS-biclags	5.14	1.00	0.50	1.76	1.00	0.49	4.37	0.92	0.14	1.66	0.97	0.25
UMIDAS-ARMA-biclags	5.21	1.01	0.40	1.77	1.01	0.39	4.34	0.91	0.11	1.65	0.97	0.22
UMIDAS-ARMA-3	5.05	0.98	0.33	1.74	0.99	0.34	4.64	0.98	0.36	1.70	1.00	0.46
AR	7.60	1.48	0.02	1.97	1.12	0.06	7.60	1.60	0.02	1.97	1.15	0.04
Forecasting Real PNFI with industrial production												
MIDAS-AR-12	32.34	1.00	NaN	4.46	1.00	NaN	30.75	1.00	NaN	4.38	1.00	NaN
MIDAS-ARMA-12	32.58	1.01	0.38	4.50	1.01	0.26	28.27	0.92	0.00	4.18	0.96	0.01
MIDAR-ARMA-3	33.12	1.02	0.22	4.48	1.01	0.35	29.53	0.96	0.26	4.29	0.98	0.25
UMIDAS-biclags	34.14	1.06	0.12	4.60	1.03	0.06	31.93	1.04	0.20	4.49	1.02	0.11
UMIDAS-ARMA-biclags	33.11	1.02	0.32	4.49	1.01	0.34	28.91	0.94	0.05	4.27	0.97	0.09
UMIDAS-ARMA-3	33.52	1.04	0.16	4.50	1.01	0.28	28.03	0.91	0.07	4.22	0.96	0.13
AR	44.91	1.39	0.01	5.11	1.15	0.01	44.91	1.46	0.01	5.11	1.17	0.01
Forecasting Real PNFI with employment												
MIDAS-AR-12	33.95	1.00	NaN	4.57	1.00	NaN	32.35	1.00	NaN	4.55	1.00	NaN
MIDAS-ARMA-12	34.92	1.03	0.16	4.58	1.00	0.33	32.59	1.01	0.29	4.55	1.00	0.49
MIDAR-ARMA-3	34.50	1.02	0.17	4.57	1.00	0.45	32.07	0.99	0.41	4.50	0.99	0.21
UMIDAS-biclags	33.69	0.99	0.14	4.54	0.99	0.12	32.73	1.01	0.35	4.58	1.00	0.35
UMIDAS-ARMA-biclags	33.91	1.00	0.46	4.52	0.99	0.10	33.05	1.02	0.25	4.61	1.01	0.18
UMIDAS-ARMA-3	34.42	1.01	0.20	4.56	1.00	0.44	31.98	0.99	0.38	4.49	0.99	0.19
AR	44.91	1.32	0.01	5.11	1.12	0.04	44.91	1.39	0.02	5.11	1.12	0.06

Note: The table reports the results on the forecasting performance of the different models one quarter ahead, when one month of information is available or all months are available. In the columns "value" we report the MSE and the MAE respectively. In the columns "ratio" we report the MSE and MAE of each model relative to the MIDAS-AR benchmark. In the columns "DM" we report the p-value of the Diebold-Mariano test. The forecasts are evaluated over the sample 1980Q1-2015Q4.

Table C.5: Forecasting U.S. GDP growth: real-time forecasts

	h_month = 0						h_month = 1						h_month = 2					
	MSE			MAE			MSE			MAE			MSE			MAE		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
Forecasting GDP with industrial production																		
MIDAS-AR-12	5.16	1.00	NaN	1.76	1.00	NaN	5.99	1.00	NaN	1.91	1.00	NaN	6.77	1.00	NaN	1.95	1.00	NaN
MIDAS-ARMA-12	5.23	1.01	0.07	1.79	1.01	0.02	6.15	1.03	0.09	1.93	1.01	0.21	6.76	1.00	0.49	1.95	1.00	0.46
MIDAR-ARMA-3	5.25	1.02	0.12	1.80	1.02	0.01	6.10	1.02	0.15	1.91	1.00	0.46	6.83	1.01	0.25	1.96	1.00	0.39
UMIDAS-biclags	5.50	1.07	0.01	1.82	1.03	0.00	6.14	1.03	0.09	1.92	1.00	0.35	6.76	1.00	0.44	1.94	0.99	0.12
UMIDAS-ARMA-biclags	5.51	1.07	0.01	1.82	1.03	0.00	6.23	1.04	0.06	1.91	1.00	0.48	6.80	1.01	0.23	1.94	0.99	0.14
UMIDAS-ARMA-3	5.32	1.03	0.02	1.80	1.02	0.01	6.16	1.03	0.06	1.92	1.01	0.27	7.32	1.08	0.04	2.01	1.03	0.09
AR	7.55	1.46	0.04	1.98	1.12	0.08	7.55	1.26	0.11	1.98	1.03	0.33	7.55	1.12	0.06	1.98	1.01	0.34
Forecasting GDP with employment																		
MIDAS-AR-12	5.29	1.00	NaN	1.80	1.00	NaN	6.73	1.00	NaN	1.93	1.00	NaN	7.68	1.00	NaN	1.98	1.00	NaN
MIDAS-ARMA-12	4.98	0.94	0.07	1.75	0.97	0.07	6.60	0.98	0.07	1.91	0.99	0.10	7.38	0.96	0.13	1.97	0.99	0.19
MIDAR-ARMA-3	4.96	0.94	0.06	1.75	0.97	0.09	6.59	0.98	0.03	1.91	0.99	0.05	7.24	0.94	0.09	1.95	0.98	0.09
UMIDAS-biclags	4.97	0.94	0.16	1.75	0.97	0.18	6.40	0.95	0.21	1.92	0.99	0.40	7.39	0.96	0.29	1.98	1.00	0.46
UMIDAS-ARMA-biclags	5.08	0.96	0.23	1.77	0.98	0.26	6.38	0.95	0.19	1.93	1.00	0.44	7.26	0.95	0.13	1.97	0.99	0.23
UMIDAS-ARMA-3	4.92	0.93	0.07	1.75	0.97	0.11	6.66	0.99	0.44	1.98	1.02	0.21	7.80	1.02	0.42	2.04	1.03	0.14
AR	7.55	1.43	0.02	1.98	1.10	0.10	7.55	1.12	0.12	1.98	1.02	0.30	7.55	0.98	0.40	1.98	1.00	0.48

Note: The table reports the results on the forecasting performance of the different models. In the columns "value" we report the MSE and the MAE respectively. In the columns "ratio" we report the MSE and MAE of each model relative to the MIDAS-AR benchmark. In the columns "DM" we report the p-value of the Diebold-Mariano test. The forecasts are evaluated on the sample 1980Q1-2015Q4.

Table C.6: Forecasting U.S. Real Gross Private Domestic Investment - Nonresidential: real-time forecasts

	h_month = 0						h_month = 1						h_month = 2					
	MSE			MAE			MSE			MAE			MSE			MAE		
	Value	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**
MIDAS-AR-12	33.22	1.00	NaN	4.59	1.00	NaN	37.29	1.00	NaN	4.95	1.00	NaN	43.04	1.00	NaN	5.27	1.00	NaN
MIDAS-ARMA-12	33.17	1.00	0.49	4.45	0.97	0.12	32.95	0.88	0.00	4.54	0.92	0.00	42.06	0.98	0.25	5.09	0.97	0.03
MIDAR-ARMA-3	33.12	1.00	0.48	4.49	0.98	0.24	34.70	0.93	0.07	4.71	0.95	0.04	41.47	0.96	0.06	5.11	0.97	0.03
UMIDAS-biclags	41.16	1.24	0.01	5.06	1.10	0.01	41.24	1.11	0.06	5.13	1.03	0.09	46.15	1.07	0.12	5.45	1.03	0.07
UMIDAS-ARMA-biclags	33.85	1.02	0.36	4.56	0.99	0.39	32.72	0.88	0.01	4.58	0.92	0.00	40.63	0.94	0.08	5.17	0.98	0.15
UMIDAS-ARMA-3	33.19	1.00	0.49	4.46	0.97	0.15	33.23	0.89	0.01	4.51	0.91	0.00	41.46	0.96	0.10	5.10	0.97	0.05
AR	54.06	1.63	0.01	5.77	1.26	0.00	54.06	1.45	0.01	5.77	1.16	0.00	54.06	1.26	0.04	5.77	1.09	0.03
Forecasting Real Gross Private Domestic Investment: Nonresidential (RINVBF) with employment																		
MIDAS-AR-12	42.02	1.00	NaN	5.16	1.00	NaN	43.68	1.00	NaN	5.15	1.00	NaN	44.13	1.00	NaN	5.16	1.00	NaN
MIDAS-ARMA-12	41.98	1.00	0.46	5.16	1.00	0.47	43.46	1.00	0.39	5.16	1.00	0.39	45.02	1.02	0.06	5.17	1.00	0.46
MIDAR-ARMA-3	42.58	1.01	0.35	5.13	0.99	0.31	43.11	0.99	0.20	5.15	1.00	0.49	44.45	1.01	0.30	5.15	1.00	0.42
UMIDAS-biclags	43.02	1.02	0.22	5.19	1.01	0.33	44.46	1.02	0.28	5.19	1.01	0.28	43.78	0.99	0.22	5.14	0.99	0.19
UMIDAS-ARMA-biclags	41.51	0.99	0.32	5.13	0.99	0.29	44.02	1.01	0.41	5.18	1.01	0.34	43.91	0.99	0.40	5.13	0.99	0.28
UMIDAS-ARMA-3	41.31	0.98	0.27	5.08	0.98	0.08	42.94	0.98	0.16	5.15	1.00	0.49	44.51	1.01	0.27	5.14	1.00	0.33
AR	54.06	1.29	0.08	5.77	1.12	0.05	54.06	1.24	0.10	5.77	1.12	0.04	54.06	1.23	0.06	5.77	1.12	0.02

Note: The table reports the results on the forecasting performance of the different models. In the columns "value" we report the MSE and the MAE respectively. In the columns "ratio" we report the MSE and MAE of each model relative to the MIDAS-AR benchmark. In the columns "DM" we report the p-value of the Diebold-Mariano test. The forecasts are evaluated on the sample 1980Q1-2015Q4.

Table C.7: Forecasting U.S. GDP deflator: real-time forecasts

	h_month = 0						h_month = 1						h_month = 2					
	MSE			MAE			MSE			MAE			MSE			MAE		
	Value	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**
Forecasting GDP deflator with CPI inflation																		
MIDAS-AR-12	0.71	1.00	NaN	0.63	1.00	NaN	0.72	1.00	NaN	0.66	1.00	NaN	0.83	1.00	NaN	0.73	1.00	NaN
MIDAS-ARMA-12	0.62	0.88	0.18	0.61	0.97	0.33	0.67	0.93	0.09	0.63	0.95	0.08	0.75	0.90	0.04	0.70	0.96	0.08
MIDAR-ARMA-3	0.69	0.97	0.41	0.62	0.99	0.46	0.64	0.88	0.02	0.62	0.93	0.04	0.95	1.13	0.21	0.75	1.04	0.32
UMIDAS-biclags	0.72	1.02	0.42	0.62	0.98	0.32	0.76	1.05	0.30	0.64	0.97	0.17	0.79	0.95	0.30	0.71	0.97	0.33
UMIDAS-ARMA-biclags	0.64	0.90	0.10	0.61	0.97	0.22	0.65	0.90	0.04	0.62	0.94	0.06	0.71	0.85	0.03	0.68	0.94	0.08
UMIDAS-ARMA-3	0.65	0.92	0.18	0.61	0.97	0.29	0.64	0.88	0.02	0.62	0.93	0.04	0.74	0.88	0.02	0.69	0.94	0.02
AR	0.97	1.36	0.02	0.75	1.19	0.02	0.97	1.34	0.05	0.75	1.13	0.06	0.97	1.16	0.16	0.75	1.03	0.33
Forecasting GDP deflator with PCE inflation																		
MIDAS-AR-12	0.74	1.00	NaN	0.63	1.00	NaN	0.75	1.00	NaN	0.65	1.00	NaN	0.98	1.00	NaN	0.75	1.00	NaN
MIDAS-ARMA-12	0.77	1.04	0.41	0.69	1.09	0.15	0.70	0.93	0.14	0.61	0.94	0.05	0.87	0.89	0.08	0.73	0.97	0.30
MIDAR-ARMA-3	0.87	1.17	0.13	0.70	1.11	0.08	0.71	0.95	0.23	0.65	1.00	0.49	0.99	1.01	0.48	0.75	1.00	0.50
UMIDAS-biclags	0.80	1.09	0.20	0.62	0.98	0.31	0.85	1.13	0.14	0.64	0.99	0.40	0.86	0.88	0.21	0.74	0.98	0.40
UMIDAS-ARMA-biclags	0.66	0.90	0.10	0.60	0.95	0.16	0.69	0.93	0.02	0.61	0.94	0.00	0.80	0.81	0.10	0.71	0.95	0.25
UMIDAS-ARMA-3	0.73	0.98	0.42	0.62	0.99	0.42	0.69	0.93	0.02	0.61	0.94	0.00	0.82	0.84	0.08	0.71	0.95	0.18
AR	1.14	1.55	0.00	0.80	1.26	0.00	1.14	1.53	0.01	0.80	1.23	0.03	1.14	1.17	0.12	0.80	1.06	0.26

Note: The table reports the results on the forecasting performance of the different models. In the columns "value" we report the MSE and the MAE respectively. In the columns "ratio" we report the MSE and MAE of each model relative to the MIDAS-AR benchmark. In the columns "DM" we report the p-value of the Diebold-Mariano test. The forecasts with CPI are evaluated on the sample 1994Q2-2015Q4, while those with PCEPI are evaluated on the sample 200Q3-2015Q4

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