Nonlinear Dynamic Factor Models with Interacting Level and Volatility

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Abstract

Volatility is an important ingredient in economic and financial decision making and yet the interaction between the levels and volatilities of macroeconomic and financial variables is not well understood. We propose a class of nonlinear dynamic factor models that has factor structures for both levels and volatilities. Both sets of latent factors are modeled jointly in an unrestricted vector autoregressive model. We develop a computationally convenient approximate filtering method for the estimation of all factors. The algorithm relies on numerical integration and can be implemented by augmenting the Kalman filter with weighted least squares regressions. The deterministic model parameters can be estimated by maximum likelihood. We derive the large sample properties of our method. The model is applied in two empirical studies. First, we consider euro area government bond yields between 2008 and 2012 and show that the volatility factor became an economically significant predictor of the yield levels in several countries. Bond purchases by the European Central Bank reduced yields but not the dispersion of pricing errors. Second, the model is applied for forecasting the levels of U.S. macroeconomic variables. We show that the inclusion of interacting volatility factors improves out-of-sample forecasts.

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1 Introduction

The presence of factor structures for the levels of macroeconomic and financial time series panels has been firmly established. Several recent studies have suggested similar structures for the volatilities of such panels.[1] It is natural to ask how the shocks that drive the level factors interact with the shocks that drive the volatility factors. For example, we may ask whether the volatility shocks have, potentially pervasive, influence over the level factors. And vice versa: whether the level shocks influence the volatility factors. We tackle this question by proposing a novel dynamic factor model where the level and volatility factors are jointly modeled in an unrestricted vector autoregressive model. The model is a natural extension of the dynamic factor model for levels.[2]

The observations of the nonlinear factor model are modeled by potentially non-Gaussian densities. The densities are parameterized by two signals: one for the level and one for the volatility. The deviation from normality is important for macroeconomic and financial panels where heavy tails and skewness are common features. The signals are modeled by factor structures to reflect the empirical regularity that levels and volatilities of different series tend to move together. The common level and volatility factors are modeled jointly in an unrestricted vector autoregressive model.

The dynamic factor model with interacting level and volatility factors requires the development of new methodology for the estimation of the factors and the model parameters. Two aspects are crucial. First, conventional methods for factor models such as principal components and maximum likelihood methods based on the Kalman filter cannot be adopted since the model is potentially non-Gaussian and the volatility factors enter the model non-linearly. Second, given that the level and volatility factors are contemporaneously and non-contemporaneously (lagged) correlated, standard Gibbs sampling routines such as those developed in Chib, Nardari and Shephard (2006) and more recently Kastner, Fruhwirth-Schnatter and Lopes (2017) also break down. In general, simulation based

[1] See for some recent examples Jurado, Ludvigson and Ng (2015), Barigozzi and Hallin (2016), Creal and Wu (2016) and Gorodnichenko and Ng (2017).

[2] Stock and Watson (2016) provide a recent review of the (structural) factor model literature.

methods, like particle filtering, Markov Chain Monte Carlo and importance sampling, are computationally expensive for the type of large dimensional panels that we consider in this paper. In addition, standard implementations may suffer from infinite variance problems, see Kohn and Scharth (2016) for a recent discussion.

Instead, we introduce a new approximate online filtering algorithm that is based on numerical integration methods. In this online efficient numerical integration (OENI) algorithm the infeasible true filtering density is replaced by an approximate filtering density from the Gaussian family where the mean and variance are chosen to minimize the divergence between the two densities. The latter criterion can be viewed as an online version of the efficient importance sampling criterion that is considered in Richard and Zhang (2007) and Koopman, Lucas and Scharth (2015). The minimum variance approximating density can be obtained by numerical integration methods that are implemented by weighted least squares regressions. As a by-product the algorithm produces an accurate approximation for the marginal likelihood that we use to obtain maximum likelihood estimates for the deterministic model parameters. We show that the approximation error is of order $\mathcal{O}_p(n^{-1})$ for all quantities of interest, where n is the cross-sectional dimension of the panel. The approximation error does not accumulate over time. The main theoretical take away from our approach is that for nonlinear panels that depend on a *small* number of common factors numerically efficient integration methods for the posterior densities are very accurate.

The proposed factor model and the accompanying estimation method can be adopted for structural analysis as well as forecasting purposes. Moreover, several new questions can be addressed using the model.

First, as a response to the problem of omitted variables in small structural vector autoregressive models it has become standard practice to include common factors from large macroeconomic panels in structural vector autoregressive models, see Bernanke, Boivin and Eliasz (2005) and Stock and Watson (2016). Our model extends this approach by also allowing the researcher to control for volatility factors from large panels. Second, the influence of shocks to volatility factors are of interest in their own right. Several existing identification approaches exists in the literature and can be adopted for our model, see Ramey (2016) and Carriero, Clark and Marcellino (2017). Third, the model facilitates testing the exogeneity of the variance shocks with respect to the mean shocks. Many recent studies allow for time-varying variances but assume that the shocks to the variance factor are uncorrelated with the mean shock, see for examples Koopman and Bos (2004) and Chib et al. (2006). In our framework a simple likelihood ratio test can asses the appropriateness of this assumption. A detailed comparison to existing models in the literature is given below.

Two empirical applications have formed the motivation for our extension of the dynamic factor model. First, a significant literature in finance explains government bond yields across different maturities and time using three common factors for the conditional mean, see e.g. Duffie and Kan (1996) and Dai and Singleton (2000). Such factors are often interpreted as level, slope, and curvature, see e.g. Diebold and Li (2006), Diebold, Rudebusch and Aruoba (2006), and Christensen, Diebold and Rudebusch (2011). Typically, common factors explain a large part of the cross-sectional variation in bond yields. During a liquidity crisis, however, substantial pricing errors can occur. Hu, Pan and Wang (2013) argue that pronounced deviations of observed yields from their model-implied counterparts are likely due to low market liquidity and an absence of arbitrage capital. The measurement error volatility factor can then be interpreted as a market (il)liquidity factor. In our empirical application we adopt the nonlinear dynamic factor model to study the interaction between the level and volatility factors for four panels of euro area government bond yields. We find that a shock to market illiquidity typically increased the level factor and reduced the (negative) slope factor. I.e., market illiquidity raised all yields, and longer-term yields by more than short-term yields. The observations that some government bond markets lacked "depth and liquidity" motivated the ECB to purchase certain government bonds within its Securities Markets Programme (SMP) between 2010 and 2012.[3] Our framework allows us to trace out the impact of SMP purchases on the term structure of bond yields while controlling

[3] We refer to Eser and Schwaab (2016), Ghysels, Idier, Manganelli and Vergote (2016) and Pooter, Martin and Pruitt (2017) for earlier work on the effects of the SMP.

for market illiquidity and other variables. We find that bond purchases for Italy and Spain substantially reduced yield levels but not pricing errors.

Second, recent interest in macroeconomics has turned to the influence of uncertainty on the levels of macroeconomic variables, see e.g. Bloom (2009) and Jurado et al. (2015). In our nonlinear factor model uncertainty is captured by the volatility factors. Uncertainty is allowed to freely interact with the levels of macroeconomic variables. By contrast, Jurado et al. (2015), Carriero et al. (2017), and Gorodnichenko and Ng (2017) need to impose a-priori restrictions on how the level and volatility factors can interact. Our framework facilitates explicit testing for the contemporaneous exogeneity of the volatility factors with respect to the level factors. In the empirical study we ask whether the volatility factors improve the prediction of the levels of macroeconomic variables. It is well known that the standard dynamic factor model for levels is hard to outperform, see Stock and Watson (2012). Nevertheless we find that adding the volatility factors reduce the mean-squared forecast errors.

The next section introduces our nonlinear dynamic factor model and clarifies how it relates to other factor models in the literature. Section 3 presents our estimation methodology. Sections 4 and 5 study euro area bond markets and the U.S. macroeconomy. Section 6 concludes and presents directions for further research.

2 Statistical model

We introduce the nonlinear dynamic factor model with interacting level and volatility factors for the $n \times 1$ observation vectors $y_t = (y_{1,t}, \ldots, y_{n,t})'$, where the index t refers to the time period. In total we have observations for $t = 1$ to T. Two sets of common factors are important in our work: (i) the r level factors $f_t = (f_{1,t}, \ldots, f_{r,t})'$ and (ii) the q log volatility factors $h_t = (h_{1,t}, \ldots, h_{q,t})'$. We summary the factors in the $(r + q) \times 1$ vector $\alpha_t = (f_t', h_t')'$.

The model for observations $y_{i,t}$ is summarized by

$$\begin{array}{llllllll} y_{i,t}|\alpha_t & \overset{i.d.}{\sim} & p(y_{i,t}|\mu_{i,t},\sigma^2_{i,t};\psi), & \mu_{i,t} & = & \lambda_i' f_t, & \sigma^2_{i,t} & = \exp(l_i' h_t), \\ \alpha_t & = & (\mathbf{I}_k - \mathbf{\Phi})\delta + \mathbf{\Phi}\alpha_{t-1} + \eta_t, & \eta_t & \overset{i.d.}{\sim} & N(0,\mathbf{\Sigma}), & & \end{array} \quad (1)$$

where $i.d.$ stands for independently distributed across i and t, and $p(y_{i,t}|\mu_{i,t},\sigma^2_{i,t};\psi)$ denotes a density with mean $\mu_{i,t}$ and variance $\sigma^2_{i,t}$. The inclusion of the parameter vector ψ allows for the density to depend on time-invariant shape parameters. Examples include parameters for skewness and tail shape. The common level factors f_t are mapped to the means $\mu_{i,t}$ by the $r \times 1$ loading vectors λ_i. The common log volatility factors h_t are mapped to the variances $\sigma^2_{i,t}$ using the $q \times 1$ loading vectors l_i and the exponential link function. The common level and volatility factors are collected in α_t and are jointly modeled as a vector autoregressive model of order one, with $(r+q) \times 1$ constant vector δ, $(r+q) \times (r+q)$ autoregressive matrix $\mathbf{\Phi}$ and $(r+q) \times 1$ vector of shocks η_t which has mean zero and positive definite variance matrix $\mathbf{\Sigma}$.

Important examples for the observation density $p(y_{i,t}|\mu_{i,t},\sigma^2_{i,t};\psi)$ include

$$y_{i,t}|\alpha_t \overset{i.d.}{\sim} N(\mu_{i,t},\sigma^2_{i,t}), \qquad y_{i,t}|\alpha_t \overset{i.d.}{\sim} t(\mu_{i,t},\sigma^2_{i,t},\nu), \qquad y_{i,t}|\alpha_t \overset{i.d.}{\sim} skt(\mu_{i,t},\sigma^2_{i,t},\xi,\nu), \quad (2)$$

where $N(\cdot,\cdot)$ denotes the normal density, $t(\cdot,\cdot,\nu)$ the student's t density with ν degrees of freedom and $skt(\cdot,\cdot,\xi,\nu)$ the skewed student's t density with skewness parameter ξ and degrees of freedom ν. Other densities could also be considered within our framework.

The key novelty of model (1) lies in its joint unrestricted treatment of the common level and log volatility factors. The specification explicitly allows for both contemporaneous and lagged interactions among the means and variances of the model.[4] Several recent studies – discussed below – argue that such interactions are important for forecasting and structural analysis. In a univariate setting the vector autoregressive model for the latent level and log

[4]We emphasize that the order of the autoregressive model is taken to be equal to one for notational convenience only. In principle any high-order VARMA(p,q) model could be considered for the common factors.

volatility was considered in Brandt and Kang (2004).[5] An independent volatility model, as in Koopman and Bos (2004) for example, is obtained when restricting $\mathbf{\Phi}$ and $\mathbf{\Sigma}$ to be block diagonal, see also Kastner et al. (2017) and references therein. In our framework no a-priori restrictions are placed on the dynamic interaction between the level and volatility factors. Volatility is thus fully endogenous and hypothesis tests can be conducted to asses the appropriateness of the exogenous volatility assumption.

Without further restrictions the parameters in the factor structures for $\mu_{i,t}$ and $\sigma^2_{i,t}$ are not identified. Several identification strategies exist. In many applications they can be formulated such that the factors have an economic interpretation, see Bai and Li (2012) for an overview of statistical identification strategies. For example, for the level factors, by selecting r vectors λ_i to form the r-dimensional identity matrix the corresponding r factors become so-called named factors and their interpretation is directly linked to the corresponding series $y_{i,t}$, see Stock and Watson (2016). Similar schemes can be considered for the log volatility factors, see Carriero et al. (2017) for an example.

At this stage we do not impose structural identification assumptions that map the reduced form shocks η_t to underlying structural shocks. In the literature several approaches exist for identifying structural shocks in vector autoregressive models, see Ramey (2016) for a recent review. In principle, any of such identification schemes could be adopted within our framework. The appropriateness depends on the application at hand.

In several empirical applications the volatility factors have important economic interpretation. We outline two of such applications and highlight how the interaction of the level and volatility factors improves inference. We return to these examples in our empirical studies below.

[5] To our knowledge this is the only paper that has considered a vector autoregressive model for the mean and variance. The difference is that their model is univariate and they rely on importance sampling methods for parameter estimation. Their method is infeasible in a multivariate setting.

2.1 Liquidity in government bond markets

Let y_t denote a vector of government bond yields with different maturities and f_t the factors that capture the level, slope and curvature of the term structure. We refer to Duffie and Kan (1996), Dai and Singleton (2000), Diebold and Li (2006), Diebold et al. (2006) and Christensen et al. (2011) as examples of term structure models that include these features. The factors – when mapped to the yields using the λ_i's – capture the model-implied value of the bonds. If arbitrage capital is abundant, then model-implied yields are likely to be close to the observed yields. During a liquidity crisis, however, market illiquidity becomes a problem and arbitrage capital dries up, see Hu et al. (2013). As a result there can be substantial deviations of observed yields from model-implied yields. It follows that the measurement error volatility factor can be interpreted as a measure of market illiquidity, see Hu et al. (2013).

Model (1) allows market liquidity, as measured by the volatility factors, to interact which the level, slope and curvature factors. This enables us to document how illiquidity interacts with the term structure of interest rates. Further, when augmenting the vector autoregressive model for the factors with additional macroeconomic, financial or monetary policy measurements it enables us to study the effects of such variables on the term structure while controlling for liquidity. Ang and Piazzesi (2003), Dewachter and Lyrio (2006), Diebold et al. (2006), Hordahl, Tristani and Vestin (2006) and Rudebusch and Wu (2008) study the impact of macroeconomic variables on the term structure, albeit without controlling for liquidity.

2.2 Macroeconomic uncertainty

A large literature in macroeconomics has developed methods for understanding how innovations to the levels of macroeconomic variables influence business cycle fluctuations. A comprehensive approach, that allow for the inclusion of many variables, is to use structural dynamic factor models, see Stock and Watson (2005), Bernanke et al. (2005) and Stock and Watson (2016). In such models the universe of macroeconomic variables is summarized by a small number of common level factors. The effects of different level shocks can then be traced out for many response variables.

Recent interest has broadened to also investigate the influence of volatility shocks on macroeconomic variables. In particular, studying the influence of volatility and its interaction with the levels has become an active research field, see e.g. Bloom (2009) and Jurado et al. (2015). The volatility factors can be interpreted, or labeled, as uncertainty factors when y_t is a vector that spans that space of macroeconomic activity, see Jurado et al. (2015). Recently, Barigozzi and Hallin (2016), Barigozzi and Hallin (2017) and Gorodnichenko and Ng (2017) proposed non-parametric methods to estimate level and volatility factors for large data panels.

In model (1), the volatility factors that capture macroeconomic uncertainty are allowed to interact freely with the level factors of economic activity. This is in contrast to Jurado et al. (2015) and Carriero et al. (2017) where the interaction is restricted in different ways. Our framework allows to investigate, pending appropriate identification assumptions, the influence of uncertainty shocks on economic activity without imposing a-priori statistical identification restrictions.[6] Moreover we can investigate the influence of other structural shocks while controlling for macroeconomic uncertainty and additionally use the uncertainty factors to improve out-of-sample forecasts.

2.3 Discussion

Model (1) is related to various existing models in the literature. This section highlights the main differences between existing modeling frameworks and our model (1).

As the examples in below (2) illustrate, a novelty is that we can use in principle any parametric density to have time-varying mean and variance.This allows for heavy-tailed and skewed distributions which are often found to be appropriate for macro-financial datasets.

[6]By statistical identification restriction we mean restrictions that are imposed to facilitate the estimation of the model parameters. Parameter estimation is often non-standard, see also the discussion in Gorodnichenko and Ng (2017). Such identification restrictions are not necessarily motivated by economic arguments.

In general, allowing for heavy-tailed distributions makes the estimates for the factors less sensitive to outliers. This is similar as what is found when comparing the standard stochastic volatility model with the student's t stochastic volatility model, see Harvey, Ruiz and Shephard (1994).

Within the class of stochastic volatility models a closely related model is the stochastic volatility in mean model, see Koopman and Sandmann (1998) for an initial contribution and Chan (2017) for an extension with time-varying parameters. In this model the mean is also determined by the volatility factor. Model (1) is different in at least three ways. First, model (1) allows for contemporaneous correlation between the shocks to the level and volatility factors. Second, lagged feedback is allowed among all level and volatility factors. Finally, model (1) allows for multiple mean and variance factors whereas the standard stochastic volatility in mean models are univariate.

Within the class of vector autoregressive models, various studies have proposed extensions with stochastic volatility, see Primiceri (2005) for the initial contribution. More recently, Carriero et al. (2017) consider a large vector autoregressive model that is augmented with stochastic volatility. They explicitly allow for lagged interaction between the mean and variance. However, contemporaneous correlation between the variance and conditional mean shocks is ruled out.

Closely related to our model are the factor models considered in Barigozzi and Hallin (2016), Barigozzi and Hallin (2017) and Gorodnichenko and Ng (2017). These studies develop non-parametric methods for the estimation of the common level and volatility factors. Our approach can be viewed as a likelihood-based approach where we explicitly model the interactions between the level and volatility factors. Creal and Wu (2016) consider a factor model where the volatility factors enter the model via the shocks to the common mean factors. In our model this is not the case as the volatility process is similar as in Jurado et al. (2015) and uncertainty is thought of as the volatility of the part of the model that is not explained by the common mean factors.

Finally, Fernandez-Villaverde and Rubio-Ramirez (2013) and Bloom, Floetotto, Jaimovich and Terry (2017) consider structural macroeconomic models where stochastic processes for the levels interact with stochastic processes for the variance. These macro models are quite far from our setting in terms of model specification, but similar estimation problems arise in these models as well.

All studies discussed here have in common that they impose some set of restrictions on the interaction between the mean and variance shocks. Model (1) does not impose such restrictions and explicitly aims to model the interactions.

3 Estimation

The estimation of the factors and parameters of model (1) is non-trivial as the model is potentially non-Gaussian and at least a subset of the latent factors – the volatility factors – enter the model non-linearly. This implies that standard filtering and smoothing methods based on the Kalman filter cannot be used. We show that when the cross-sectional dimension of the data $n \to \infty$ then an approximate conditional density based on online efficient numerical integration converges to the true conditional density of α_t.

To outline the general estimation problem let the deterministic model parameters of model (1) be included in θ. This parameter vector includes the shape parameters ψ as well as all other deterministic model parameters. The conditional filtered mean for functions of the common factors can be expressed as

$$\mathrm{E}_p(h(\alpha_t)|Y_t;\theta) = \int_{\alpha_t} h(\alpha_t)p(\alpha_t|Y_t;\theta)\mathrm{d}\alpha_t \tag{3}$$

where $h(\alpha_t)$ is some function of the factors and $Y_t = \{y_1,\ldots,y_t\}$. For example $h(\alpha_t) = \alpha_t$ gives the filtered mean of the factors. It follows that, in order to obtain conditional mean estimates, we must know the density $p(\alpha_t|Y_t;\theta)$. However, our model specification implies that this density has no closed form analytical expression and we must resort to either approximation methods or simulation methods to solve the integral. We emphasize that the presence of contemporaneous correlation between the common mean and variance factors rules out simulation methods that treat the mean factors conditional on the variance factors (and vice versa). This feature of our model excludes computationally convenient iterative approaches that were considered in for example Chib et al. (2006) and Carriero, Clark and Marcellino (2016). In general, simulation-based methods are computationally expensive for the type of large panels that we consider.

Instead, we develop a filtering algorithm that is based on online efficient numerical integration. The method is similar in spirit when compared to the online Gaussian Laplace approximation algorithm of Koyama, Castellanos, Shalizi and Kass (2010), but with the important difference that we compute the approximating filtering density using efficient numerical integration. This approximation is more accurate when compared to the Laplace approximation, see Richard and Zhang (2007), and has the additional benefits that it can be implemented by weighted least squares regressions and delivers a smooth marginal likelihood. The latter is important for the estimation of the deterministic model parameters θ, which can now be carried out using the maximum likelihood method.

3.1 Online Efficient Numerical Integration

Let $p(\alpha_t|Y_t;\theta)$, $p(\alpha_{t+1}|Y_t;\theta)$ and $p(\alpha_t|Y_T;\theta)$ denote the filtered, predictive and smoothed true posterior densities. Their efficient numerical integrated counterparts are denoted by $\hat{p}(\alpha_t|Y_t;\theta)$, $\hat{p}(\alpha_{t+1}|Y_t;\theta)$ and $\hat{p}(\alpha_t|Y_T;\theta)$. These approximating densities are chosen to belong to the Gaussian family. The means and variances of the predictive and filtered densities are denoted by $\hat{a}_{t|t-1} = \mathrm{E}_{\hat{p}}(\alpha_t|Y_{t-1};\theta)$, $\hat{\mathbf{V}}_{t|t-1} = \mathrm{Var}_{\hat{p}}(\alpha_t|Y_{t-1};\theta)$, $\hat{a}_{t|t} = \mathrm{E}_{\hat{p}}(\alpha_t|Y_t)$ and $\hat{\mathbf{V}}_{t|t} = \mathrm{Var}_{\hat{p}}(\alpha_t|Y_t)$. We aim to recursively obtain these estimates in a way such that the approximation errors relative to the true conditional densities are bounded and do not accumulate over time. Importantly, we do not use simulation methods such as importance sampling to correct for the approximation. The filtered and predictive estimates are obtained recursively as follows.

Algorithm 1: Online Efficient Numerical Integration (OENI)

(i) Initialize; set $t = 1$, $\hat{a}_{1|0} = \delta$ and $\hat{\mathbf{V}}_{1|0} = (\mathbf{I}_k - \mathbf{\Phi\Phi}')^{-1}\mathbf{\Sigma}$

(ii) Filter:

$$\begin{aligned}\hat{a}_{t|t} &= \hat{a}_{t|t-1} + \hat{\mathbf{V}}_{t|t-1}\mathbf{R}'(\mathbf{R}\hat{\mathbf{V}}_{t|t-1}\mathbf{R}' + \hat{\mathbf{C}}_t^{-1})^{-1}(\hat{\mathbf{C}}_t^{-1}\hat{b}_t - \mathbf{R}\hat{a}_{t|t-1})\\ \hat{\mathbf{V}}_{t|t} &= \hat{\mathbf{V}}_{t|t-1} - \hat{\mathbf{V}}_{t|t-1}\mathbf{R}'(\mathbf{R}\hat{\mathbf{V}}_{t|t-1}\mathbf{R}' + \hat{\mathbf{C}}_t^{-1})^{-1}\mathbf{R}\hat{\mathbf{V}}_{t|t-1}\end{aligned}$$

where $\mathbf{R} = \text{diag}(\mathbf{\Lambda}, \mathbf{L})$, with $\mathbf{\Lambda} = (\lambda_1, \ldots, \lambda_n)'$ and $\mathbf{L} = (l_1, \ldots, l_n)'$.[7] The estimated coefficient $\hat{\mathbf{C}}_t$ and $\hat{b}_t$ are determined as explained below.

(iii) Predict;

$$\begin{aligned}\hat{a}_{t+1|t} &= \mathbf{\Phi}\hat{a}_{t|t}\\ \hat{\mathbf{V}}_{t+1|t} &= \mathbf{\Phi}\hat{\mathbf{V}}_{t|t}\mathbf{\Phi}' + \mathbf{\Sigma}\end{aligned}$$

(iv) Increase $t = t + 1$ and go to step (ii)

The recursions above look similar to the standard Kalman filter recursions. The important difference is that the filtering step (ii) is based on efficient numerical integration. In particular the $2n \times 2n$ coefficient matrix $\hat{\mathbf{C}}_t$ and the $2n \times 1$ vector $\hat{b}_t$ are chosen such that the variance of $\log\left(p(y_t|\alpha_t;\theta)/\hat{p}(y_t|\alpha_t;\theta)\right)$ is minimized. Since the true density is independent across i we have $p(y_t|\alpha_t;\theta) = \prod_{i=1}^n p(y_{i,t}|z_{i,t};\theta)$, where $z_{i,t} = (\mu_{i,t}, \log\sigma_{i,t}^2)'$. This enables us to choose the approximating Gaussian density as follows

$$\hat{p}(y_t|\alpha_t;\theta) = \prod_{i=1}^n \hat{p}(y_{i,t}|z_{i,t};\theta), \qquad \hat{p}(y_{i,t}|z_{i,t};\theta) = \exp\left(a_{i,t} + b_{i,t}'z_{i,t} - \frac{1}{2}z_{i,t}'\mathbf{C}_{i,t}z_{i,t}\right), \tag{4}$$

where the 2×1 and 2×2 matrices $b_{i,t}$ and $\mathbf{C}_{i,t}$ contain the coefficients that minimize the variance of $\log\left(p(y_{i,t}|z_{i,t};\theta)/\hat{p}(y_{i,t}|z_{i,t};\theta)\right)$. Formally we obtain $b_{i,t}$ and $\mathbf{C}_{i,t}$ from

$$\{b_{i,t}, \mathbf{C}_{i,t}\} = \arg\min_{\tilde{b}_{i,t}, \tilde{\mathbf{C}}_{i,t}} \int_{z_{i,t}} \left[\log\left(p(y_{i,t}|z_{i,t};\theta)/\hat{p}(y_{i,t}|z_{i,t};\theta)\right)\right]^2 \hat{p}(z_{i,t}|Y_{t-1};\theta)\mathrm{d}z_{i,t}. \tag{5}$$

[7] Here $\text{diag}(\cdot,\cdot)$ denotes the diagonal concatenation, e.g. $\text{diag}(\mathbf{A}, \mathbf{B}) = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix}$.

The optimal coefficients $b_{i,t} = (b_{i,t,1}, b_{i,t,2})$ and $\mathbf{C}_{i,t} = [\mathbf{C}_{i,t,11}, \mathbf{C}_{i,t,12}; \mathbf{C}_{i,t,21}, \mathbf{C}_{i,t,22}]$ are collected in $b_t = (b_{1,t,1}, \ldots, b_{n,t,1}, b_{1,t,2}, \ldots, b_{n,t,2})'$ and

$$\mathbf{C}_t = \begin{bmatrix} \mathbf{C}_{t,11} & \mathbf{C}_{t,12} \\ \mathbf{C}_{t,21} & \mathbf{C}_{t,22} \end{bmatrix} \qquad \mathbf{C}_{t,lk} = \mathrm{diag}(\mathbf{C}_{1,t,lk}, \ldots, \mathbf{C}_{n,t,lk})$$

for $l, k = 1, 2$. Our approximation is based on the efficient criterion function (5) and can be viewed as an online version of the efficient importance sampling approach adopted in Richard and Zhang (2007) and Koopman et al. (2015). Since the log density $\log p(y_{i,t}|z_{i,t};\theta)$ is nonlinear in $z_{i,t}$ and potentially non-Gaussian the integral in (5) cannot be solved analytically. Instead we follow the exposition in Monahan (2001) and use Gaussian-quadrature for the evaluation of the integral, see also Koopman et al. (2015).

To outline the numerical integration procedure consider a set of abscissa $\{v_l\}_{l=1}^{L}$ with associated Gauss-Hermite weights $h(v_l)$ for $l = 1, \ldots, L$. The numerical implementation of the minimization (5) is given by

$$\{\hat{b}_{i,t}, \hat{\mathbf{C}}_{i,t}\} = \arg \min_{\tilde{b}_{i,t}, \tilde{\mathbf{C}}_{i,t}} \sum_{l,k=1}^{L} w_{l,k} \left[\log \left(p(y_{i,t}|z_{i,t}^{l,k};\theta) / \hat{p}(y_{i,t}|z_{i,t}^{l,k};\theta) \right) \right]^2 \tag{6}$$

with $w_{l,k} = h(v_l)h(v_k)\exp(\frac{1}{2}v_l^2)\exp(\frac{1}{2}v_k^2)$ and $z_{i,t}^{l,k} = \mathbf{R}_i \hat{a}_{t|t-1} + (\mathbf{R}_i \hat{\mathbf{V}}_{t|t-1} \mathbf{R}_i')^{1/2} v_{l,k}$, with $v_{l,k} = (v_l, v_k)'$ and $\mathbf{R}_i = \mathrm{diag}(\lambda_i', l_i')$. Notice that we do not integrate out $z_{i,t}$ directly but first rescale $z_{i,t}$ such that $\hat{p}(z_{i,t}|Y_{t-1};\theta)$ becomes equal to one. Since $\log \hat{p}(y_{i,t}|z_{i,t}^{l,k};\theta)$ is a linear function of the coefficients in $b_{i,t}$ and $\mathbf{C}_{i,t}$ the minimization problem in (10) is equal to a standard weighted least squares problem. The estimated coefficients $\hat{b}_{i,t}$ and $\hat{\mathbf{C}}_{i,t}$ replace $b_{i,t}$ and $\mathbf{C}_{i,t}$ in equation (4) and Algorithm 1 is based on this approximating density.

The smoothing density can be obtained by the usual backward recursions. For $t = n, \ldots, 1$ we may compute

$$\hat{p}(\alpha_t|Y_T;\theta) = \hat{p}(\alpha_t|Y_{t-1};\theta) \int_{\alpha_{t+1}} \frac{\hat{p}(\alpha_{t+1}|Y_T;\theta) p(\alpha_{t+1}|\alpha_t;\theta)}{\hat{p}(\alpha_{t+1}|Y_t;\theta)} \, \mathrm{d}\alpha_{t+1} \tag{7}$$

which is available entirely in closed form. The details are given in Durbin and Koopman (2012, Chapter 4) and for convenience we include them in the appendix.

3.2 Univariate implementation

In this section we show that there exists a univariate implementation of the online efficient numerical integration method. The main motivation for a univariate approach is similar to Koopman and Durbin (2000) who provide a univariate implementation for the Kalman filter for multivariate state space models. The benefit of the univariate approach is two-fold. First, similar as in Koopman and Durbin (2000), it is faster when compared to the standard implementation discussed in Algorithm 1. This follows as it avoids the inversion of matrices whose sizes are proportional to n. Second, it yields a convenient way of constructing the marginal likelihood using numerical integration. The marginal likelihood can, as explained in detail in the next section, be used to obtain maximum likelihood estimates for the parameters θ.

Let $\alpha_{i,t} = (f'_{i,t}, h'_{i,t})'$, where $f_{i,t}$ and $h_{i,t}$ have the same dimensions as f_t and h_t. We can write model (1) alternatively as follows

$$
\begin{aligned}
&y_{i,t}|\alpha_{i,t} \overset{i.d.}{\sim} p(y_{i,t}|\mu_{i,t}, \sigma^2_{i,t}; \psi), \qquad \mu_{i,t} = \lambda'_i f_{i,t}, \qquad \sigma^2_{i,t} = \exp(l'_i h_{i,t}), \\
&\alpha_{i,t} = \begin{cases} (\mathbf{I}_k - \mathbf{\Phi})\delta + \mathbf{\Phi}\alpha_{n,t-1} + \eta_t, \quad \eta_t \sim N(0, \mathbf{\Sigma}) & \text{if} \quad i = 1 \\ \alpha_{i-1,t} & \text{if} \quad i = 2, \dots, n \end{cases} .
\end{aligned} \tag{8}
$$

We are interested in the approximate filtering density $\hat{p}(\alpha_t|y_{i,t}, \dots, y_{1,t}, Y_{t-1}; \theta)$, where we also condition on the variables $y_{i,t}, \dots, y_{1,t}$. Instead of updating the factors every time period we now update the factors for every new (scalar) observation $y_{i,t}$ that comes in. Denote the predictive and filtered means and variance by $\hat{a}_{1,t|t-1} = \mathrm{E}_{\hat{p}}(\alpha_t|Y_{t-1}; \theta)$, $\hat{\mathbf{V}}_{1,t|t-1} = \mathrm{Var}_{\hat{p}}(\alpha_t|Y_{t-1}; \theta)$, $\hat{a}_{i,t|t} = \mathrm{E}_{\hat{p}}(\alpha_t|y_{i,t}, \dots, y_{1,t}, Y_{t-1}; \theta)$ and $\hat{\mathbf{V}}_{i,t|t} = \mathrm{Var}_{\hat{p}}(\alpha_t|y_{i,t}, \dots, y_{1,t}, Y_{t-1}; \theta)$. The online efficient numerical integration recursions are as follows.

Algorithm 2: Univariate Online Efficient Numerical Integration (U-OENI)

(i) Initialize; set $i = 1$ and $t = 1$, $\hat{a}_{1,1|0} = \delta$ and $\hat{\mathbf{V}}_{1,1|0} = (\mathbf{I}_k - \mathbf{\Phi}\mathbf{\Phi}')^{-1}\mathbf{\Sigma}$

(ii) Filter: for $i = 1, \ldots, n$

- if $i = 1$

$$\hat{a}_{i,t|t} = \hat{a}_{1,t|t-1} + \hat{\mathbf{V}}_{1,t|t-1}\mathbf{R}_i'(\mathbf{R}_i\hat{\mathbf{V}}_{1,t|t-1}\mathbf{R}_i' + \hat{\mathbf{C}}_{i,t}^{-1})^{-1}(\hat{\mathbf{C}}_{i,t}^{-1}\hat{b}_{i,t} - \mathbf{R}_i\hat{a}_{1,t|t-1})$$
$$\hat{\mathbf{V}}_{i,t|t} = \hat{\mathbf{V}}_{1,t|t-1} - \hat{\mathbf{V}}_{1,t|t-1}\mathbf{R}_i'(\mathbf{R}_i\hat{\mathbf{V}}_{1,t|t-1}\mathbf{R}_i' + \hat{\mathbf{C}}_{i,t}^{-1})^{-1}\mathbf{R}_i\hat{\mathbf{V}}_{1,t|t-1}$$

- if $i = 2, \ldots, n$

$$\hat{a}_{i,t|t} = \hat{a}_{i-1,t|t} + \hat{\mathbf{V}}_{i-1,t|t}\mathbf{R}_i'(\mathbf{R}_i\hat{\mathbf{V}}_{i-1,t|t}\mathbf{R}_i' + \hat{\mathbf{C}}_{i,t}^{-1})^{-1}(\hat{\mathbf{C}}_{i,t}^{-1}\hat{b}_{i,t} - \mathbf{R}_i\hat{a}_{i-1,t|t})$$
$$\hat{\mathbf{V}}_{i,t|t} = \hat{\mathbf{V}}_{i-1,t|t} - \hat{\mathbf{V}}_{i-1,t|t}\mathbf{R}_i'(\mathbf{R}_i\hat{\mathbf{V}}_{i-1,t|t}\mathbf{R}_i' + \hat{\mathbf{C}}_{i,t}^{-1})^{-1}\mathbf{R}_i\hat{\mathbf{V}}_{i-1,t|t}$$

where $\mathbf{R}_i = \text{diag}(\lambda_i', l_i')$. The estimated coefficients $\hat{\mathbf{C}}_{i,t}$ and $\hat{b}_{i,t}$ are determined as explained below.

(iii) Predict;

$$\begin{aligned}\hat{a}_{1,t+1|t} &= \mathbf{\Phi}\hat{a}_{n,t|t}\\ \hat{\mathbf{V}}_{1,t+1|t} &= \mathbf{\Phi}\hat{\mathbf{V}}_{n,t|t}\mathbf{\Phi}' + \mathbf{\Sigma}\end{aligned}$$

(iv) Increase $t = t + 1$ and go to step (ii)

The coefficients $\hat{b}_{i,t}$ and $\hat{\mathbf{C}}_{i,t}$ in the filtering step (ii) are obtained by a similar criterion as before. In particular,

$$\{b_{i,t}, \mathbf{C}_{i,t}\} = \arg\min_{\tilde{b}_{i,t},\tilde{\mathbf{C}}_{i,t}} \int_{z_{i,t}} \left[\log\left(p(y_{i,t}|z_{i,t};\theta)/\hat{p}(y_{i,t}|z_{i,t};\theta)\right)\right]^2 \hat{p}(z_{i,t}|Y_{t-1}, y_{i-1,t}, \ldots, y_{1,t};\theta)\text{d}z_{i,t}. \tag{9}$$

The integrating density is given by $\hat{p}(z_{i,t}|Y_{t-1};\theta) \sim N(\mathbf{R}_i\hat{a}_{1,t|t-1}, \mathbf{R}_i\hat{\mathbf{V}}_{1,t|t-1}\mathbf{R}_i')$ for $i = 1$ and by $\hat{p}(z_{i,t}|Y_{t-1}, y_{i-1,t}, \ldots, y_{1,t};\theta) \sim N(\mathbf{R}_i\hat{a}_{i-1,t|t}, \mathbf{R}_i\hat{\mathbf{V}}_{i-1,t|t}\mathbf{R}_i')$ for $i = 2, \ldots, n$. The integral in (9) cannot be computed analytically. We therefore resort again to numerical integration using Gaussian quadrature.

The numerical implementation of the minimization (9) is given by

$$\{\hat{b}_{i,t}, \hat{\mathbf{C}}_{i,t}\} = \arg \min_{\tilde{b}_{i,t}, \tilde{\mathbf{C}}_{i,t}} \sum_{l,k=1}^{L} w_{l,k} \left[\log p(y_{i,t}|z_{i,t}^{l,k};\theta)/\hat{p}(y_{i,t}|z_{i,t}^{l,k};\theta) \right]^2 \tag{10}$$

with $w_{l,k} = h(v_l)h(v_k)\exp(\frac{1}{2}v_l^2)\exp(\frac{1}{2}v_k^2)$. For $i = 1$ we have $z_{i,t}^{l,k} = \mathbf{R}_i\hat{a}_{1,t|t-1} + (\mathbf{R}_i\hat{\mathbf{V}}_{1,t|t-1}\mathbf{R}_i')^{1/2}v_{l,k}$, with $v_{l,k} = (v_l, v_k)'$. For $i = 2, \ldots, n$ we have $z_{i,t}^{l,k} = \mathbf{R}_i\hat{a}_{i-1,t|t-1} + (\mathbf{R}_i\hat{\mathbf{V}}_{i-1,t|t-1}\mathbf{R}_i')^{1/2}v_{l,k}$.

3.3 Likelihood evaluation

We adopt the maximum likelihood method for the estimation of the time-invariant parameters θ. This is a common estimation approach for dynamic factor models when the levels of the data are driven by common factors, see e.g. Doz, Giannone and Reichlin (2012), Bai and Li (2012), Jungbacker and Koopman (2014), Banbura and Modugno (2014) and Bai and Li (2016). We show that the output of the univariate online numerical integration algorithm can be used to obtain an efficient approximation of the marginal likelihood.

In particular, we approximate the marginal likelihood by

$$\begin{aligned} \hat{p}(y;\theta) &= \prod_{t=1}^{T}\prod_{i=1}^{n} p(y_{i,t}|y_{i-1,t},\ldots,y_{1,t},Y_{t-1};\theta) \\ &= \prod_{t=1}^{T}\prod_{i=1}^{n} \int_{z_{i,t}} p(y_{i,t}|z_{i,t};\theta)\hat{p}(z_{i,t}|y_{i-1,t},\ldots,y_{1,t},Y_{t-1};\theta)\ \mathrm{d}z_{i,t} \end{aligned} \tag{11}$$

where both $p(y_{i,t}|z_{i,t};\theta)$ and $\hat{p}(z_{i,t}|Y_{t-1}, y_{i-1,t}, \ldots, y_{1,t};\theta)$ are known, but since $p(y_{i,t}|z_{i,t};\theta)$ is non-Gaussian and nonlinear with respect to the variance factors we cannot compute this integral in closed form. Instead, we notice that since the integral is low (two)-dimensional we can use numerical integration to obtain an arbitrarily accurate estimate. In particular,

$$\hat{\hat{p}}(y;\theta) = \prod_{t=1}^{T}\prod_{i=1}^{n}\sum_{l,k=1}^{L} w_{l,k}p(y_{i,t}|z_{i,t}^{l,k};\theta) \tag{12}$$

with $z_{i,t}^{l,k} = \mathbf{R}_i\hat{a}_{1,t|t-1} + (\mathbf{R}_i\hat{\mathbf{V}}_{1,t|t-1}\mathbf{R}_i')^{1/2}v_{l,k}$ and $v_{l,k} = (v_l, v_k)'$ for $i = 1$. For $i = 2, \ldots, n$ we have $z_{i,t}^{l,k} = \mathbf{R}_i\hat{a}_{i-1,t|t-1} + (\mathbf{R}_i\hat{\mathbf{V}}_{i-1,t|t-1}\mathbf{R}_i')^{1/2}v_{l,k}$.

3.4 Accuracy of Online Efficient Numerical Integration

This section studies the regularity conditions that ensure that the approximating densities and the marginal likelihood are close to their population counterparts. The next theorem establishes that the estimated conditional mean and variance for α_t are close to their population counterparts based on model (1).

Theorem 1. *Let y_t be generated by model (1) and assume that $p(y_t|\alpha_t;\theta)$ is five times continuously differentiable, $\frac{1}{n}\sum_{i=1}^{n}\lambda_i\lambda_i' \rightarrow \mathbf{D}_\lambda$ with $\mathbf{D}_\lambda$ of full rank, and $\frac{1}{n}\sum_{i=1}^{n} l_i l_i' \rightarrow \mathbf{D}_l$ with $\mathbf{D}_l$ of full rank. We have that*

(i) $\hat{a}_{t|t} = \mathrm{E}_p(\alpha_t|Y_t;\theta) + \mathcal{O}_p\left(\frac{1}{n}\right)$

(ii) $\hat{a}_{t+1|t} = \mathrm{E}_p(\alpha_{t+1}|Y_t;\theta) + \mathcal{O}_p\left(\frac{1}{n}\right)$

(iii) $\hat{V}_{t|t} = \mathrm{Var}_p(\alpha_t|Y_t;\theta) + \mathcal{O}_p\left(\frac{1}{n}\right)$

(iv) $\hat{V}_{t+1|t} = \mathrm{Var}_p(\alpha_{t+1}|Y_t;\theta) + \mathcal{O}_p\left(\frac{1}{n}\right)$

where $\hat{a}_{t|t}$, $\hat{a}_{t+1|t}$, $\hat{V}_{t|t}$ and $\hat{V}_{t+1|t}$ are computed as in Algorithm 1.

The theorem implies that the errors due to the efficient numerically integrated Gaussian density approximation vanish as n grows large. In addition, the errors do not accumulate over time. The theorem takes model (1) as correctly specified. It would be of interest to study the behavior of the approximate filter for misspecified models. We leave this to future work. Theorem 1 applies equally for the univariate implementation since Algorithm 2 computes the same quantities as Algorithm 1, e.g. $\hat{a}_{t|t} \equiv \hat{a}_{n,t|t}$, $\hat{a}_{t+1|t} \equiv \hat{a}_{1,t|t}$, $\hat{V}_{t|t} \equiv \hat{V}_{n,t|t}$ and $\hat{V}_{t+1|t} \equiv \hat{V}_{1,t|t}$.

4 Euro area sovereign bond yields

This section applies our model (1) to vectors y_t of euro area sovereign bond yields. We consider bonds issued by the four largest euro area countries: Germany, France, Italy and

Spain. We first study the interaction among the mean and variance factors. The interpretation of the variance factor as an (il)liquidity factor allows us to study how market illiquidity interacts with the term structure factors. We then use the model to study the effects of ECB bond purchases on both the term structure and market liquidity. For this we augment the process for the factors α_t with additional variables as in Diebold et al. (2006). The application illustrates how model (1) can be adopted to conduct a relevant structural analysis.

4.1 Yield and SMP data

We obtain zero-coupon government bond yields from Thomson/Reuters. The sample starts on 1 October 2008 after the collapse of Lehman brothers and ends before the allotment of the ECB's first very long term refinancing operation (VLTRO) on 21 December 2012, see Eser and Schwaab (2016). We consider the 12, 24, 36, 48, 60, 72, 84, 96, 108, and 120 month maturity sampled at the daily frequency. This yields $T = 840$ for $n = 10$ yields.

Figure 1 plots the yield data for our four countries. Table 1 provides descriptive statistics for the yields. All yield curves are upward sloping on average. The unconditional dispersion of yields decreases with maturity. For Italy and Spain, a brief period of yield curve inversion is visible during the euro area sovereign debt crisis in late 2011. Italy and Spain were relatively more affected by the crisis than France and Germany. The latter may even have benefited from flight-to-safety effects during the crisis, see e.g. ECB (2014).

For our structural analysis we consider data on SMP bond purchases by country at a daily frequency. Bond purchases are entered at par values. Assets were purchased in over-the-counter dealer markets via non-anonymous trades. As a result, market participants quickly learned that SMP-related trades were taking place on intervention days. Figure 2 plots weekly total purchases across all five countries as well as their accumulated book value over time. Noticeably, the daily purchase data are unevenly spread over time. The largest purchases occurred after the introduction of the SMP on 10 May 2010 and after its cross-sectional expansion to include Italy and Spain on 8 August 2011. No German and French bonds were purchased within the SMP.

4.2 Interaction of term structure and illiquidity factors

We consider model (1) with a Student's t density for the bond yields. This implies that ψ now includes a (common) degrees of freedom parameter that captures the thickness of the tails of the observation densities. We consider a model with $r = 3$ level factors and $q = 1$ volatility factor. The choice of $r = 3$ is standard in the literature, see e.g. Diebold and Li (2006) and Diebold et al. (2006). The choice of $q = 1$ is motivated by a preliminary data analysis. To facilitate the interpretation of the mean factors as level, slope and curvature factors we adopt the Nelson-Siegel structure for the loadings of the level factors. The ith loading vector is therefore given by

$$\lambda_i = \left(1, \frac{1 - e^{-\kappa\tau_i}}{\kappa\tau_i}, \frac{1 - e^{-\kappa\tau_i}}{\kappa\tau_i} - e^{-\kappa\tau_i}\right),$$

where τ_i is the maturity of the ith yield.

The decay parameter κ is estimated along with the other time-invariant parameters by maximum likelihood as outlined in Section 3. We implement the filtering algorithm using 20 weights for the numerical integration procedure and for the evaluation of the marginal likelihood. The standard errors of the parameters are computed by inverting the negative Hessian evaluated at the maximum likelihood estimates.

The estimated conditional mean and volatility factors are plotted in Figure 3. The conditional mean factors are approximately similar across countries until mid-2010, and diverge considerably afterwards. There are pronounced differences in the estimated volatility factors as well. Estimated volatility is typically low for France and Germany during the entire sampling period. By contrast, the volatility estimates for Italy and Spain fluctuate heavily. For Italy, volatility doubles in the course of the sovereign debt crises.

Figure 4 plots the impulse responses that are obtained from a one standard deviation shock to the volatility factor. This allows us to study the interaction between the level and volatility factors. The impulse response is identified by a recursive scheme where the volatility factor is ordered last. This means that the level, slope and curvature factors do not respond contemporaneously to a volatility shock. We find, consistently across countries, that volatility increases the level factor and reduces the (negative) slope factor. The effect of the volatility shock on the curvature factor is mixed and is typically not significantly different from zero. The impulse responses suggest that market illiquidity was a main determinant of euro area sovereign bond yields at all maturities, and tended to increase long-term yields more than short-term yields. The observation that some government bond markets lacked "depth and liquidity" motivated the ECB to undertake its Securities Markets Program between 2010 and 2012. This is where we turn next.

4.3 Impact of ECB asset purchases

This section studies the effect of ECB bond purchases within the SMP on the term structure of sovereign bond yields as well as on the associated liquidity factors. To this end we follow Diebold et al. (2006) and augment the factor process with additional conditioning variables. Let p_t denote the amount of bond purchases in €billion for either Italy or Spain on day t. In addition, we denote by x_t a 2×1 vector that contains additional variables. Vector x_t contains a country-specific credit default swap (CDS) spread as well as the euro area three-month overnight index swap (OIS) rate. We include the CDS spread to capture country-specific default risk perceptions during the sovereign debt crisis. The OIS rate summarizes the monetary policy stance as e.g. implied by other conventional and unconventional monetary policies at the time. The state equation becomes

$$\alpha_t = (p_t, f_t', h_t', x_t')', \qquad \alpha_t = (\mathbf{I} - \mathbf{\Phi})\delta + \mathbf{\Phi}\alpha_{t-1} + \eta_t, \qquad \eta_t \sim N(0, \mathbf{\Sigma}), \tag{13}$$

where the interpretation of the time-invariant coefficients is unchanged.

We note that SMP purchases are ordered first in equation (13). We also order these purchases first in a Choleski decomposition to identify the VAR structural shocks. Proceeding in this way implicitly assumes that the purchase amounts are *predetermined* with respect to the yields and the other variables. Eser and Schwaab (2016) argue that substantial coordination within the Eurosystem required the intervention decisions and purchase amounts to be predetermined at the daily frequency. Such coordination was required since it was in practice not the ECB as a single institution, but the whole Eurosystem, consisting of the ECB and the (then) 17 NCBs, which jointly implemented the SMP and undertook the purchases. As a large number of institutions were involved, a strategy for the day had been, as a rule, discussed and fixed before markets open. The strategies were generally not conditional on yield developments during the upcoming trading day. We refer to Eser and Schwaab (2016) for the institutional detail.

While purchases had been fixed before markets opened, purchases were still determined against the backdrop of an escalating sovereign debt crisis. As a result, purchases were only observed during an intense crisis, whose symptoms included high CDS spreads, pronounced market illiquidity, and volatile yields. Each of these elements is controlled for in our VAR specification.

Figure 5 plots the impulse response functions of a €1 bn shock to bond purchases on the level and volatility factors. Figure 6 maps these impulse responses to the yield curve at different day-ahead horizons. We find that bond purchases lowered yields across the entire maturity spectrum. Yields decreased slightly more at the medium-to-long end than the short end. The impact estimates of -2 bps per €1 bn of purchases is in the ballpark of the estimates reported in the literature.[8] In addition, Figure 6 suggests that the initial impact (solid black line) is fairly persistent, and still present and economically significant a few weeks (dotted lines) after an intervention.

While purchases lowered yield levels, Figure 5 also suggests that purchases did not lower the dispersion of pricing errors. We conjecture that this finding is related to the communication strategy adopted by the ECB at the time. On the two key announcement dates

[8]E.g., Eser and Schwaab (2016) report an initial impact estimate of -1.4 bps per €1 bn of purchases for Italy and -5.8 bps per €1 bn of purchases for Spain.

for the SMP, the ECB announced that it would undertake bond purchases and what their objective would be, but did not disclose the total amounts that would be spent, nor a time frame over which the program would be active, nor which countries or set of securities would be targeted. While reducing risk premia overall, the program therefore also likely added to pricing uncertainty in the cross section of the bonds.

5 Uncertainty in macroeconomics

5.1 Data and preliminary analysis

We consider a panel of quarterly macroeconomic and financial time series. The series span a wide selection of macroeconomic and financial variables and is similar as in Carriero et al. (2017). The series are summarized in Table 2. In total we have $n = 29$ series. The sampling period is from 1959-Q1 until 2017-Q2 and spans a total of $T = 234$ time periods. Each series is transformed to ensure stationarity and standardized to have mean zero and unit variance.

A preliminary factor analysis – summarized in Figure 7 – reveals the following. The eigenvalue ratio estimator of Ahn and Horenstein (2013) indicates that two common mean factors are appropriate for this panel. [9] To get an indication for the number of log variance factors we square the principal components residuals (obtained using the first two principal components) and apply the log transformation. Next, we apply the eigenvalue ratio estimator of Ahn and Horenstein (2013) to this panel of volatility proxies, see also Barigozzi and Hallin (2016) and Barigozzi and Hallin (2017). The eigenvalues and their ratios for this panel are shown in the bottom row of Figure 7. The ratio estimate indicates that 1 common factor is present. Admittedly, for the variance factors the evidence is less strong and a panel with $q = 2$ common log variance factors can also be considered.

In Table 2 we show the R^2's of the regression of the series on the two principal components for the mean and the R^2's of the regression of the volatility proxies on the first

[9] This is confirmed by the edge-estimator of Onatski (2010) and several Bai and Ng (2002) information criteria.

principal component for the panel of volatility proxies. We find that the principal components explain approximately 45% of the mean variance and about 16% of the variation of the volatility proxies. Financial series and series for real economic activity are explained quite well, whereas series related to prices are hard to explain.

5.2 Interaction level factors and uncertainty

Next, we estimate the parameters of model (1) using the online efficient numerical integration approach. The estimated factors that are computed based on the maximum likelihood estimates for the deterministic parameters are shown in Figure 8. For the level factors we find that the two factors have very different dynamics. The first level factor evolves slowly over time and can be viewed as a business cycle factor.[10] The factor reaches its lowest point around 2009 and the slow recovery from the crisis is clearly visible. It loads most heavily on the real-variables such as housing starts and permits, industrial production and income. The second level factor is noisy and corresponds mainly to the financial variables such as the industry portfolios and the excess returns.

The volatility factor is also pro cyclical and peaks during recessions. The largest peak is for the recent financial crisis but also the period after 2000 shows a longer period of increased uncertainty. For comparison purposes we also show the uncertainty factor from Jurado et al. (2015) in the same plot. The pattern of this factor is similar which confirms the interpretation of this factor as capturing uncertainty.

To investigate the interaction among the level and volatility factors we compute the impulse responses of a shock to the volatility factors on the level factors. The impulse responses are shown in Figure 9. We find no significant responds for the first level factor which mainly reflects the movements of the real variables in the panel. The response on the second level factor, which corresponds mainly with the financial variables, is significant and negative for about 8 quarters.

[10] The autoregressive coefficient associated with this series is high (around 0.9).

5.3 Forecasting using uncertainty factors

Next, we discuss the results from an out-of-sample forecasting study. The set-up is as follows. We consider model (1) for different combinations of level and volatility factors. For each model we forecast $h = 1, 2, 3$ and 4 quarters ahead using a recursive window that starts from 1989-Q4. Hence the first forecast is for 1990-Q1 and the last forecast is constructed for 2017-Q2. In total we have 113 out-of-sample forecasts for each model. We compute the mean squared error for each forecast and show in Table 3 the averages over all forecasts.

Results to be added

6 Conclusions

In this paper we have introduced a nonlinear dynamic factor model for panels of observations that admitted a factor structure for both the levels and volatilities. A key novelty in our framework was the interaction between the common level and volatility factors via an unrestricted vector autoregressive model. To facilitate the estimation of the common factors we developed an approximate online filtering algorithm. The algorithm relied on replacing the infeasible filtering density with a Gaussian density that was chosen to minimize the variance between the two densities. The properties on the filter are attractive for large panels. The model is applied to panels of government bond yields and a panel of macroeconomic variables. We illustrated its usefulness for structural analysis and forecasting.

References

Ahn, S. C. and Horenstein, A. R.: 2013, Eigenvalue Ratio Test for the Number of Factors, *Econometrica* **80**, 1203–1227.

Ang, A. and Piazzesi, M.: 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics* **50**, 745–787.

Bai, J. and Li, K.: 2012, Statistical Analysis of Factor Models of High Dimension, *The Annals of Statistics* **40**, 436–465.

Bai, J. and Li, K.: 2016, Maximum Likelihood Estimation and Inference for Approximate Factor Models of High Dimension, *The Review of Economics and Statistics* **98**, 298–309.

Bai, J. and Ng, S.: 2002, Determining the Number of Factors in Approximate Factor Models, *Econometrica* **70**, 191–221.

Banbura, M. and Modugno, M.: 2014, Maximum Likelihood Estimation of Factor Models on Datasets with Arbitrary Pattern of Missing Data, *Journal of Applied Econometrics* **29**, 133–160.

Barigozzi, M. and Hallin, M.: 2016, Generalized dynamic factors models and volatilities: recovering the market volatility shocks, *Econometrics Journal* **19**, 33–60.

Barigozzi, M. and Hallin, M.: 2017, Generalized dynamic factors models and volatilities estimation and forecasting, *Journal of Econometrics* . forthcoming.

Bernanke, B. S., Boivin, J. and Eliasz, P.: 2005, Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach, *Quarterly Journal of Economics* **120**, 387–422.

Bloom, N.: 2009, The impact of uncertainty shocks, *Econometrica* **77**, 623–685.

Bloom, N., Floetotto, M., Jaimovich, N. and Terry, S.: 2017, Really uncertain business cycles. Working paper: available from https://people.stanford.edu/nbloom/research.

Brandt, M. W. and Kang, Q.: 2004, On the relationship between the conditional mean and volatility of stock returns: A latent VAR approach, *Journal of Financial Economics* **72**, 217–257.

Carriero, A., Clark, T. and Marcellino, M.: 2016, Common Drifting Volatility in Large Bayesian VARs, *Journal of Business and Economics Statistics* **34**, 375–390. forthcoming.

Carriero, A., Clark, T. and Marcellino, M.: 2017, Measuring uncertainty and its impact on the economy, *Review of Economics and Statistics* . forthcoming.

Chan, J.: 2017, The Stochastic Volatility in Mean Model with Time-Varying Parameters: An Application to Inflation Modeling , *Journal of Business and Economic Statistics* **35**, 17–28.

Chib, S., Nardari, F. and Shephard, N.: 2006, Analysis of high dimensional multivariate stochastic volatility models, *Journal of Econometrics* **134**, 341371.

Christensen, J. H. E., Diebold, F. X. and Rudebusch, G. D.: 2011, The affine arbitrage-free class of Nelson-Siegel term structure models, *Journal of Econometrics* **164**, 4–20.

Creal, D. and Wu, C.: 2016, Monetary policy uncertainty and economic fluctuations, *International Economic Review* . forthcoming.

Dai, Q. and Singleton, K.: 2000, Specification analysis of affine term structure models, *Journal of Finance* **55**, 1943–1978.

Dewachter, H. and Lyrio, M.: 2006, Macro Factors and the Term Structure of Interest Rates, *Journal of Money, Credit and Banking* **38**, 119–140.

Diebold, F. X. and Li, C.: 2006, Forecasting the Term Structure of Government Bond Yields, *Journal of Econometrics* **130**, 337–364.

Diebold, F. X., Rudebusch, G. D. and Aruoba, B.: 2006, The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach, *Journal of Econometrics* **131**, 309–338.

Doz, C., Giannone, D. and Reichlin, C.: 2012, A Quasi Maximum Likelihood Approach for Large Approximate Dynamic Factor Models, *Review of Economics and Statistics* **94**, 1014–1024.

Duffie, D. and Kan, R.: 1996, A yield-factor model of interest rates, *Mathematical Finance* **6**, 379–406.

Durbin, J. and Koopman, S. J.: 2012, *Time Series Analysis by State Space Methods*, Oxford University Press, Oxford.

ECB: 2014, The determinants of euro area sovereign bond yield spreads during the crisis, ECB Monthly Bulletin article, May 2014.

Eser, F. and Schwaab, B.: 2016, The yield impact of central bank asset purchases: the case of the ECB's Securities Markets Programme, *Journal of Financial Economics* **119**(1), 147–167.

Fernandez-Villaverde, J. and Rubio-Ramirez, J. F.: 2013, Macroeconomics and volatility: Data, models, and methods, *Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Economet- ric Society*, Cambridge University Press, Cambridge.

Ghysels, E., Idier, J., Manganelli, S. and Vergote, O.: 2016, A high frequency assessment of the ECB Securities Markets Programme, *Journal of the European Economic Association* . forthcoming.

Gorodnichenko, Y. and Ng, S.: 2017, Level and volatility factors in macroeconomic data, *Journal of Monetary Economics* . forthcoming.

Harvey, A., Ruiz, E. and Shephard, N.: 1994, Multivariate stochastic variance models, *The Review of Economic Studies* **61**, 247–264.

Hordahl, P., Tristani, O. and Vestin, D.: 2006, A joint econometric model of macroeconomic and term structure dynamics, *Journal of Econometrics* **131**, 405–444.

Hu, G. X., Pan, J. and Wang, J.: 2013, Noise as information for illiquidity, *Journal of Finance* **68**, 2341–2382.

Jungbacker, B. and Koopman, S. J.: 2014, Likelihood-based Analysis for Dynamic Factor Analysis for Measurement and Forecasting, *Econometrics Journal* **17**, forthcoming.

Jurado, K., Ludvigson, S. C. and Ng, S.: 2015, Measuring uncertainty, *American Economic Review* **105**, 1117–1216.

Kastner, G., Fruhwirth-Schnatter, S. and Lopes, H. F.: 2017, Efficient bayesian inference for multivariate factor stochastic volatility models, *Journal of Computational and Graphical Statistics* **0**(0), 1–13.

Kohn, R. and Scharth, M.: 2016, Particle Efficient Importance Sampling, *Journal of Econometrics* **190**, 133–147.

Koopman, S. J. and Bos, C. S.: 2004, State space models with a common stochastic variance, *Journal of Business and Economic Statistics* **22**, 346–357.

Koopman, S. J. and Durbin, J.: 2000, Fast filtering and smoothing for multivariate state space models, *Journal of Time Series Analysis* **21**, 281–296.

Koopman, S. J., Lucas, A. and Scharth, M.: 2015, Numerically Accelerated Importance Sampling for Nonlinear Non-Gaussian State Space Models, *Journal of Business and Economic Statistics* **33**, 114–127.

Koopman, S. J. and Sandmann, G.: 1998, Estimation of stochastic volatility models via monte carlo maximum likelihood, *Journal of Econometrics* **87**, 271–301.

Koyama, S., Castellanos, L., Shalizi, C. R. and Kass, R. E.: 2010, Approximate Methods for State-Space Models, *Journal of the American Statistical Association* **105**, 170–180.

Monahan, J. F.: 2001, *Numerical Methods of Statistics*, Cambridge University Press, Cambridge.

Onatski, A.: 2010, Determining the Number of Factors from Empirical Distribution of Eigenvalues, *Review of Economics and Statistics* **92**, 1004–1016.

Pooter, M. D., Martin, R. F. and Pruitt, S.: 2017, The liquidity effects of official bond market intervention, *Journal of Financial and Quantitative Analysis, forthcoming* .

Primiceri, G. E.: 2005, Time varying structural vector autoregressions and monetary policy, *The Review of Economic Studies* **72**, 821–852.

Ramey, V.: 2016, Macroeconomic Shocks and Their Propagation, *in* J. B. Taylor and H. Uhlig (eds), *Handbook of Macroeconomics*, Elsevier, Amsterdam, North Holland.

Richard, J. F. and Zhang, W.: 2007, Efficient High-Dimensional Importance Sampling, *Journal of Econometrics* **141**, 1385–1411.

Rudebusch, G. D. and Wu, T.: 2008, A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy, *The Economic Journal* **118**, 906–926.

Stock, J. H. and Watson, M. W.: 2005, Implications of dynamic factor models for var analysis. Working paper: available from www.princeton.edu/ mwatson/.

Stock, J. H. and Watson, M. W.: 2012, Generalized Shrinkage Methods for Forecasting Using Many Predictors, *Journal of Business and Economic Statistics* pp. 481–493.

Stock, J. H. and Watson, M. W.: 2016, Factor Models and Structural Vector Autoregressions in Macroeconomics, *in* J. B. Taylor and H. Uhlig (eds), *Handbook of Macroeconomics*, Elsevier, Amsterdam, North Holland.

Appendix

6.1 Proof of Theorem 1

To be added

6.2 Smoothing

Let $\hat{a}_{t|T} = \mathrm{E}_{\hat{p}}(\alpha_t|Y_T;\theta)$ and $\hat{V}_{t|T} = \mathrm{Var}_{\hat{p}}(\alpha_t|Y_T;\theta)$. These quantities can be computed by the backward recursions based on the output of Algorithm 1.

Algorithm 3: Smoothing recursions

(i) Initialize; set $t = T$, $\hat{\mathbf{N}}_T = 0$ and $\hat{r}_T = 0$

(ii) Smooth:

$$
\begin{aligned}
r_{t-1} &= \mathbf{R}'(\mathbf{R}\hat{\mathbf{V}}_{t|t-1}\mathbf{R}' + \hat{\mathbf{C}}_t^{-1})^{-1}(\hat{\mathbf{C}}_t^{-1}\hat{b}_t - \mathbf{R}\hat{a}_{t|t-1}) + \mathbf{L}_t' r_t \\
\mathbf{N}_{t-1} &= \mathbf{R}'(\mathbf{R}\hat{\mathbf{V}}_{t|t-1}\mathbf{R}' + \hat{\mathbf{C}}_t^{-1})^{-1}\mathbf{R} + \mathbf{L}_t'\mathbf{N}_t\mathbf{L}_t \\
\hat{a}_{t|T} &= \hat{a}_{t|t-1} + \hat{\mathbf{V}}_{t|t-1} r_{t-1} \\
\hat{\mathbf{V}}_{t|T} &= \hat{\mathbf{V}}_{t|t-1} - \hat{\mathbf{V}}_{t|t-1}\mathbf{N}_{t-1}\hat{\mathbf{V}}_{t|t-1}
\end{aligned}
$$

where $\mathbf{R} = \mathrm{diag}(\mathbf{\Lambda}, \mathbf{L})$, with $\mathbf{\Lambda} = (\lambda_1, \ldots, \lambda_n)'$ and $\mathbf{L} = (l_1, \ldots, l_n)'$, and $\mathbf{L}_t = \mathbf{I} - \mathbf{\Phi}\hat{\mathbf{V}}_{t|t-1}\mathbf{R}'(\mathbf{R}\hat{\mathbf{V}}_{t|t-1}\mathbf{R}' + \hat{\mathbf{C}}_t^{-1})^{-1}$.

(iii) Decrease $t = t - 1$ and go to step *(ii)*

We refer to Durbin and Koopman (2012, Chapter 4) for the derivation. The univariate implementation for the smoothing recursions is as follows.

Algorithm 4: Smoothing recursions (univariate)

(i) Initialize; set $t = T$, $\hat{\mathbf{N}}_{n,T} = 0$ and $\hat{r}_{n,T} = 0$

(ii) Smooth:

- for $i = n, \ldots, 1$

$$\begin{aligned}
r_{i-1,t} &= \mathbf{R}_i'(\mathbf{R}_i\hat{\mathbf{V}}_{i-1,t|t}\mathbf{R}_i' + \hat{\mathbf{C}}_{i,t}^{-1})^{-1}(\hat{\mathbf{C}}_{i,t}^{-1}\hat{b}_{i,t} - \mathbf{R}_i\hat{a}_{i-1,t|t}) + \mathbf{L}_{i,t}'r_{i,t} \\
\mathbf{N}_{i-1,t} &= \mathbf{R}_i'(\mathbf{R}_i\hat{\mathbf{V}}_{i-1,t|t}\mathbf{R}_i' + \hat{\mathbf{C}}_{i,t}^{-1})^{-1}\mathbf{R}_i + \mathbf{L}_{i,t}'\mathbf{N}_{i,t}\mathbf{L}_{i,t}
\end{aligned}$$

$$\begin{aligned}
r_{n,t-1} &= \mathbf{\Phi}'r_{0,t} \\
\mathbf{N}_{n,t-1} &= \mathbf{\Phi}'\mathbf{N}_{0,t}\mathbf{\Phi}
\end{aligned}$$

$$\begin{aligned}
\hat{a}_{t|T} &= \hat{a}_{1,t|t-1} + \hat{\mathbf{V}}_{1,t|t-1}r_{0,t-1} \\
\hat{\mathbf{V}}_{t|T} &= \hat{\mathbf{V}}_{1,t|t-1} - \hat{\mathbf{V}}_{1,t|t-1}\mathbf{N}_{0,t-1}\hat{\mathbf{V}}_{1,t|t-1}
\end{aligned}$$

(iii) Decrease $t = t - 1$ and go to step (ii)

6.3 Numerical Integration

To be added

Table 1: Descriptive Statistics European Yields

Country	Maturity	Mean	Sd	Skew	Kurt	Min	Max
France	2	1.00	0.51	2.08	8.09	0.23	3.62
	5	2.52	0.46	0.22	3.40	1.60	4.10
	10	3.52	0.38	-0.49	2.94	2.55	4.45
Germany	2	0.91	0.53	1.55	6.10	0.03	3.30
	5	2.15	0.55	-0.23	2.95	0.86	3.82
	10	3.05	0.50	-0.69	2.86	1.72	4.21
Italy	2	1.87	1.15	2.07	8.13	0.75	8.39
	5	3.71	0.99	1.70	6.20	2.64	7.90
	10	4.75	0.67	1.57	5.64	3.87	7.35
Spain	2	1.73	1.03	1.49	5.46	0.50	6.07
	5	3.71	0.82	0.71	2.50	2.69	6.38
	10	4.69	0.73	0.79	2.43	3.82	6.91

We report the mean, standard deviation, skewness, kurtosis, minimum and maximum statistics for the two-, five- and ten-year maturities for France, Germany, Italy and Spain. The sample is daily data from 1 October 2008 to 20 December 2012.

Table 2: Macroeconomic and Financial Time Series

Series	Transformation	R^2-mean	R^2-variance
All Employees: Total non-farm	4	0.743	0.060
IP Index	4	0.788	0.059
Capacity Utilization: Manufacturing	3	0.707	0.187
Help wanted to unemployed ratio	3	0.629	0.086
Unemployment rate	3	0.692	0.040
Real personal income	4	0.290	0.092
Weekly hours: goods-producing	1	0.097	0.134
Housing starts	2	0.282	0.536
Housing permits	2	0.275	0.467
Real consumer spending	4	0.351	0.011
Real manufacturing and trade sales	4	0.680	0.082
Orders for durable goods	4	0.437	0.070
Avg. hourly earnings, goods-prod.	5	0.006	0.026
PPI, finished goods	5	0.004	0.150
PPI, commodities	5	0.002	0.097
PCE price index	5	0.018	0.154
Federal funds rate	3	0.265	0.160
S&P 500	4	0.884	0.153
Spread, Baa-10y Treasury	1	0.252	0.109
Excess return	1	0.972	0.243
SMB FF factor	1	0.252	0.100
HML FF factor	1	0.131	0.361
Momentum factor	1	0.103	0.331
Spread book-to-market	1	0.219	0.280
Industry portfolio 1	1	0.851	0.174
Industry portfolio 2	1	0.796	0.149
Industry portfolio 3	1	0.819	0.041
Industry portfolio 4	1	0.667	0.178
Industry portfolio 5	1	0.858	0.198
Total	-	0.451	0.163

The transformation codes indicate: 1. no transformation, 2. $\log y_{i,t}$, 3. $\Delta y_{i,t}$, 4. $\Delta \log y_{i,t}$ and 5. $\Delta^2 \log y_{i,t}$. The R^2 for the mean is obtained by regressing each series $y_{i,t}$ on the first two principal components of the panel for y_t. The R^2 for the variance is obtained by regressing each volatility proxy $\hat{r}_{i,t} = \log(y_{i,t} - \hat{\lambda}_i' \hat{f}_t)^2$ with $\hat{\lambda}_i$ and $\hat{f}_t$ estimated by PCA using 2 factors on the first principal component of the panel for $\hat{r}_t$.

Table 3: Out of Sample Forecasting Results

$h=1$	$r=1$	$r=2$	$r=3$
$q=0$			
$q=1$			
$q=2$			
$q=3$			

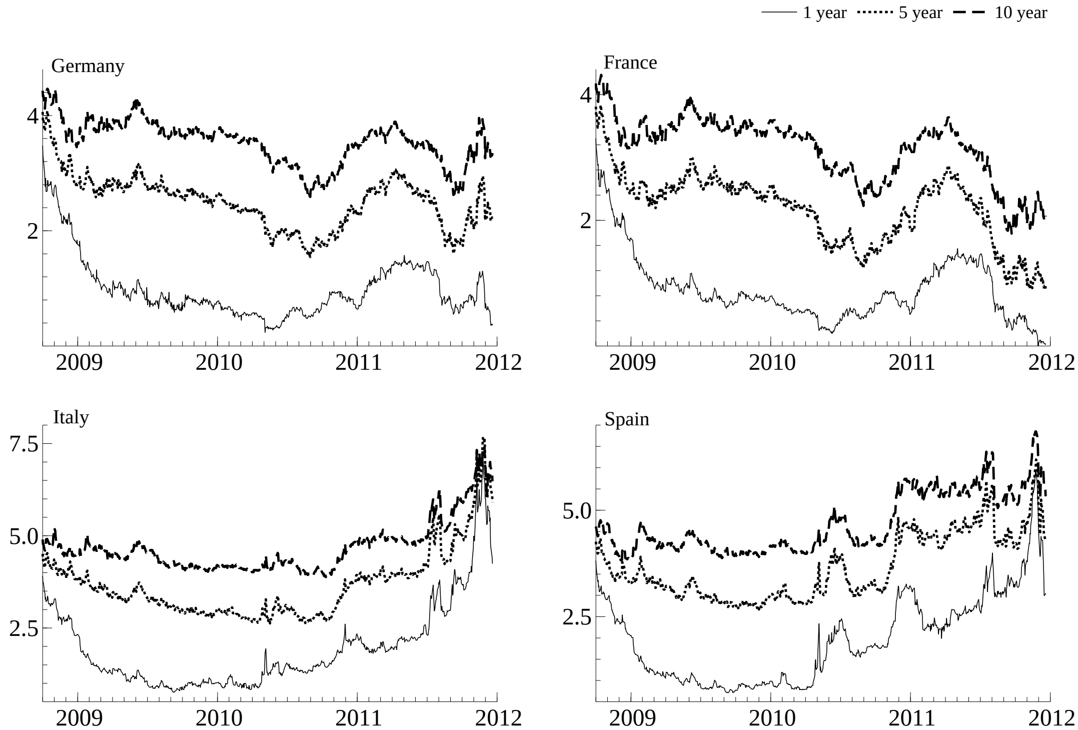
$h=2$	$r=1$	$r=2$	$r=3$
$q=0$			
$q=1$			
$q=2$			
$q=3$			

$h=3$	$r=1$	$r=2$	$r=3$
$q=0$			
$q=1$			
$q=2$			
$q=3$			

$h=4$	$r=1$	$r=2$	$r=3$
$q=0$			
$q=1$			
$q=2$			
$q=3$			

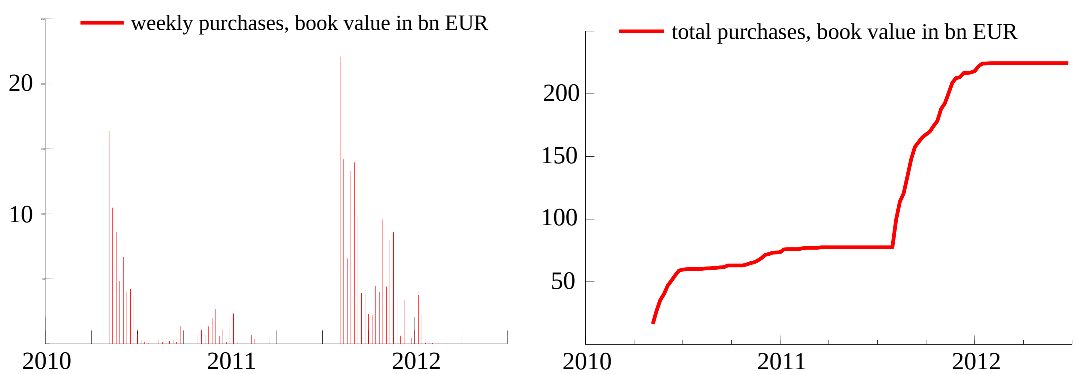
Average mean squared errors for different forecasting horizons and different combinations of level and volatility factors.

Figure 1: Government Bond Yields



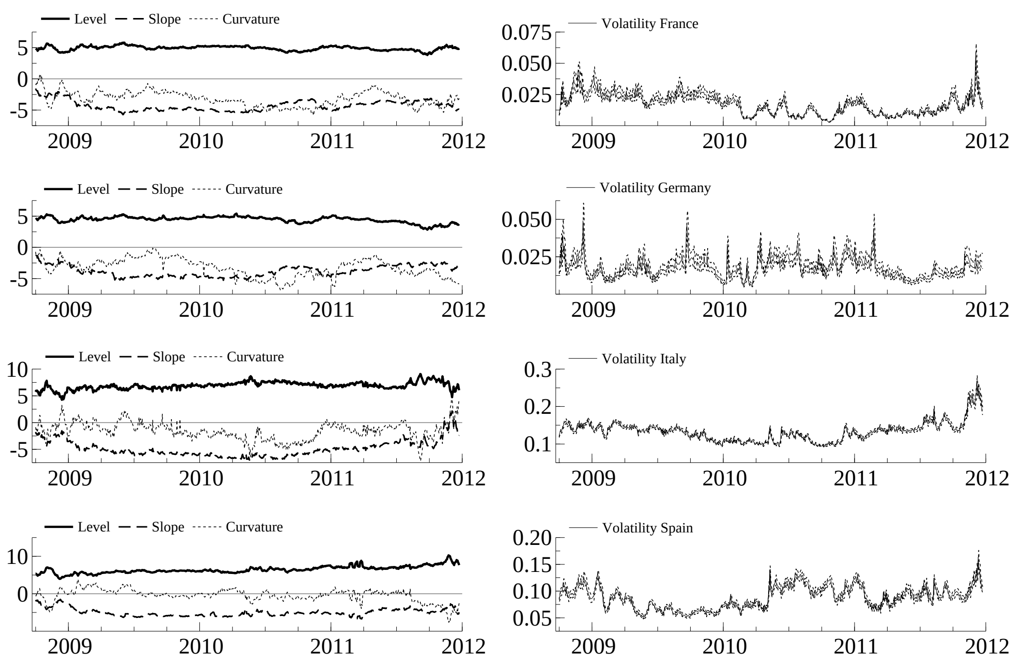
The figure shows the daily government bond yields with 1, 5 and 10 year maturities for France, Germany, Italy and Spain between 01-10-2008 and 20-12-2011.

Figure 2: Bond Market Purchases SMP Programme



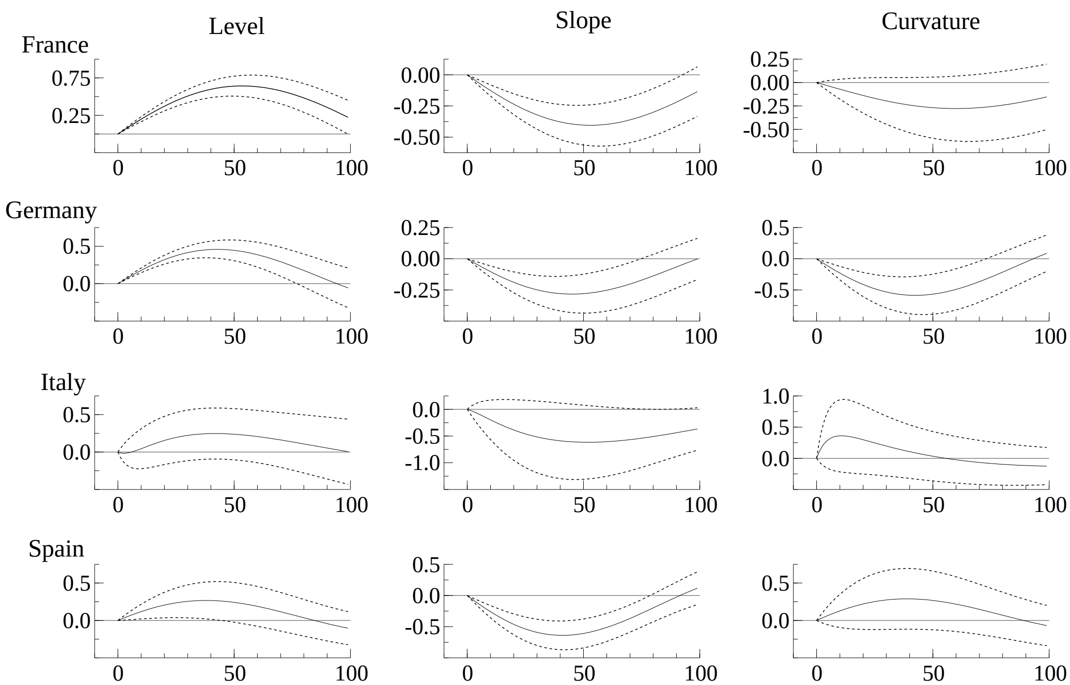
Weekly and total SMP purchase amounts. The figure plots the book value of settled SMP purchases as of the end of a given week. We report weekly purchases across countries (left panel) as well as the cumulative amounts (right panel). Maturing amounts are excluded.

Figure 3: Estimated Factors for Government Bond Yields



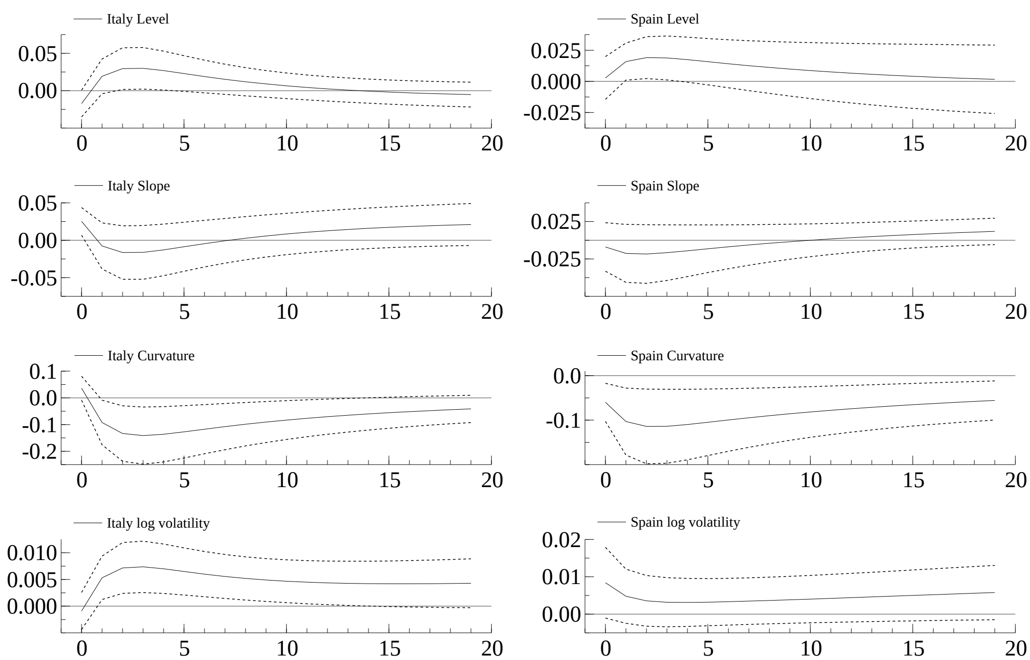
We show the conditional mean estimates for the common mean and variance factors together with their 95% confidence bounds.

Figure 4: Impulse Responses from Volatility Shocks on Yield Curve Factors



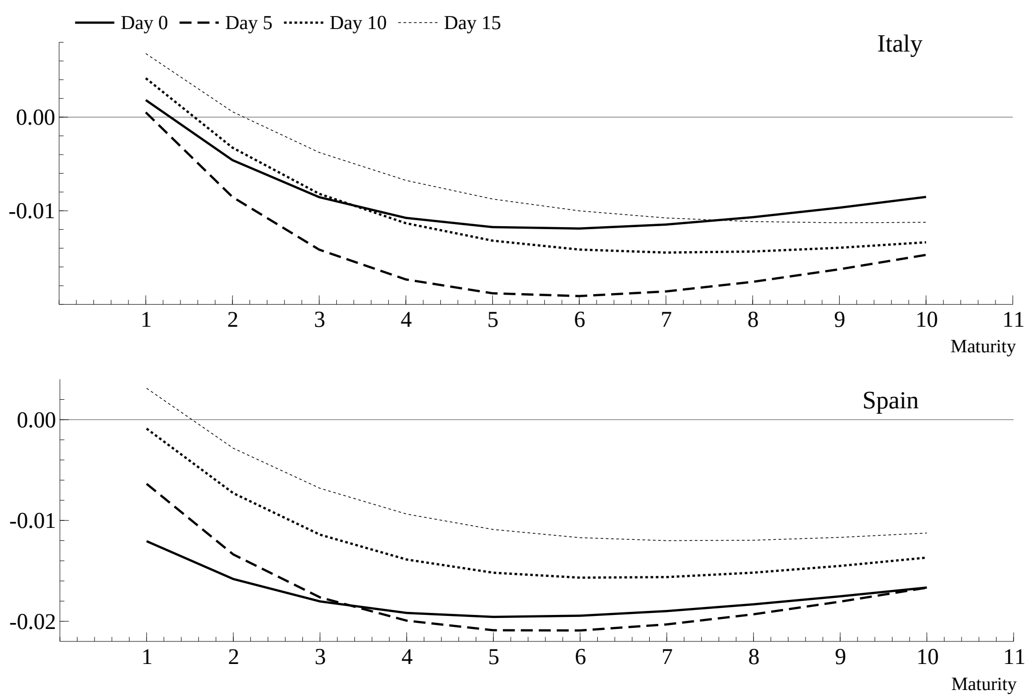
We show the impulse responses of a one standard deviation shock to the volatility factor on the level, slope and curvature factors together with their 95% confidence bounds.

Figure 5: Impulse Responses from Purchases on Yield Curve Factors



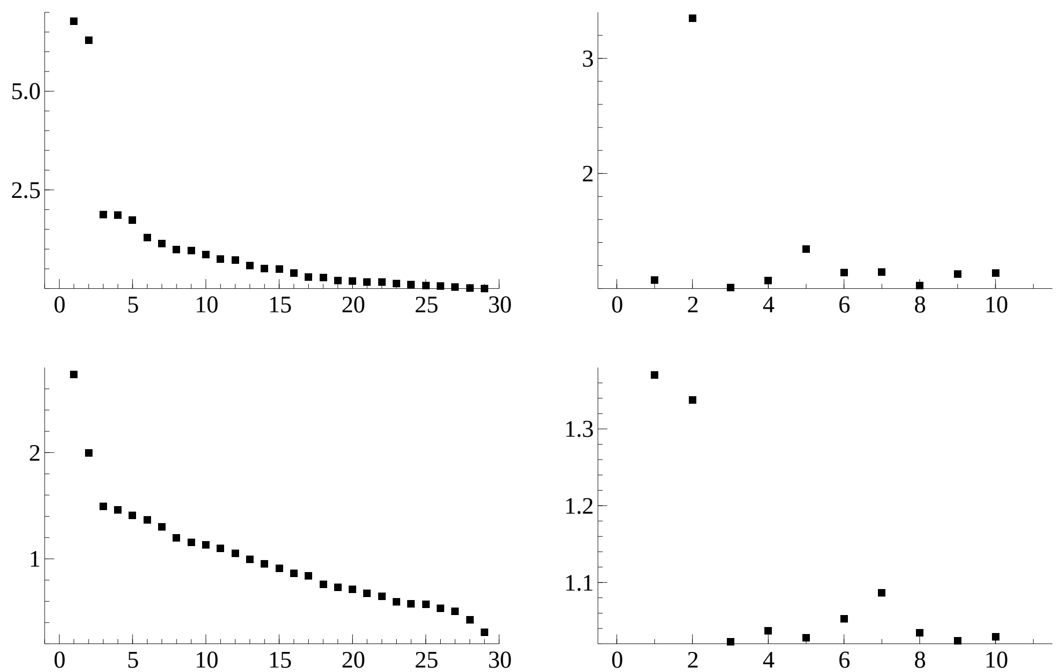
We show the impulse responses of a one billion euro shock to the purchases on the level, slope and curvature factors together with their 95% confidence bounds.

Figure 6: Effects of Purchases on Yields with Different Maturities



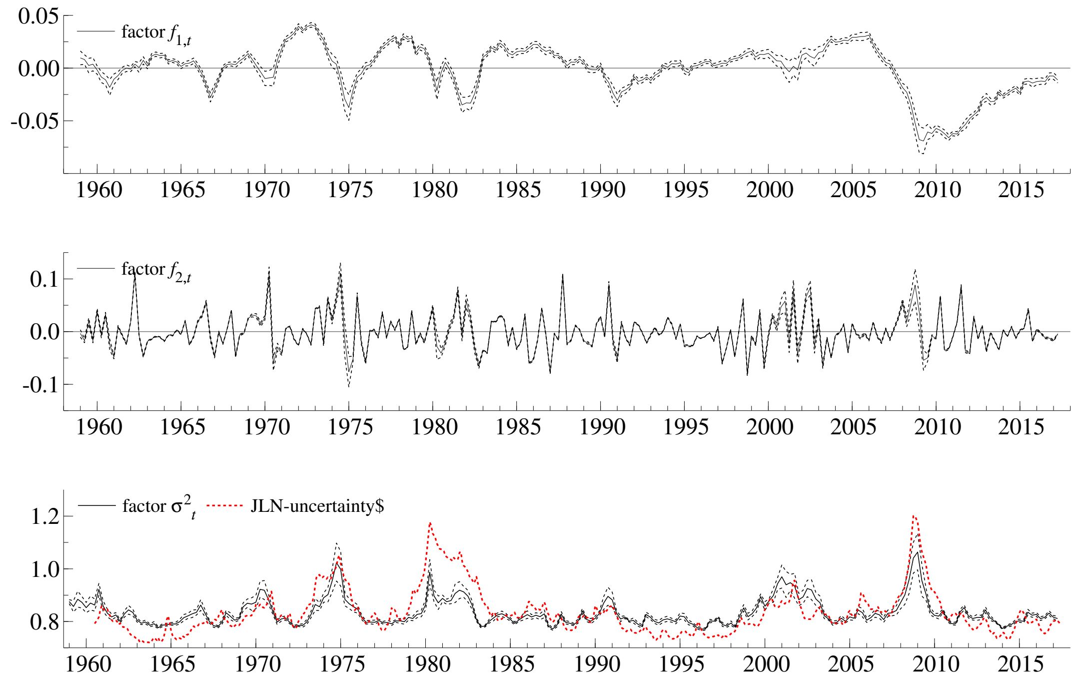
We show the impulse responses for different days of a one billion euro shock to the purchases on the yields for different maturities.

Figure 7: Eigenvalue results for Macroeconomic and Financial Panel



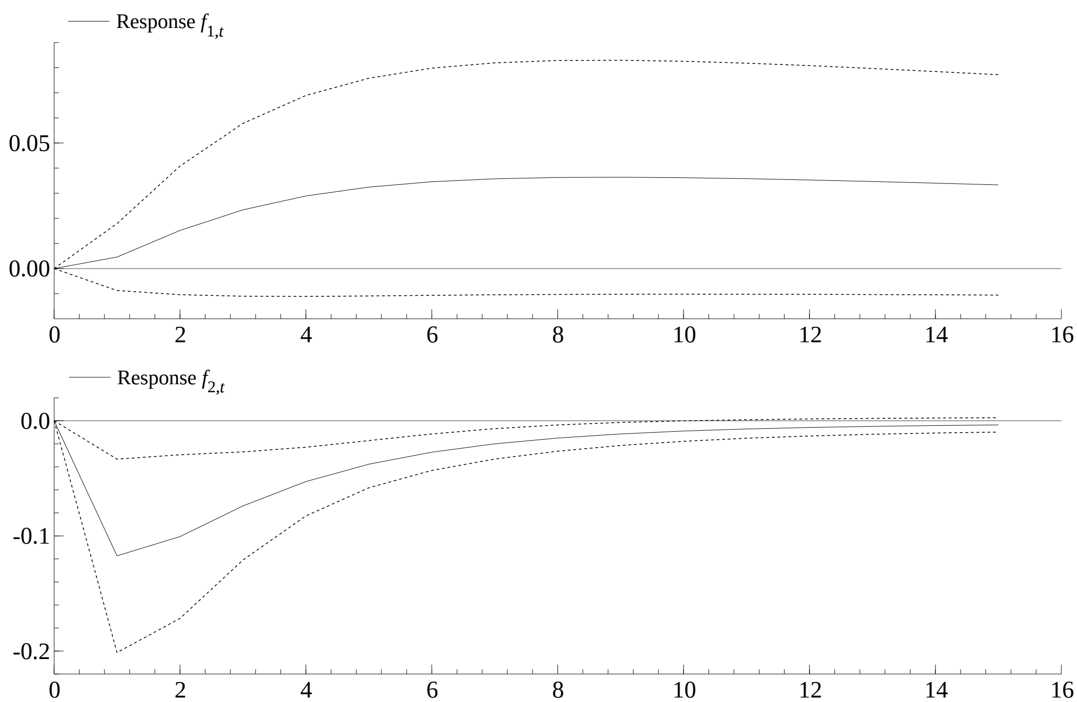
We show the ordered eigenvalues and eigenvalue ratios for $\frac{1}{T}\sum_{t=1}^{T} y_t y_t'$ (top row) and $\frac{1}{T}\sum_{t=1}^{T} \hat{r}_t \hat{r}_t'$ (bottom panel) where $\hat{r}_{i,t} = \log(y_{i,t} - \hat{\lambda}_i' \hat{f}_t)^2$ with $\hat{\lambda}_i$ and $\hat{f}_t$ estimated by PCA using 2 factors.

Figure 8: ESTIMATED FACTORS FOR MACROECONOMIC AND FINANCIAL PANEL



We show the conditional mean estimates for the common mean and variance factors together with their 95% confidence bounds.

Figure 9: Impulse Responses from Volatility Shocks on Level Factors



We show the impulse responses of a one standard deviation shock to the volatility factor on the level factors together with their 95% confidence bounds.