Global Banking: Endogenous Competition and Risk Taking*

Ester Faia
Goethe University Frankfurt and CEPR

Gianmarco Ottaviano
London School of Economics, University of Bologna, CEP and CEPR

First draft: October 2014. This draft: August 2017.

Abstract

Direct involvement of global banks in local retail activities through a "bricks and mortar" business model can reduce risk-taking by promoting local competition. We develop this argument through a dynamic model in which multinational banks may choose to operate in different imperfectly competitive national markets through the horizontal expansion of their deposit and loan activities. In making this choice, banks compare charter values and entry barriers. When foreign operations entail additional monitoring costs, multinational banks face predatory lending incentives that are stronger the smaller their market shares are. The model generates predictions that are consistent with the "bricks and mortar" argument as long as the expansionary impact of competition on multinational banks' aggregate future discounted profits through larger scale is strong enough to offset its parallel contractionary impact through lower loan-deposit return margin. This effect is stronger with perfectly than imperfectly correlated loans' risk, with exogenous than endogenous exit, with horizontal than vertical expansion (cross-border lending).

JEL: G21, G32, L13.

Keywords: *global bank, oligopoly, oligopsony, endogenous risk taking, expectation of rents extraction, appetite for leverage.*

---

*We thank Javier Suarez for very useful comments. We also thank for comments participants at various conferences and seminars. We gratefully acknowledge financial support from the EU-FP7 grant MACFINROBODS-SSH-2013-612796. The paper is also recipient of the Baffi-Bocconi prize. Soeren Karau provided excellent research assistance. Correspondence to: Ester Faia, Goethe University Frankfurt, Theodor-W. Adorno Platz 3, 60323 Frankfurt am Main, Germany; e-mail: faia@wiwi.uni-frankfurt.de. Gianmarco Ottaviano: London School of Economics and Political Science, Department of Economics, Houghton Street, London WC2A 2AE, U.K.; e-mail: g.i.ottaviano@lse.ac.uk.

1 Introduction

Banking globalization has been blamed for generating and propagating risk in the run up to the financial crisis (Rajan [37]). More recently, however, it has been suggested that direct involvement of global banks in local retail activities through a 'bricks and mortar' business model can reduce risk-taking by promoting local competition (IMF [28]). If confirmed, this could represent a major develpment in terms of global financial stability given that, while cross-border lending has diminished since the crisis, banks' globalization through 'bricks and mortar' has remained sustained (Claessens and van Horen [12] and [13]).

Against this background, a still small but growing empirical literature has recently started to study the impact of banks' geographical expansion on credit conditions and financial stability, paying due attention to issues related to identification and reverse causation. Evidence shows that the presence of foreign banks helps reduce the cost of credit, hence risk-taking, the more so the lower the entry barriers, and thus the wider the scope for competition (Claessens et al. [15]; Berger et al. [4]; Giannetti and Ongena [22]). For US banks expanding across US states, Goetz, Laeven and Levine [23] and Levine, Lin and Xie [30] find that geographic expansion reduces banks' riskiness thanks to better asset diversification. Faia, Ottaviano and Sanchez-Arjona [21] reach similar conclusions in the case of European banks expanding across European countries.

The dataset collected by Faia, Ottaviano and Sanchez-Arjona [21] covers the openings by the 15 European G-SIBs (i.e. Global Systemically Important Banks, as listed by the Basel Committee for Banking Supervision) from 2005 to 2014. For these banks, the authors compute various risk indicators and test the impact of banks' foreign expansion on both individual bank risk (measured through CDS prices or loan loss provisions over assets) and systemic risk (measured with metrics of marginal capital short-fall or CoVaR). They find that foreign expansion through 'bricks and mortar' reduces all risk measures. Figures 1 and 2 provide a visual representation of two key patterns emerging from their dataset. First, as shown in Figure 1, banks with a larger number of foreign openings are associated with lower risk (measured here by the log growth in CDS prices).Second, as shown in Figure 2, more competitive markets feature a larger number of openings by all banks,

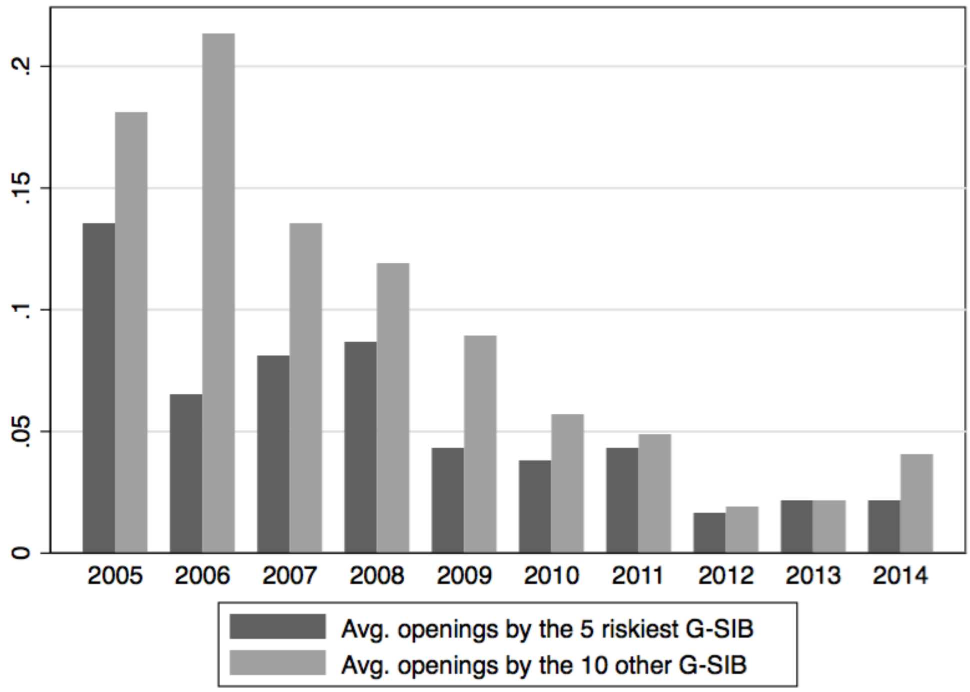

Figure 1: Share of openings for the 15 banks classified as GSIBs by the Basel Committee for Banking Supervision. The relation is derived for the 5 riskier groups and the remaining 10 groups. The share of openings is divided for the 5 riskiest banks and for remaining ones. European data. Source: Faia, Ottaviano and Sanchez-Arjona [21].

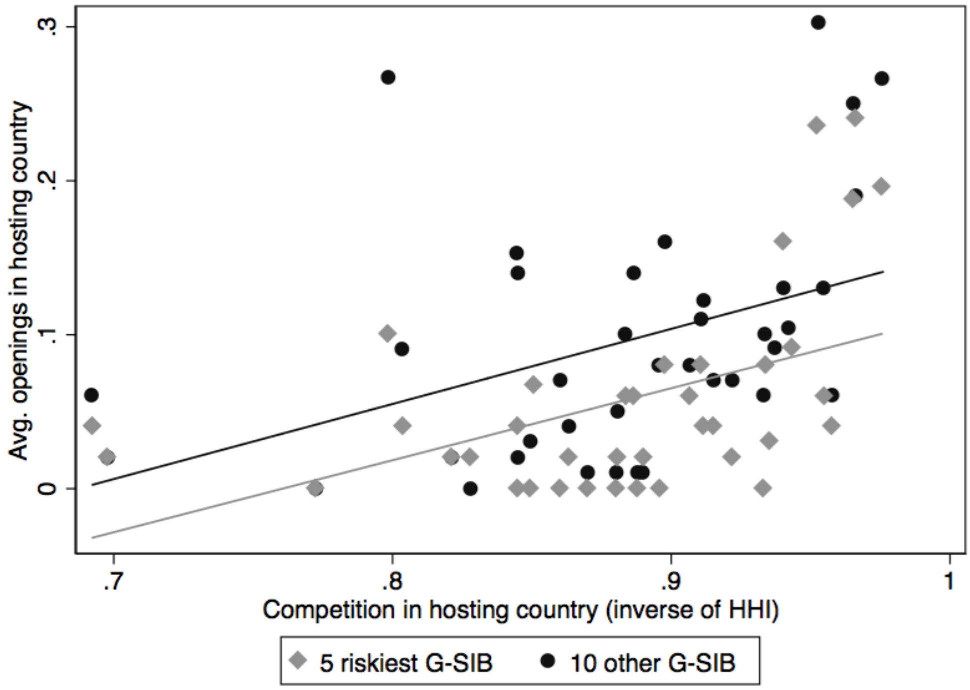

Figure 2: Relation between foreign expansion (average number of openings) and competition in host country for the 15 banks classified as GSIBs by the Basel Committee for Banking Supervision. The relation is derived for the 5 riskier groups and the remaining 10 groups. European data. Source: Faia, Ottaviano and Sanchez-Arjona [21].

but disproportionally by less risky banks.

While the patterns depicted in these figures are only correlations, they are consistent with foreign expansion having a negative impact on banks' systemic risk. To formalize and question this argument, we develop a dynamic entry model in open economy, in which banks can decide to operate in different countries by setting up local subsidiaries (or branches). In doing so, they face a fixed entry cost to create their headquarters and a fixed setup cost for each local subsidiary they open. Banks raise deposits from households and extend loans to firms. Deposits are fully ensured in each country. Banks pay the corresponding insurance fees and provide monitoring services on loans that firms use to finance risky projects under limited liability. There is moral hazard in that higher project returns are associated with higher probability of project failure but limited liability implies that firms underweight the downside with respect to banks.

National markets are segmented: banks cannot move funds across borders, and can raise deposits and extend loans only through local subsidiaries. However, monitoring loans in a country in which banks are not headquartered is more costly to them due to lower relationship lending ability. Each national market is imperfectly competitive with banks facing Cournot competition in both deposits (*oligopsony*) and loans (*oligopoly*). Households and firms have no market power, which allows banks to extract rents from the spread between the interest rate on loans and the interest rate on deposits, with the former above and the latter below their respective perfectly competitive levels. These rents generate profits that may make it worthwhile for banks to enter and operate in the different national markets. This happens as long as banks' future discounted profits (*charter value*) exceed entry and setup costs. The additional cost of monitoring foreign loans leads to *predatory banking*, whereby banks penetrate the foreign market by accepting a lower loan-deposit spread than in their domestic market. Predatory banking incentives are stronger the smaller a bank's foreign market share relative to the domestic one.[1]

The interest rate on loans determines the risk appetite of firms, with higher loan rates inducing more risk-shifting under moral hazard (Stiglitz and Weiss [40]; Jensen and Meckling [25]). Therefore, banks' decisions on entry, deposits demanded and loans supplied drive the risk-return profile of firms' selected projects. In particular, by changing the number and the composition of incumbent banks, entry affects the intensity of competition in the banking sector and the loan rates on offer. The degree of competition is thus endogenous and feeds back into firms' *endogenous risk-taking*. This happens through different channels. For example, as additional banks enter, more competition in deposits reduces banks' oligopsonistic power, increasing the amount of deposits raised and the interest rate paid on them for given loan rate (*deposit rate channel*); more competition in loans reduces banks' oligopolistic power, increasing the amount of loans extended and decreasing the interest rate requested on them for given deposit rate (*deposit rate channel*); these two effects combined reduce the loan-deposit spread, thereby decreasing banks' profits and charter value (*charter value channel*); as charter value falls, banks' entry eventually stops. When banks' entry is initially triggered by lower monitoring cost on foreign loans, more competition is

[1] This is akin to 'dumping' in international trade (Brander and Krugman [6]).

accompanied by a rebalancing of market shares between domestic and foreign banks that reduces the scope for predatory banking (*predatory banking channel*). Whether firms' risk-taking eventually decreases or increases depends on whether the interest rate on loans rises or falls, which itself depends on whether the compression of the loan-deposit spread dominates or is dominated by the rising interest rate on deposits. The end result hinges on the specific functional forms of the demand of loans, the supply of deposits and the relation between project return and risk. We show, however, that for empirically relevant and generally accepted functional forms the compression of the loan-deposit spread prevails.[2]

We reach this conclusion through an analytical and numerical investigation of the model's behavior. In particular, banks' entry in foreign markets increases competition and reduces risk-taking as long as the expansionary impact of competition on multinational banks' aggregate profits through larger scale is strong enough to offset its parallel contractionary impact through lower loan-deposit rate spread. Under this condition, endogenous competition exerts a discipline role and induces banks to make firms behave more cautiously, despite the presence of a deposit insurance would in itself foster banks' risk-taking.[3]. We consider two scenarios: a deteministic 'long-run' scenario, in which the tradeoff faced by firms between project risk and return is time invariant; and a stochastic 'short-run' scenario, in which the tradeoff is affected by productivity shocks, such that a positive shock increases the probability of project success for any given return.

In our focal exercise we look at the effects of the aforementioned fall in the additional costs of monitoring foreign loans, through which we want to capture an exogenously driven increase in banking globalization. In the long-run scenario, lower foreign monitoring costs lead to an increase in the number of multinational banks as well as in the total amount of deposits and loans in each national market. It also leads to higher interest rate on deposits, lower loan-deposit spread and lower interest rate on loans. As a result, firms select projects with lower return but higher success rate. Hence, more involvement of multinational banks in local retail activities does reduce risk-taking by

[2] We follow Boyd and De Nicolo [5] and Martinez-Miera and Repullo [31] in assuming linear functional forms for the demand of loans, the supply of deposits and the relation between projects' returns and risk, an assumption compatible with decreasing hazard rates.

[3] See, e.g., Merton [34].

promoting local competition. Comparing different versions of the model, we find that this effect is stronger with perfectly than with imperfectly correlated loans' risk, with exogenous than with endogenous exit, and with horizontal than with vertical expansion (*cross-border lending*). In the short-run scenario, more banking globalization has a stabilizing effect, dampening the responses of all endogenous variables to productivity shocks. In the numerical simulations we pay special attention to parameters' calibration based on micro banking data and estimation through the method of moments, performing sensitivity checks for several alternative parameter value configurations.

The rest of the paper is organized as follows. Section 2 sets our contribution in the context of the existing literature. Section 3 describes our model of multinational banking. Section 4 solves the model and studies its predictions, both analytically and numerically, in the long-run and the short-run scenarios. Section 5 presents variants of the model allowing for cross-border lending, asymmetric country shocks, endogenous exit, and systemic risk. Section 6 concludes.

2 Related Literature

Our paper is primarily connected to the theoretical banking literature that studies the role of competition for risk-taking. This literature generally employs models with Cournot-Nash competition, but mainly focuses on static models with exogenous exit and no heterogeneity across markets (closed economy). Allen and Gale [1] analyze the link between deposit competition and banks' choice of the risk-return profile of their investment porfolio. Higher competition induces banks to increase rates on deposits to entice investors. Differently, Boyd and De Nicolo [5] build a model with competition in the loan market (as opposed to deposit market) and show that, as competition rises, banks apply lower loan rates that induce firms to select less risky projects. Their result is challenged by Martinez-Miera and Repullo [31] when banks' probability of default is allowed to depend on a common latent factor (Vasicek [42]). Focusing on competition in the loan market, they show that whether risk-taking increases or decreases with competition depends on the correlation of funded projects, which in turn is driven by the latent factor. Given the conflicting conclusions of these and other papers not mentioned for brevity, whether more competition increases or decreases banks'risk-taking remains an open question.

Differently from this literature, we address that question from a specific and topical viewpoint, that of increased competition driven by the activities of global banks. This leads us to consider a number of additional channels through which competition affects risk-taking. First, we consider a dynamic environment where banks' entry decisions depend on the comparison of charter values (as captured by the sum of future discounted profits) with entry costs. Entry makes competition endogenous and generates a feedback loop with endogenous risk-taking. Consistently with Keeley [29], as competition intensifies, banks see their profits shrinking. However, differently from that paper, in our model tougher competition also improves project selection, thus raising future discounted profits and reducing banks' risk. Second, our multinational banks face a cost structure conducive to *predatory banking* in foreign markets akin to what Brander and Krugman [6] call *dumping* in international trade. In the presence of higher monitoring costs on foreign loans, banks are willing to accept lower profit margins in foreign markets in order to penetrate them. This possibility has not been formalized before in the banking literature. Third, an important role in our model is played by the impact of competition on project *selection*. This aspect parallels the idea recently advanced in the trade literature that tougher competition associated with globalization leads to selection of the best performing firms (Melitz [33]; Melitz and Ottaviano [32]). This literature contributed to shift the focus of the determinants of international trade from the country level to the firm level. Analogously, our approach shifts the focus of the determinants of capital flows from the country to the bank level. In so doing, as the trade literature, we model endogenous entry (as well as endogenous exit) and industry dynamics, while dealing with banks rather than firms opens up the additional dimension of endogenous risk-taking. Also Corbae and D'Erasmo [16] study the link between competition and risk-taking in a dynamic entry model, but they do so in closed economy. Moreover, differently from ours, in their model banks are monopolistic competitive, hence there is no strategic interaction; and they focus on competition in the loan market, while we take into account also competition in the deposit market. As their analysis does not consider the possibility that banks might enter heterogeneous markets, it does not feature predatory banking.

Very few papers analyze the theoretical underpinnings of global banking. Bruno and Shin [7] build a model of the international banking system where global banks raise short term funds ('deposits') at worldwide level, but interact with local banks for the provision of loans. Differently from us, they focus on banks' leverage cycle. Niepman [36] proposes a perfectly competitive model of banking across borders, in which the pattern of foreign bank asset and liability holdings emerges endogenously because of international differences in relative factor endowments and banking efficiency. Competition and risk-shifting are not part of the analysis. De Blas and Russ [17] investigate whether foreign participation in the banking sector increases real output. Using a general equilibrium model of heterogeneous Bertrand-competitive lenders and a simple search process, they show that lending-to-deposit rate spreads can increase with FDI whereas the lending rates remain largely unchanged or even fall. They also contrast the competitive effects from cross-border bank takeovers with those of cross-border lending. Differently from us, they do not emphasize risk-shifting in the presence of limited liability.[4]

Finally, supplementing what we already discussed in the Introduction, our paper is also related to the emerging empirical literature on the role of global banks in the recent crisis. For instance, Cetorelli and Goldberg [10] and [11] study liquidity management by global banks during the Great Recession and focus on the interaction with the monetary policy transmission mechanism. As they consider banks that are already global, they do not investigate the factors that might induce banks to enter foreign markets.[5] Claessens and van Horen [14] highlight the observed asymmetric reactions of cross-border lenders and multinational banks to negative shocks in foreign markets, with the former typically retreating more than the latter. Our model provides a theoretical underpinning to these empirical findings.

3 A Dynamic Model of Multinational Banking

We consider an imperfectly competitive banking sector with endogenous entry that operates in two symmetric national markets, called H and F. Banks raise deposits from households under oligop-

[4] See Hale and Russ [18] for a recent overview of related works.
[5] See also the papers in Buch and Goldberg [9] for a recent overview.

sony and extend loans to firms under oligopoly for their investment projects. While households are risk averse, firms are risk lovers due to moral hazard induced by limited liability, which gives them risk-shifting incentives. The role of banks is to provide monitoring services on loans and insurance on deposits. Full insurance, however, implies that also banks face risk-shifting incentives.

Banks are headquartered in only one of the two markets but can operate in both. However, when a bank operates in the market it is not headquartered in, it faces an additional monitoring cost on loans $\mu > 0$. Entry is endogenous as determined by banks' forward-looking decisions trading off the total sum of future discounted profits and a fixed entry cost $\kappa > 0$, which subsumes a bank entry cost $\kappa^b > 0$ and a subsidiary setup cost $\kappa^d > 0$ for each market the bank operates in ($\kappa = \kappa^b + 2\kappa^d$). We use $N^a_{t,H}$ and $N^a_{t,F}$ to denote the numbers of active banks that at any time t are headquartered in H and F respectively, and $N^a_t = N^a_{t,H} + N^a_{t,F}$ to denote the resulting total number of active banks.

Henceforth, as the two national markets are symmetric, for conciseness of exposition we will focus on the description of market H with analogous expressions holding for market F.

3.1 Banks' Entry and Exit

In each period t the number of active banks is determined endogenously by entry and exit as follows. Entry requires establishing a headquarter in one of the two national markets and a subsidiary in each market at the overall fixed cost $\kappa > 0$. A constant discount factor $\beta \in (0, 1)$ captures the exogenous per period opportunity cost associated with financing κ in an un-modelled international capital market. The fact that β is constant means that the two national banking markets we focus on are 'small' with respect to the international capital market and thus financing conditions in the latter are not affected by banks decisions in the former. Banks become active as soon as they enter. Exit happens exogenously and does not entail additional costs. In each period banks face an exogenous death rate $\varrho \in (0, 1)$.[6]

Accordingly, active banks consist of incumbents that survived from the previous period and new entrants. If we use $N_{t-1,H}$ and $N^e_{t,H}$ to denote the numbers of incumbent and entrant banks

[6] An extension of the model with endogenous exit is discussed in Section 5.3.

headquartered in H in period t, we have that the corresponding number of active banks is:

$$N_{t,H}^{a} = N_{t-1,H} + N_{t,H}^{e} = \frac{N_{t,H}}{1-\varrho}. \tag{1}$$

Note that, due to exogenous death, the number of incumbents in any period is only a share $1-\varrho$ of the number of active banks in the previous period.

In deciding whether to enter or not, banks compare the fixed entry cost κ with the total present expected value of future per-period profits over an infinite time horizon, taking into account the exogenous exit probabilities. The sum of future expected profits weighted by the exit rates can be written recursively using the Bellman operators. If we use $V_{t,H}$ to denote the value of being active at time t for a bank headquartered in H, we can write the total sum of its future discounted profits recursively as:

$$V_{t,H} = \Pi_{t,HH} + \Pi_{t,HF} + \beta(1-\varrho)\mathbb{E}_t \left\{V_{t+1,H}\right\}. \tag{2}$$

where $\Pi_{t,HH}$ and $\Pi_{t,HF}$ denote the per-period profits that a bank headquartered in H earns in period t from operations in markets H and F respectively, and $\mathbb{E}_t$ denotes the conditional expectation operator given information at time t. As entry happens instantaneously, the model features no transitional dynamics. Free entry therefore implies that in any instant t the value of being active equals the overall entry cost: $V_{t,H} = \kappa$. This condition highlights the role of banks' charter value for risk-taking and competition. Any decision made by banks on loans affects their current and future rents, which in turn affect their entry decision. We will return to this point later on.

We will consider two cases, a stochastic environment and a deterministic environment in which banks' profits per period are constant and equal to the annuity value of that cost:

$$\Pi_{HH} + \Pi_{HF} = \left[1-\beta(1-\varrho)\right]\kappa, \tag{3}$$

which shows that the larger are the fixed entry cost κ, the opportunity cost β of financing entry and the death rate ϱ, the larger profits have to be in order to justify entry. Analogous results hold for banks headquartered in country F.

3.2 Banks, Firms and Depositors

Banks act as intermediaries between depositors and borrowers ('firms'), acting as oligopsonist vis-à-vis the former and as oligopolist vis-à-vis the latter. In both cases they behave as Cournot-Nash competitors. For simplicity, we assume that: (i) firms do not have internal funds and banks are their only source of funds; (ii) banks can only finance firms using own deposits; (iii) depositors can only use their funds for deposits. The absence of bank equity in the model is compensated by assuming that banks pay a fee to the deposit insurance fund, which in the pecking order is the first loss absorber. Furthermore, we assume that both home and foreign banks can finance home firms using local deposits. This assumption reflects well the reality of the 'bricks and mortar' business model, in which liquidity cannot be moved easily across branches/subsidiaries. Banks optimize in each destination markets separately ('market segmentation') but markets will be linked through the banks' free entry condition. Note that firms' and banks' optimizations (as well as strategic interactions among banks) take place within a period, hence in what follows we will leave the time index implicit.

3.2.1 Deposit Supply

While banks and firms are risk neutral, depositors are risk averse households with concave utility function in their consumption. Deposits are insured by banks at a flat rate deposit insurance premium $\xi > 0$. This implies that in market H the total supply of deposits D_H^T as well as the return on deposits r_H^D do not depend on the riskiness of banks' portfolios: depositors only care about the expected return of deposits, as they will not bear banks' asset losses due to the insurance. Notice that the presence of the insurance, by expanding the bank's limited liability region, also contributes to the banks' risk-taking incentives (we will come back to this aspect later on). Thus, the (inverse) supply of deposits can be characterized as a return function of D_H^T only. This function $r_H^D = r^D\left(D_H^T\right)$ is assumed to satisfy $r^D(0) \geq 0$ and to be twice differentiable with $r^{D\prime}\left(D_H^T\right) > 0$ and $r^{D\prime\prime}\left(D_H^T\right) \geq 0$. Using D_{HH} and D_{FH} to denote the deposits raised by home and foreign banks respectively, we have $D_H^T = D_{HH} + D_{FH}$.

Notice that households could potentially invest in firms' projects by themselves. In this case, however, they would receive a risky return. By investing in insured banks' deposits, they receive instead a fixed return, which better suits their risk averse preferences. Hence, in addition to monitoring loans, a key function of banks in the model is that of risk insurance providers. Risk neutral banks collect deposits, invest them in risky assets by diversifying and provide a fixed returns to risk averse depositors. Importantly, the deposit insurance plays the role of bank capital in our model: the insurance fee is proportional to assets and the insurance fund is the first in the pecking order of loss absorbing assets.

3.2.2 Loan Demand

Firms' projects are funded by banks. In each national market firms have access to a set of constant-return risky technologies ('projects') with fixed output normalized to 1. For market H, projects are indexed r_H^I yielding ar_H^I with probability $p(r_H^I, a)$ for $r_H^I \in [0, \overline{r}^I]$ and 0 otherwise, where a is an aggregate shock.[7] We assume that this shock is common across markets in order to insulate our analysis of the effects of global banking on risk-taking channeled through competition from those channeled through risk diversification.[8]

Probability $p(r_H^I, a)$ satisfies $p(0, a) = 1$, $p(\overline{r}^I, a) = 0$, $p_1(r_H^I, a) < 0$, $p_{11}(r_H^I, a) \leq 0$ for all $r_H^I \in [0, \overline{r}^I]$ so that $p(r_H^I, a)ar_H^I$ is strictly concave in r_H^I. It also satisfies $p_2(r_H^I, a) > 0$ and $p_{12}(r_H^I, a) \geq 0$. Accordingly, for given a, the probability of success decreases more than proportionately as projects' returns increase, while it (weakly) increases as a increases. Moreover, the positive impact of larger a on r_H^I is (weakly) stronger for larger r_H^I so that higher return projects with lower probability of success benefit (weakly) more than proportionately from favourable aggregate shocks. The choice of projects by firms is unobservable to banks, which can only observe (at no cost) whether projects have been successful ($r_H^I > 0$) or not ($r_H^I = 0$).

As firms are risk neutral, in each national market the total demand of loans $L_H^T = L_{HH} + L_{FH}$ (with L_{HH} and L_{FH} denoting the supply of loans from home and foreign banks respectively) as well as their return r_H^L do not depend on the riskiness of firms' projects. The (inverse) demand of

[7] Under this assumption all projects succeed with probability $p(r^I, a)$. An extension of the model allowing for imperfect correlation of projects' outcomes and systemic risk is presented in Section 5.4.

[8] An extension of the model with asymmetric country shocks and risk diversification is discussed in Section 5.2.

loans can then be characterized as a return function of L_H^T only. This function $r_H^L = r^L\left(L_H^T\right)$ is assumed to satisfy $r^L(0) > 0$ and to be twice differentiable with $r^{L\prime}\left(L_H^T\right) < 0$, $r^{L\prime\prime}\left(L_H^T\right) \leq 0$ and $r^L(0) > r^D(0)$.[9]

Finally, as banks can only finance loans through deposits and firms can only finance projects through bank loans, the total amounts of firms' investments I_H^T, banks' loans L_H^T and deposits D_H^T have to be the same: $I_H^T = L_H^T = D_H^T$, where the total amount of investments financed by home and foreign banks is $I_H^T = I_{HH} + I_{FH}$.

3.2.3 Investment and Risk

We introduce moral hazard by assuming that firms have limited liability in that they repay their loans only if their projects are successful. Those elements imply that firms have an incentive to risk-shifting, the more so the higher the cost of credit. We follow in this respect the tradition of Stiglitz and Weiss [40] and Jensen and Meckling [25]. This implies that, given risk neutrality, a firm (in the H market) chooses r_H^I in order to maximize expected per period profits:

$$p(r_H^I, a)(ar_H^I - r_H^L), \tag{4}$$

as failure happens with probability $1 - p(r_H^I, a)$ but does not require any loan repayment.[10] Note that, given the monotonic relation between $p(r_H^I, a)$ and r_H^I, choosing r_H^I is equivalent to choosing $p(r_H^I, a)$. In this respect, firms choose the 'risk-return profile' of investments for given return on loans r_H^L (and given a).

The first order condition for a firm maximizing (4) is:

$$p(r_H^I, a)a + p_1(r_H^I, a)(ar_H^I - r_H^L) = 0, \tag{5}$$

which shows that firms trade off higher return $(p(r_H^I, a)a > 0)$ and lower success probability $(p_1(r_H^I, a)(ar_H^I - r_H^L) < 0)$. Making the dependence of r_H^L on L_H^T explicit allows us to rewrite (5)

[9] Additional details on how to microfound these properties can be found in Appendix A.

[10] We could alternatively assume that firms earn a fixed amount $(1 - c)$ with probability $1 - p(r_H^I, a_H)$. This, however, would not change the main incentives faced by firms and banks. Indeed, in case of failure firms would be unable to repay the loans, banks would repossess the amount left $(1 - p(r_H^I, a_H))(1 - c)$ and firms would receive zero. The proceeds earned by banks would then enter banks' profits and their first order conditions would be simply scaled up by $(1 - p(r_H^I, a_H))(1 - c)$.

as:

$$\frac{p(r_H^I, a)a}{p_1(r_H^I, a)} + ar_H^I = r^L\left(L_H^T\right), \tag{6}$$

which expresses the return on investment r_H^I (and thus also risk $1 - p(r_H^I, a)$) as an implicit function of aggregate loans L_H^T with exogenous parameter a. In particular, (6) shows that, by affecting L_H^T, banks indirectly command the return-risk profile chosen by firms. Specifically, given the functional properties of $r^L\left(L_H^T\right)$ and $p(r_H^I, a)$, a contraction in bank credit (smaller L_H^T) induces firms to select a more 'aggressive' investment profile characterized by higher return and higher risk (i.e., larger r_H^I and larger $p(r_H^I, a)$).[11] Larger a has the same qualitative effects on firms' choice due to its disproportionate boost to high-return high-risk projects.[12] Hence, by disproportionately boosting the probability on the upper tail of the projects' returns distribution, larger a increases firms' 'exuberance'. The choice of firms in the F market is equivalent.

3.3 Banks' Competition

As banks can only finance local loans by own local deposits, in market H the loans $L_{r,HH}$ ($L_{r,FH}$) of any home (foreign) bank r have to exactly match its deposits $D_{r,HH}$ ($D_{r,FH}$). This implies $L_{r,HH} = D_{r,HH}$ ($L_{r,FH} = D_{r,FH}$) with $D_{HH} = \sum_{r=1}^{N_H} D_{r,HH}$ ($D_{FH} = \sum_{r=1}^{N_H} D_{FH}$) so that $L_{r,HH}$ or $D_{r,HH}$ ($L_{r,FH}$ or $D_{r,FH}$) can be equivalently chosen as a home (foreign) bank's choice variable. In what follows, we will choose $L_{r,HH}$ ($L_{r,FH}$). Then, Cournot-Nash behavior requires each home (foreign) bank r to take into account its individual impacts through L_H^T on both the return on deposits $r^D\left(L_H^T\right) = r^D\left(D_H^T\right)$ and the return on loans $r^L\left(L_H^T\right)$ when choosing its amount of loans $L_{r,HH}$ ($L_{r,FH}$).

Each period t starts with a certain number of incumbent banks operating in both markets. The timing of ensuing events for market H is as follows. First, the aggregate shock a is realized. Second, based on the number of incumbents and the realization of a, new banks may decide to enter bringing the total number of active banks to $N^a = N/\left(1-\varrho\right)$ with $N_H^a = N_H/\left(1-\varrho\right)$ and $N_F^a = N_F/\left(1-\varrho\right)$ (see the law of motion (1)). Third, active banks simultaneously choose the amounts of loans $L_{r,HH}$

[11] The crucial restriction here is $p_{11}(r_H^I, a_H) < 0$.
[12] The crucial restriction here is $p_{12}(r_H^I, a_H) \geq 0$.

$(L_{r,FH})$ in market H separately from market F (due to their segmentation). Aggregation of these simultaneous individual decisions up to L_H^T determines loans and deposits returns r_H^L and r_H^D. Fourth, based on r_H^L and the realization of a, firms design their risk-return profiles by choosing r_H^I or equivalently $p(r_H^I, a)$. Fifth, uncertainty over projects' outcomes is resolved. Successful firms repay their loans and, whatever happens, depositors receive return r_H^D thanks to full insurance. Finally, exogenous exit takes place at rate ϱ. Surviving banks become the incumbents at the beginning of the next period.

Given this timing, the backward solution of the model requires us first to characterize the Cournot-Nash equilibrium of loan extension (deposit collection) for given numbers of active banks and then to endogenize those numbers through the entry condition (3).

3.3.1 Profit Maximization

Due to market segmentation, banks maximize profits independently in the two markets. In the case of market H, a bank r headquartered in H chooses $L_{r,HH}$ to maximize

$$\Pi_{r,HH} = p(r_H^I, a) \left(r^L \left(L_H^T \right) L_{r,HH} - r^D (D_H^T) D_{r,HH} - \xi D_{r,HH} \right),$$

whereas a bank s headquartered in F chooses $L_{s,FH}$ to maximize

$$\Pi_{s,FH} = p(r_H^I, a) \left(r^L \left(L_H^T \right) L_{s,FH} - r^D (D_H^T) D_{s,FH} - \xi D_{s,FF} - \mu L_{s,FH} \right),$$

subject to the constraint that local loans must match local deposits:

$$L_{r,HH} = D_{r,HH},\ L_{s,FH} = D_{s,FH}$$

as well as to the firms' first order condition (6), which implicitly defines the return of investment chosen by firms as a function of the loan rate: $r_H^I = r^I \left(r^L \left(D_H^T \right) \right)$. In doing so, banks are aware that their individual decisions affect aggregate loans (deposits):

$$\begin{aligned} L_H^T &= \sum_r L_{r,HH} + \sum_s L_{s,FH} \\ D_H^T &= \sum_r D_{r,HH} + \sum_s D_{s,FH} \end{aligned}$$

with $L_H^T = D_H^T$.

The first order condition for domestic bank r in its domestic market H is:

$$\begin{aligned} \frac{d\Pi_{r,HH}}{dL_{r,HH}} &= p(r_H^I, a)\left(r^L\left(L_H^T\right) - r^D(L_H^T) - \xi\right) + \\ &+ p(r_H^I, a)\left(r^{L\prime}\left(L_H^T\right) - r^{D\prime}(L_H^T)\right) L_{r,HH} + \\ &+ p_1(r_H^I, a) r^{I\prime}\left(r^L\left(L_H^T\right)\right) r^{L\prime}\left(L_H^T\right)\left(r^L\left(L_H^T\right) - r^D(L_H^T) - \xi\right) L_{r,HH} = 0 \end{aligned} \tag{7}$$

After the first equality, the first term is the ‘scale effect’. It is positive and represents the marginal gain from increasing one unit of bank scale (as measured by the total amount of loans and deposits). The second term is the ‘competition effect’. It is negative and captures the impacts of larger bank scale on deposit return ($r^{D\prime}\left(L_H^T\right) > 0$) and loan return ($r^{L\prime}\left(L_H^T\right) < 0$). More deposits and loans lead to a rise in the rate on deposits and a fall in the rate on loans. The third and last term is the ‘risk-taking effect’. It is positive and captures the effects of competition on the risk-return investment profile of firms. More loans decrease the loan rate and this in turn induces firms to select profiles associated with lower return and higher probability of success.

The profit maximizing choice of loans by foreign bank s in its foreign market H satisfies an analogous first order condition:

$$\begin{aligned} \frac{d\Pi_{s,FH}}{dL_{s,FH}} &= p(r_H^I, a)\left(r^L\left(L_H^T\right) - r^D(L_H^T) - \xi\right) + \\ &+ p(r_H^I, a)\left(r^{L\prime}\left(L_H^T\right) - r^{D\prime}(L_H^T)\right) L_{r,FH} + \\ &+ p_1(r_H^I, a) r^{I\prime}\left(r^L\left(L_H^T\right)\right) r^{L\prime}\left(L_H^T\right)\left(r^L\left(L_H^T\right) - r^D(L_H^T) - \xi - \mu\right) L_{s,FH} = 0, \end{aligned} \tag{8}$$

which differs from (7) only due to the presence of the additional monitoring cost μ. Analogous conditions hold for market F.

3.3.2 Cournot-Nash Equilibrium

We focus on a symmetric outcome in which in each market all home banks achieve the same scale $L_{r,HH} = L_{s,FF} = \ell$ and all foreign banks achieve the same scale $L_{s,FH} = L_{r,HF} = \ell^*$. In this case, in each market total loans (and deposits) are:

$$L^T = \frac{N}{1-\varrho}\left(\ell + \ell^*\right). \tag{9}$$

For given N, in each market the Cournot-Nash equilibrium (in any period t) is characterized by the solution of the following system of two equations in the two unknown scales ℓ and ℓ^*:

$$\begin{aligned} & p(r^I, a)\left(r^L\left(L^T\right) - r^D(L^T) - \xi\right) + \\ & + p(r^I, a)\left(r^{L\prime}\left(L^T\right) - r^{D\prime}(L^T)\right)\ell + \\ & + p_1(r^I, a) r^{I\prime}\left(r^L\left(L^T\right)\right) r^{L\prime}\left(L^T\right)\left(r^L\left(L^T\right) - r^D(L^T) - \xi\right)\ell = 0 \end{aligned} \tag{10}$$

and

$$\begin{aligned} & p(r^I, a)\left(r^L\left(L^T\right) - r^D(L^T) - \xi\right) + \\ & + p(r^I, a)\left(r^{L\prime}\left(L^T\right) - r^{D\prime}(L^T)\right)\ell^* + \\ & + p_1(r^I, a) r^{I\prime}\left(r^L\left(L^T\right)\right) r^{L\prime}\left(L^T\right)\left(r^L\left(L^T\right) - r^D(L^T) - \xi - \mu\right)\ell^* = 0, \end{aligned} \tag{11}$$

where, exploiting symmetry between markets, we have dropped the market index from all variables.

With explicit time dependence reinstated to avoid confusion, the values of ℓ_t and ℓ_t^* that solve system (10)-(11) determine the maximized values of domestic profits Π_t and foreign profits Π_t^*. These are the same for all banks ($\Pi_{t,HH} = \Pi_{t,FF} = \Pi_t$ and $\Pi_{t,HF} = \Pi_{t,FH} = \Pi_t^*$) and are functions of the number of active banks N_t^a. In turn, the equilibrium number of active firms is pinned down by the free entry condition described in Section 3.1, which with symmetry becomes

$$\Pi_t + \Pi_t^* = \left[1 - \beta(1 - \varrho)\right]\kappa \tag{12}$$

in the determinist environment and

$$V_t = \Pi_t + \Pi_t^* + \beta(1 - \varrho)\mathbb{E}_t\left\{V_{t+1}\right\} = \kappa \tag{13}$$

in the stochastic environment. Finally, the equilibrium values of ℓ_t, ℓ_t^* and N_t^a determine the equilibrium deposit return r_t^D, loan return r_t^L, and risk-return profile $(r_t^I, p(r_t^I, a_t))$. Given the number of incumbents, they also determine the equilibrium number of entrants by (1). The fact that the equilibrium of the two national markets can be characterized by such a parsimonious set of equations is obviously due to the assumption that the two markets are symmetric.

4 Qualitative and Quantitative Implications

Below we assess the qualitative and quantitative channels of our model by relying on analytical and numerical results. For the analytical results we focus on the 'long-run' relation between banks' competition and risk-taking, with no productivity shocks ($a = 1$) and entry conditions as in equation (12). While these results also require no additional monitoring cost for foreign loans ($\mu = 0$), we also provide a numerical solution of the long-run equilibrium using Newton-Raphson iterative methods. This allows us to check which analytical results obtained for $\mu = 0$ keep on holding for a wide range of this parameter.

Next we consider a 'short-run' environment which is stochastic (with productivity shocks following an AR(1) process) as well as dynamic (with entry conditions as in (13) and law of motion for the number of banks as in (1)). In this case we rely on numerical results obtained from empirically grounded calibration and estimation of shocks and parameters. Calibration for both the long- and short-run simulation exercises is based on a combination of micro data and method of moments estimation. The targets for data matching and estimation are given by both average long-run values and business cycle industry statistics. A detailed discussion is provided in Section 4.3 devoted to the dynamic stochastic simulations. The long-run results in Section 4.2 are based on the same calibration.

4.1 Functional forms

To investigate the equilibrium behavior of the model, we select specific functional forms that comply with the properties detailed in Section 3.2. In the wake of Boyd and De Nicolo [5] and Martinez-Miera and Repullo [31], we assume that the demand of loans and the supply of deposits take the following forms:

$$\begin{array}{lll} r^L\left(L_t^T\right) = \frac{a_t}{\alpha} - \beta_1 L_t^T & \text{with} & \beta_1 > 0, \\ r^D(D_t^T) = \gamma D_t^T & \text{with} & \gamma > 0. \end{array} \tag{14}$$

We also assume that investment projects succeed with probability:

$$p(r_t^I, a_t) = \begin{cases} a_t\left(1 - \alpha r_t^I\right) & \text{for } r^I \in [0, 1/\alpha] \\ 0 & \text{otherwise} \end{cases}. \tag{15}$$

Hence, for given returns, larger a_t increases the demand of loans by (14), the productivity of projects by (4) as well as their success probability by (15). Accordingly, we will refer to larger (smaller) a_t as better (worse) 'investment climate'. Differently, larger α decreases loan demand as well as projects' success probability without affecting their productivity. We assume that projects are symmetric and perfectly correlated.[13]

4.2 Deterministic Equilibrium

We characterize the deterministic equilibrium in two steps. First, we provide an analytical assessment for the simpler case in which $\mu = 0$. Then, we assess the role of banking globalization (as captured by an reduction of μ) through numerical simulations.

As with $a_t = 1$ all variables are constant, we drop the time subscript. We can then use (14) and (15) with $a = 1$ and $D^T = D^T$ to rewrite firms' first order condition (6) as:

$$r^I = \frac{1}{\alpha} - \frac{\beta_1}{2} L^T, \tag{16}$$

with associated success probability:

$$p = \frac{\alpha\beta_1}{2} L^T. \tag{17}$$

These expressions show that more loans (and thus more deposits) make firms choose investments with lower return and higher probability of success (i.e. with more cautious risk-return profile). As for banks' first order conditions, (10) and (11) can be rewritten respectively as

$$L^T \left[\frac{1}{\alpha} - (\beta_1 + \gamma) L^T - \xi\right] + \left[\frac{1}{\alpha} - 2(\beta_1 + \gamma) L^T - \xi\right] \ell = 0 \tag{18}$$

and

$$L^T \left[\frac{1}{\alpha} - (\beta_1 + \gamma) L^T - \xi - \mu\right] + \left[\frac{1}{\alpha} - 2(\beta_1 + \gamma) L^T - \xi - \mu\right] \ell^* = 0, \tag{19}$$

where we again focus on the symmetric Cournot-Nash equilibrium, in which in both national markets all home banks choose the same amount of loans ℓ_{ss} and all foreign banks choose the same amount of loans ℓ^*_{ss}. Henceforth, we will use subscript ss to denote the values of all variables in

[13] We will relax this assumption in Section 5.4.

the deterministic equilibrium. Note that conditions (18) and (19) imply that in such equilibrium, foreign banks facing the additional monitoring cost $\mu > 0$ end up being smaller than their home competitors. Indeed, for any given L^T, if the (18) holds for $\ell = \ell_{ss}$, then (19) can hold only for $\ell^* = \ell^*_{ss} < \ell_{ss}$. Moreover, larger μ is associated with smaller ℓ^*_{ss} relative to ℓ_{ss}, with ℓ^*_{ss} going to zero for large enough μ. To summarize, when foreign banks face an additional monitoring cost, they are smaller than their home competitors. The more so, the higher the monitoring cost. When the monitoring cost is high enough, foreign banks do not operate in the home market.

Having discussed the role of $\mu > 0$, in order to further understand the role of the other parameters of the model, it is useful to focus on the special case in which foreign banks face no additional monitoring cost ($\mu = 0$). In this case, (18) and (19) are identical and can be solved for:

$$L^T_{ss}(N^T_{ss}) = N^T_{ss} d_{ss}(N^T_{ss}) = \frac{\frac{1}{\alpha} - \xi \, \frac{\frac{N^T_{ss}}{1-\varrho}}{\frac{N^T_{ss}}{1-\varrho}+2}}{\beta_1 + \gamma \, \frac{\frac{N^T_{ss}}{1-\varrho}+1}{\frac{N^T_{ss}}{1-\varrho}+2}} \tag{20}$$

with $N^T_{ss}(1-\varrho)$ denoting the total number of active banks and $N_{ss}/(1-\varrho) = N^T_{ss}(1-\varrho)/2$ denoting the common number of home and foreign banks. Expression (20) shows that, as the number of active banks $N^T_{ss}(1-\varrho)$ increases, total loans $L^T_{ss}(N^T_{ss})$ also increase. Expressions (16) and (17) then imply that, when more banks are active, firms target projects with lower return $r^I_{ss}(N^T_{ss}) = 1/\alpha - \beta_1 L^T_{ss}(N^T_{ss})/2$ and higher success probability $p_{ss} = \alpha\beta_1 L^T_{ss}(N^T_{ss})/2$. This is the net outcome of two opposing forces. On the one hand, increasing the number of banks strengthens banks' competition for deposit funds, weakening their oligopsony power in the deposits market and thus raising the return on deposits as well as the total amount of deposits. For a given spread of the loan rate over the deposit rate $r^L - r^D$, a larger number of active banks would increase the deposit rate r^D, therefore inducing firms to take more risk as r^L would also increase. On the other hand, a larger number of active banks also strengthens competition in loans provision, weakening their oligopoly power in the loan market and thus reducing the return on loans r^L for any given deposit rate r^D. Under the assumptions embedded in the chosen functional forms, the downward pressure on the loan rate dominates the upward pressure on the deposit rate, which induces firms to reduce return and risk. Hence, more competition due to a larger number of home and foreign banks makes firms target investments with lower return and lower probability of failure.

Thus far we have taken the number of active banks as exogenously given. Free entry implies, however, that this number is endogenously determined by (12):

$$\pi_{ss}(N_{ss}^T) = \frac{\alpha\beta_1\left(\frac{1}{\alpha}-\xi\right)^3}{(\beta_1+\gamma)^2} \frac{\left(\frac{N_{ss}^T}{1-\varrho}+1\right)^2}{\frac{N_{ss}^T}{1-\varrho}\left(\frac{N_{ss}^T}{1-\varrho}+2\right)^3} = \left[1-\beta(1-\varrho)\right]\kappa. \tag{21}$$

Implicit derivation of (21) shows that stronger demand of loans by firms and higher success rate of their investments (as captured by lower α) cause a rise in the number of active banks given $dN_{ss}^T/d\alpha < 0$. This is accompanied by a higher number of entrants as in equilibrium (1) implies $N_{e,ss}^T = \varrho N_{ss}^T/(1-\varrho)$. By (20), larger N_{ss}^T leads to a rise in both total and per-bank loans: $dL_{ss}^T/d\alpha < 0$ and $d\ell_{ss}/d\alpha < 0$. Then, by (14), falling α and rising L_{ss}^T lead (on net) to higher rates on deposits and loans: $dr_{ss}^D/d\alpha < 0$ and $dr_{ss}^L/d\alpha < 0$. Finally, by (16) and (17), falling α and rising L_{ss}^T also determine (on net) a rise in firms' success rate and in their return on investment: $dp_{ss}/d\alpha < 0$ and $dr_{ss}^I/d\alpha < 0$. Hence, stronger demand of loans by firms and higher success rate of their investments lead to an expansion of the banking sector along both the extensive margin (number of active banks) and the intensive margin (deposits and loans per bank). Returns to deposits, loans and investment all rise. Firms target less risky projects.

The effects of lower insurance premium ξ are similar, though less complex as they are channeled only through smaller N_{ss}^T and L_{ss}^T (as ξ appears only in (20) and (21)). Those of lower entry cost κ are also similar but even more straightforward as they are channeled only through N_{ss}^T (as κ appears only in (21)).

When banks face additional monitoring costs for their foreign operations, we have to resort to numerical investigation as analytical results are hard to obtain for $\mu > 0$. In particular, we compute the deterministic equilibrium through Newton-Raphson iterations of the model's system of equations. Our 'endogenous risk' refers to the overall default probability $1-p(r^I, a)$. As projects are perfectly correlated across firms, this probability corresponds also to the aggregate default risk, hence to endogenous systemic risk.[14]

[14] More generally, however, when projects are imperfectly correlated across firms, systemic risk is not necessarily equivalent to $1-p(r^I, a)$. Martinez-Meira and Repullo [31] show how the aggregate endogenous risk metric shall change when idiosyncratic project failures are driven by a latent factor à la Vasicek [42] and projects are imperfectly

Figure 3 describes how 'banking globalization' (lower μ) affects all the endogenous variables in the model under our calibration.[15] In the panels of this figure the different variables are reported on the vertical axis, while μ increases rightward along the horizontal axis. Hence, the effects of banking globalization can be read moving leftward. Indeed, as μ falls, the number of banks rises. Furthermore, the figure show that falling μ is accompanied also by an increase in the market share of foreign banks. Deposits and loans per capita increase for foreign banks and fall for domestic banks (second panel in the right column). Intensified competition leads to an increase in the total amount of deposits and loans, a decrease in the return on loans and an increase in the return on deposits. As a consequence, the spread between loan and deposit rates shrinks. As for firms, lower loan rates make them more cautious, targeting projects with lower return and higher probability of success. Despite more caution, the spread between the returns on investment and loans increases, whereas the spread between the returns on loans and deposits decreases. Finally, note that for all values of μ the spread between loan and deposit rates is smaller for foreign than home banks once the monitoring cost is netted out. This reveals that banks practice 'dumping' in the sense of Brander and Krugman [6]: they are willing to accept a lower spread for their foreign operations than for their domestic ones and thus do not pass on the full additional costs of foreign operations to their customers. This happens as banks perceive higher elasticities of loans demand and deposits supply in their foreign market given that their market share is smaller there, and explains why costly cross-hauling of identical banking services by banks headquartered in different national markets arises in equilibrium despite additional monitoring costs. The partial absorption of the additional monitoring costs by foreign banks becomes less pronounced as μ falls, driving the perceived elasticities of loans demand and deposits supply in their foreign market closer to the ones in their home market.

correlated. In Section 5.4 we will show that changing the risk metric can quantitatively affect the responses of the risk variables, but does not change the agents' optimization behavior and the incentives behind the model mechanics. For this reason, in this section we focus on the simpler limiting case of perfectly correlated projects as our baseline.

[15] See Section 4.3 for details on the calibration exercise and Table 1 for the resulting calibrated parameters.

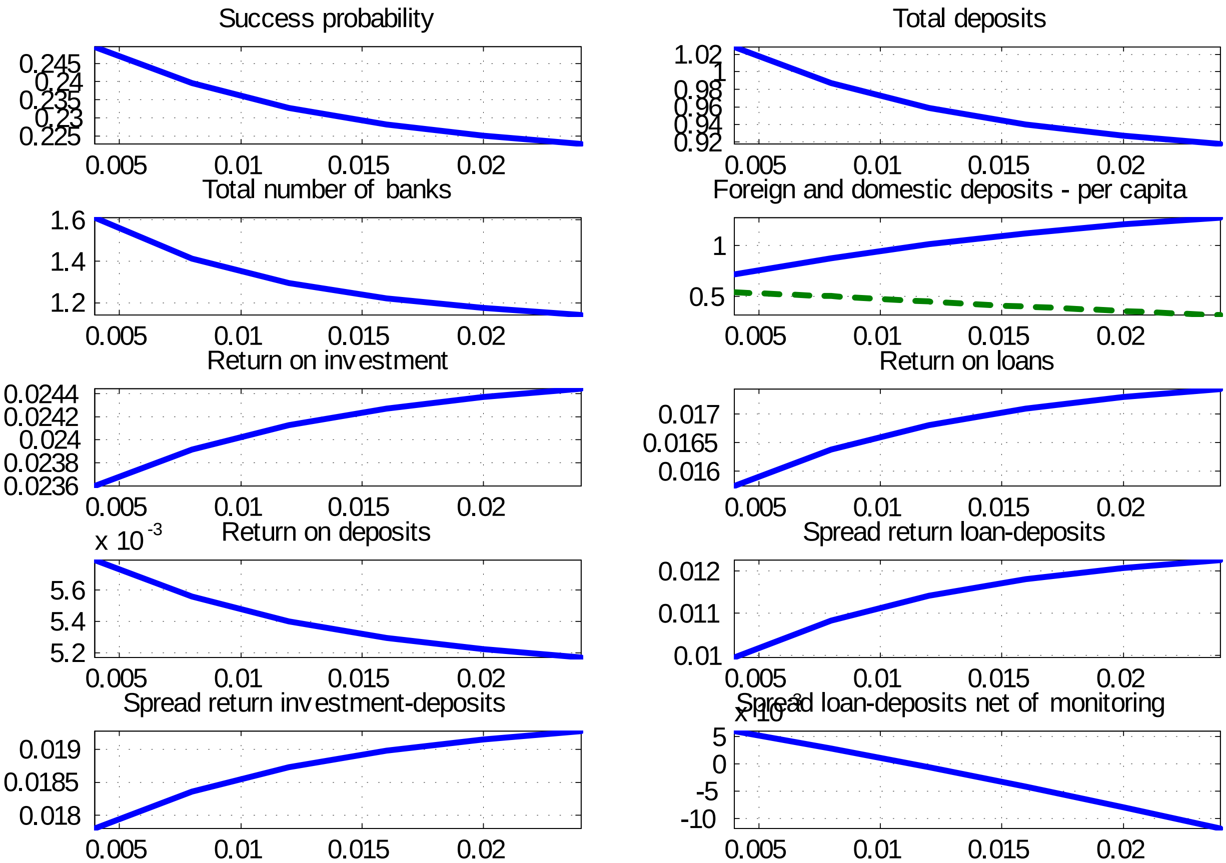

Figure 3: Steady state values of selected variables when changing monitoring cost, μ.

4.3 Stochastic Equilibrium

We now investigate how the banking sector reacts when the investment climate is subjected to productivity shocks modelled through a Markov stationary process. Specifically, we choose an autoregressive process and look at the impulse responses of endogenous risk (firm default probability), bank entry, deposits/loans of domestic and foreign banks, and the return on loans. We simulate how these responses change depending on the monitoring cost (μ), the deposit insurance premium (ξ), the entry cost (κ) and the demand of loans (α). To make the implications of the model as realistic as possible, we determine the model parameters' baseline values through a combination of calibration and estimation based of the method of moments. An important aspect of the dynamic stochastic analysis is that expectations about banks' future profits (charter value) will play a role.

4.3.1 Calibration

Most parameters in the calibration are set primarily to match average long-run values of all variables in the model. Table 1 shows the calibrated parameters, Table 2 the implied long-run steady-state values of the different variables. The remaining parameters are estimated so as to target the second moment of the entry rate in the banking industry.

In detail, in log-linear terms the productivity shock is assumed to follow the AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t^a$. It is calibrated based on sectoral data following Iacoviello and Neri [27], who estimate a persistence of 0.95 and a standard deviation of 0.01 for consumption-good producing technology in a multi-sector model of the US economy. The discount factor β is set so as to imply a 4% annual risk-free interest rate. The calibration of the intermediation spread, $r^L - r^D$, follows Repullo and Suarez [38], who report an annual spread of roughly 4% based on FDIC statistics for US banks. This is achieved by setting α, γ and β_1 in the model so as to obtain a steady-state bank margin of 3.98%. The calibration of the insurance cost ξ is based on FDIC insurance fees (insurance assessment rates). These range from 2.5 to 10 basis points for a typical bank, but can go up to 45 basis point depending on a bank's risk characteristics, in particular its equity ratio.[16] Since in our model banks do not have equity as an additional loss absorber, we set ξ to the FDIC's maximum

[16] See See https://www.fdic.gov/bank/analytical/qbp/2015dec/dep4c.html..

Table 1: **Calibration of parameters.**

Parameter	Mnemonics	Value
Discount factor	β	0.99
Functional form $p(L^T, a)$	α	31.80
Functional form of r^L	β_1	0.0153
Functional form of r^D	γ	0.0056
Persistence of productivity	ρ_a	0.95
Standard deviation of productivity	σ_a	0.01
Monitoring cost	μ	0.004
Exit probability	ϱ	0.0125
Insurance fee	ξ	0.0011
Entry cost	κ	0.10
Persistence of entry cost	ρ_κ	0.95
Monitoring cost	σ_κ	0.01

fee of 45 basis points annually. The value for μ is based on data from banks' loan loss provisions. In the euro area, these amounted to 40 basis points of assets on average for the pre-crisis period (1991-2003), hence we set to 0.004.[17] In the model ϱ determines the ratio of entering to active banks (entry rate). This rate can be calculated based on the bank ownership database of Claessens and Van Horen [13]. In doing so, we count all foreign offices of US banks in the database in a given year (N_t) and define the number of entering banks as all banks that become active in a given year and were inactive in the respective country in the preceding year (N_t^e). The entry rate is then calculated as N_t^e/N_{t-1} and found to have a pre-crisis (1996-2007) average of 5.1% and a standard deviation of 3.3%. The former can be matched in the model with an appropriate choice of ϱ, which we set to 0.0125. In order to match the latter, we resort to the calibration of the following process:

$$\kappa_t = (1 - \rho_k)\overline{\kappa} + \rho_\kappa \kappa_{t-1} + \varepsilon_t^k. \tag{22}$$

Specifically, after normalizing $\overline{\kappa}$ to 0.1, we estimate ρ_κ and the associated standard deviation σ_κ via a grid search minimizing the squared distance between the entry rate volatility implied by the model and the one observed in the data. Table 3 reports the corresponding results.

[17] See https://www.ecb.europa.eu/pub/pdf/other/mb200403f ocus02:en:pdf?600fe329cdc84c23b26b8c432b83a75d:

Table 2: **Steady-state values.**

Description	Variable	Value
Success probability	$p(L^T)$	0.25
Loan return	r^L	0.0157
Deposit return	r^D	0.0058
Project return	r^I	0.0236
Bank pro ts domestic	Π	0.0016
Bank pro ts abroad	Π^*	0.0007
Number of banks (normalized)	N	0.8047
Number of entering banks (normalized)	N^e	0.0102
Bank value (normalized)	V	0.1
Deposits domestic	ℓ	0.7170
Deposits abroad	ℓ^*	0.5449
Total deposits	L^T	1.0285

Table 3: **Entry rate moments in benchmark model.**

Variable	Model	Data
Entry rate mean	5.06	5.1
Entry rate std. dev.	3.25	3.3

4.3.2 Simulation

To assess how the banking sector reacts to changes in the investment climate (a_t), we present impulse responses of selected variables to a 1% aggregate productivity shock common to the two national markets. We perform stochastic simulations using higher order Taylor expansions of our model around the deterministic equilibrium.[18] Parameters are calibrated as described in the previous section. Our focal exercise on the effects of banking globalization looks at how impulse responses change for different values of the foreign monitoring cost (μ). Nonetheless, before describing the corresponding results, we discuss also those related to different values of the entry cost (κ), the insurance premium (ξ) and the parameter regulating the demand of loans as well as the success rate of projects (α). This will give insight on the how various channels operate in the model. In Appendix B we provide the full set of equations used in the simulations of the dynamic model. Here we present the main findings.

[18] See Judd [26].

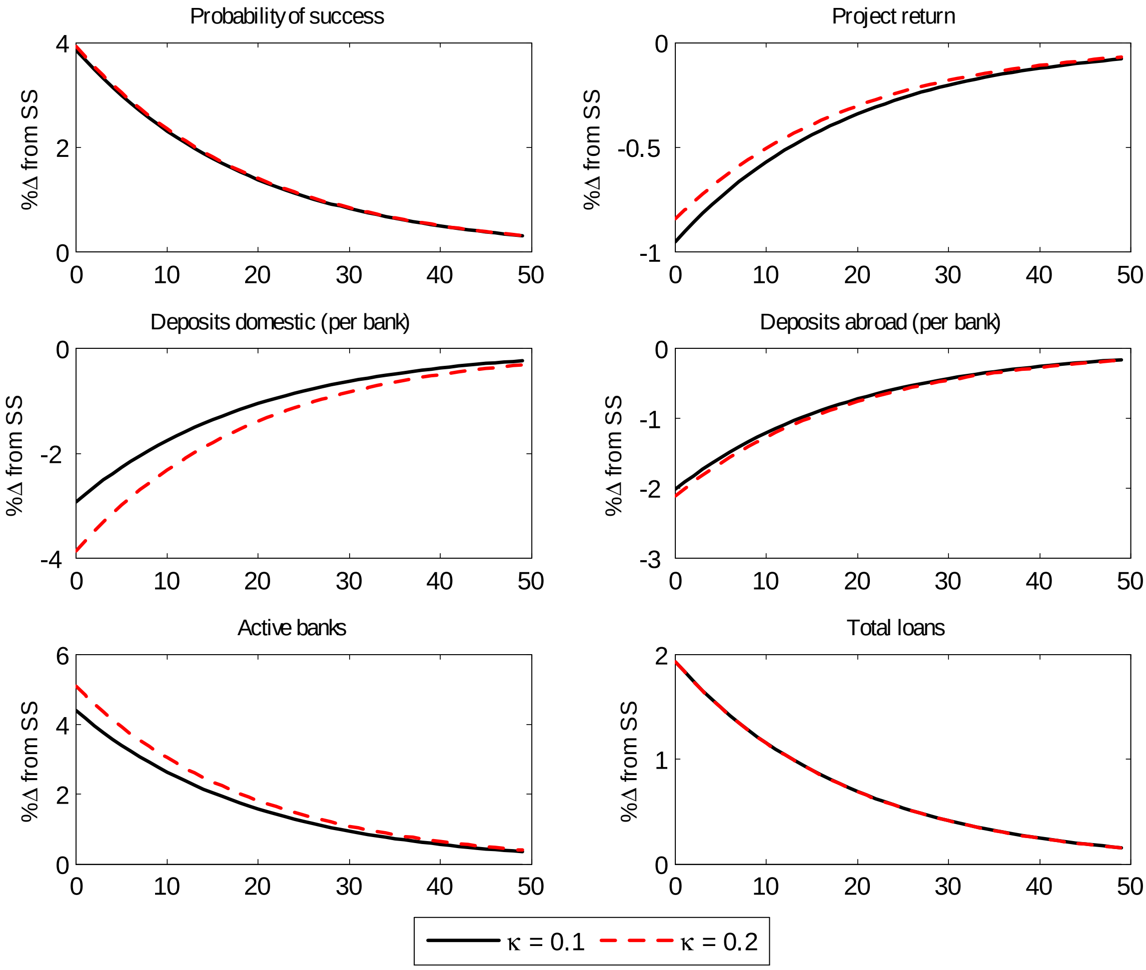

Figure 4: Impulse responses of selected variables to a positive productivity shock for different values of κ.

Figure 4 shows the impulse responses for higher and lower values of the entry cost κ.

In both cases an increase in productivity increases project success probability (first panel to the left). This comes at the cost of lower project returns (first panel to the right). Despite lower margins, improved project selection, due to the more cautious risk-return profile, increases banks' profits (today and in the future), hence their scale. The associated rise in banks' charter value increases the number of active banks (last panel to the left), which in turn increases aggregate credit supply (last panel to the right). Increased competition due to a larger number of banks reduces the market share of each of them (second left and right panels). Overall, the transmission mechanism shows that lower default probability (bank risk) is associated with higher competition. Finally, as loan supply rises, banks increase their deposit demand. To do so, they offer higher deposit rates, which in turn translates into higher loan rates (both not shown for brevity). The bank's response is weaker for foreign than for domestic deposits as the former face additional monitoring costs. All variables react more to the productivity shock when the entry cost is higher. The only exception is project return, for which the opposite holds.

An alternative way to interpret the role of the entry cost is to resort upon *option value theory.* Banks enter when their future sum of discounted profits equates the entry cost. By solving recursively equation (2) we can express this condition as:

$$\kappa = \mathbb{E}_t \left\{ \sum_{z=t}^{\infty} (\beta(1-\varrho))^{z-t} \left(p(r_t^I, a_t) \left(r_t^L \left(L_t^T \right) - r_t^D (L_t^T) - \xi \right) (\ell_t + \ell_t^*) - \mu \ell_t^* \right) \right\} \tag{23}$$

The *option value* of opening a new branch or subsidiary is given by the discounted sum of future banks' rents, hence it also captures the charter value of banks. The higher the entry barrier is, the higher are the rents a bank extracts to satisfy condition (23). Rents' extraction is reflected in the fact that the bank sets lower loan rates (and achieves larger market shares) by a larger extent when κ is larger. In other words, banks' predatory incentives are stronger when the entry cost is higher. This effect holds for all banks, indeed higher entry costs induce higher overall profits and induce more banks to enter (last panel on the left). Once again and as before, effects are asymmetric and weaker for foreign operations.

Figure 5 examines the impulse responses for higher and lower values of the insurance premium ξ.

The qualitative patterns are the same as before for all variables. All variables react more to the productivity shock when the insurance premium is higher with the only exception of project return, for which the opposite holds. The size of the insurance premium changes the extent of the risk-taking channel. When banks pay a higher premium, they effectively bear a higher share of the losses as they materialize. Hence, a higher insurance premium disciplines banks and reduces the extent of risk-taking. This explains why, with higher values of ξ the increase in project success rate (the fall in endogenous risk) is larger, while the fall in project return is more muted.

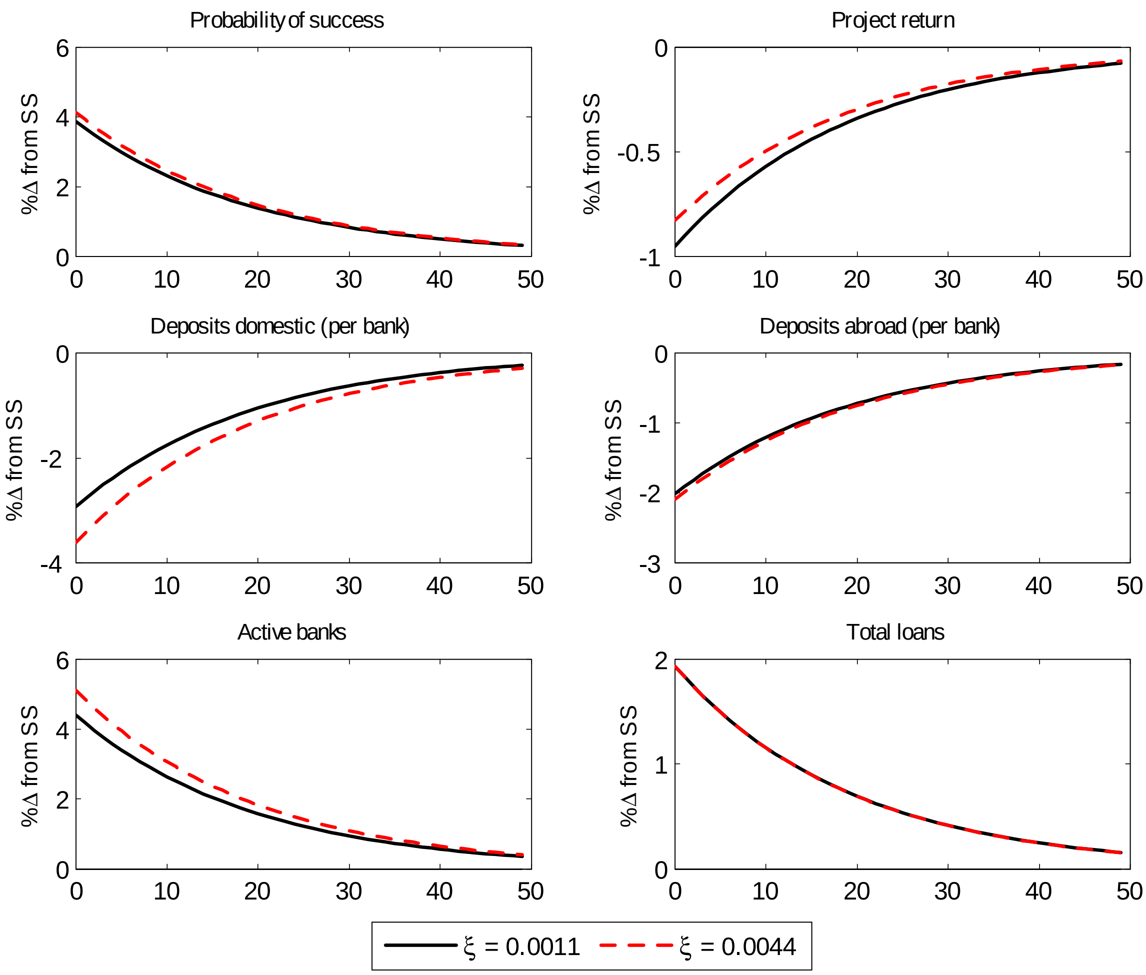

Figure 5: Impulse responses of selected variables to a positive productivity shock for different values of ξ.

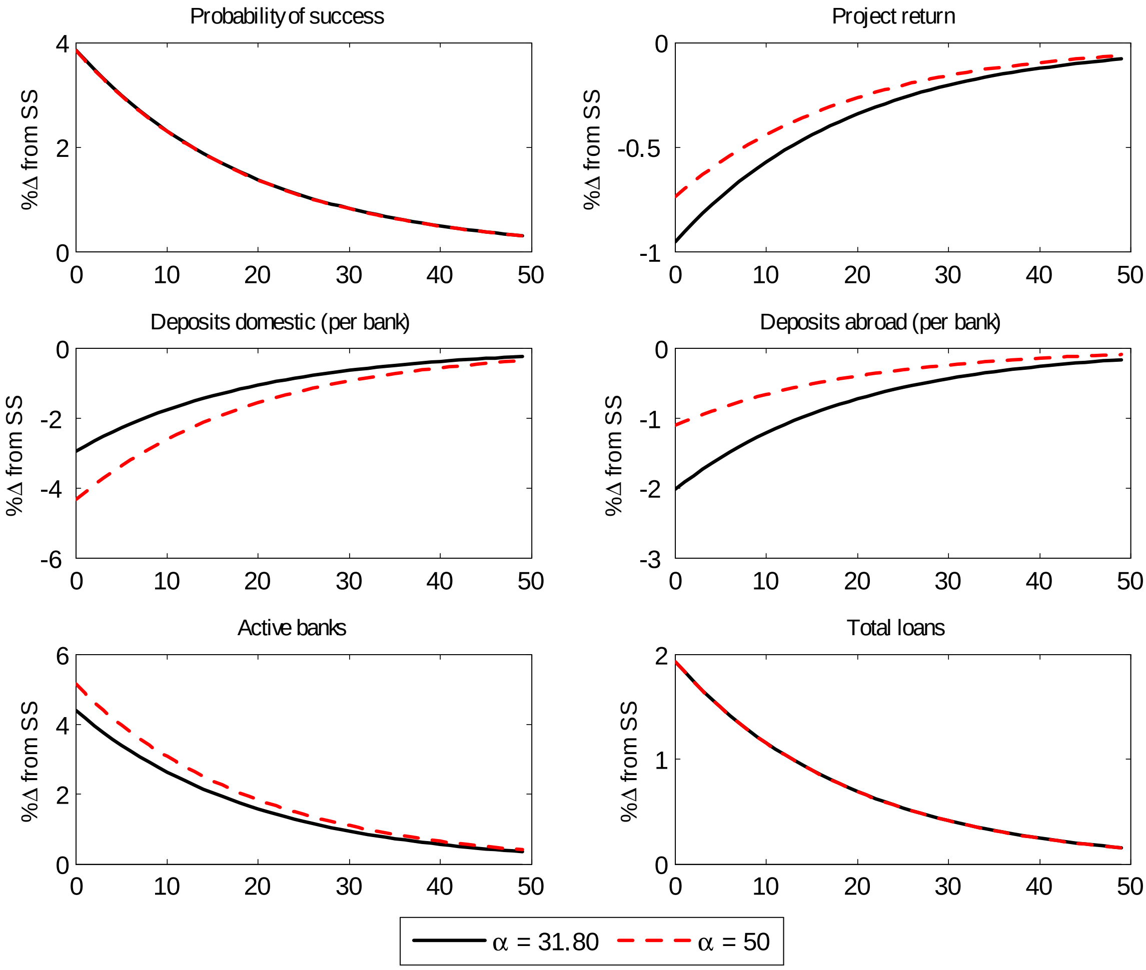

Figure 6: Impulse responses of selected variables to a positive productivity shock for different values of α.

Figure 6 shows the response to the positive productivity shock for higher and lower values of α.

The responses of the selected variables are to be interpreted as before. Higher α increases the number of active banks and total loan supply, but reduces the share of domestic banks. Higher loan supply implies that the fall in project return (second panel to the right) is more muted. Since projects are marginally more profitable, the overall number of active banks raise.

Finally, Figure 7 shows the response for different values of the foreign monitoring cost μ with lower values capturing more bank globalization.

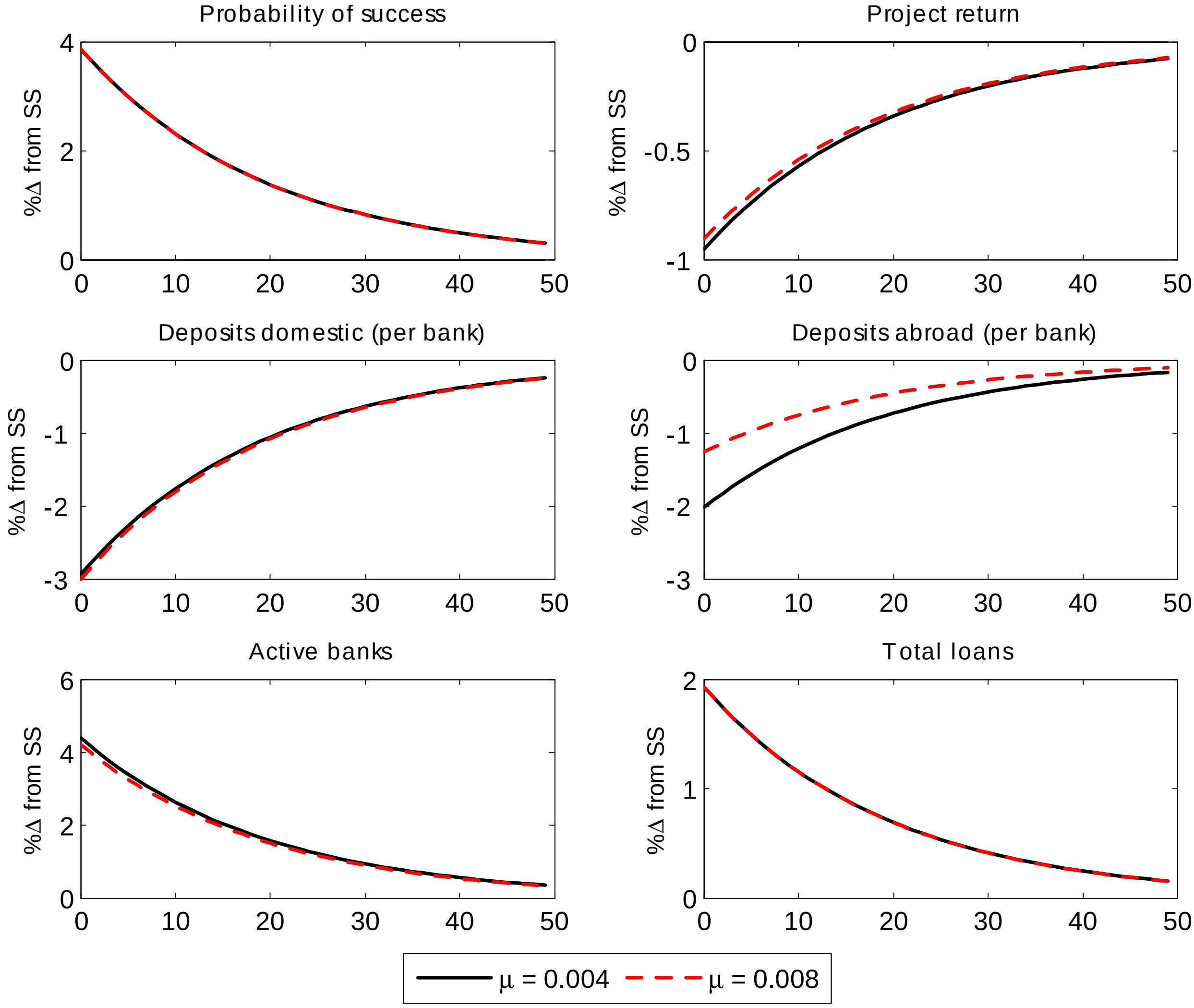

Figure 7: Impulse responses of selected variables to a positive productivity shock for different values of μ ($\xi = 0.53$).

Once again the qualitative transmission of shocks is confirmed. Lower monitoring costs increase banks' market shares abroad (second left and right panels). This reduces the incentives for predatory banking through which banks accept lower profit margins abroad in order to penetrate foreign markets. Moreover, with lower monitoring costs, the number of active banks and the retun on projects react more to the productivity shock.

We can summarize the channels at work as follows. First, an improvement in the investment climate (positive productivity shock), by fostering entry and competition in loan markets, reduces risk-shifting incentives and induces banks to select portfolios of investments with higher probability to succeed (see also Boyd and De Nicolo [5]). As a result, a better investment climate leads to a fall in risk. Second, due to additional monitoring costs on foreign loans, banks behave in foreign markets in a predatory way ('dumping'), accepting lower profit margins abroad than at home (see Brander and Krugman [6] for a similar effect in the trade literature). This effect in isolation would reduce banks' margins for their non-defaulting loans (see Martinez-Miera and Repullo [31]) and jeopardize their portfolios' sustainability, thereby increasing their risk. Third, as entry is endogenous, shifts in the loan curves also change banks' relative market shares. By reducing loan rates, banking globalization increases foreign market shares. Overall, this dampens the fall in per period banks' margins for non-defaulting loans. The increase in market shares raises the value of a bank that continues to do business in the future (i.e. its 'charter value'; see Vives [43]), and this ends up reducing its overall risk.

5 Further Issues and Extensions

While the model presented in the previous section is already quite rich in ingredients and implications, there are additional issues worth exploring. First, internationalization for banks can take place in different forms. So far we have explored the possibility of multinational banking, which materializes through the opening of branches or subsidiaries in a foreign country that raise deposits and extend loans locally. An alternative to this business model is cross-border lending whereby banks foreign operations are restricted to loan provision. The difference between these business models might be relevant in terms of risk-taking behavior. Our results in Section 4 show that expansion by multinationals can indeed reduce risk-taking. It is worth examining whether expansion through cross-border activity can lead to different conclusions. We do so in Section **??** where we show that cross-border lending is associated with more risk-taking than multinational banking.

Second, a reason for banks to enter foreign markets is that this amplifies the scope of their investment possibilities and allows them to improve risk-sharing. This can happen to the extent that countries experience asymmetric and partially correlated shocks rather than symmetric shocks as in Section 4. Section 5.2 studies how the implications of our model change when we accommodate asymmetric shocks across countries. It shows that, for a given degree of correlation, the qualitative responses of variables to a productivity shock discussed in Section 4 are confirmed. However, the amplitude of these responses depends on the degree of correlation with more correlation leading to smaller changes in aggregate profits, in aggregate loans/deposits, in the number of active banks as well as in risk-taking.

Third, so far we have assumed that banks choose endogenously whether to enter, but that exit is determined by exogenous factors. In practice, however, the choice to exit is also determined endogenously and is affected by relocation or other adjustment costs with banks choosing to remain operative as long as their future discounted profits (charter values) are larger than the exit cost. In this respect, an interesting case arises when the entry cost is smaller than the exit cost. In such case there is a region of inaction in the space of shocks: for some realizations of the shocks banks' total discounted profits are lower than the entry cost but still higher than the exit cost. When this happens, there is neither exit of incumbent banks nor entry of new banks even though the free entry conditions do not hold with equality. Only when total discounted profits become low enough to fall short of the exit cost, do incumbents leave the market. This inertia associated with exit decisions may be important as it endogenously affects competition and thus risk-taking in the banking sector. It is, therefore, important to assess the robustness of our results in Section 4 to an alternative specification of the model that includes endogenous exit decisions. This is what we do in Section 5.3, which shows that, when endogenous exit is associated with liquidity shocks, the patterns described in Section (4) are confirmed in qualitative terms. Endogenous exit adds, however, an extra selection mechanism that dampens the reactions of all variables to productivity shocks.

Finally, the measure of bank risk we have considered so far is based on the assumption that all projects succeed with probability $p(r^I, a)$ (and fail conversely). Moreover, the fact that the realization of the aggregate productivity shock is observed before any decision is made by firms and banks implies that the probability of banks' portfolio failure (the metric for banks' systemic risk) is equal to the simple average of the probability of project failure, which is obviously again $p(r^I, a)$. In reality such an extreme risk correlation across projects is hardly observed and aggregate shocks occur also after banks have made their portfolio decisions, in which case banks' portfolio may fail ex post despite the control banks have on $p(r^I, a)$ through the loan rate ex ante. It is thus of interest to check how our findings change when projects have less extreme degrees of risk correlation and additional shocks happen after banks have already made irreversible portfolio decisions. Section 5.4 extends the model in this direction to allow for imperfect correlation of projects' outcomes due to common (systematic) and idiosyncratic ex post shocks. It shows that the result of Section 4 that banks' competition decreases risk applies to the case of imperfectly correlated projects' returns if the expansionary impact of competition on active banks' profits through total loans and deposits is strong enough to offset its parallel contractionary impact through the lending-to-deposit rate spread.

5.1 Cross-Border Lending??

The business model of multinational banks is one in which internationalization takes place through horizontal expansion, while the business model of cross-border lending is one in which internationalization takes place through vertical integration. We assume that, differently from multinational banks, cross-border lenders have a lighter foreign presence. This can be captured by a lower setup cost for foreign operations, which we normalize to zero. Accordingly, the overall fixed cost of a cross-border lender is $\kappa - \kappa^d$, where κ and κ^d are the overall fixed cost and the subsidiary setup cost of a multinational bank respectively.

A cross-border lender r headquartered in market H raises deposits $D_{r,H}$ in its domestic market and allocates them to domestic loans $L_{r,HH}$ and foreign loans $L_{r,HF}$. We use $D_{r,HH}$ and $D_{r,FH}$ to denote the complementary amounts of deposits allocated to loans in H and F respectively, so that we have $D_{r,HH} = L_{r,HH}$, $D_{r,FH} = L_{r,FH}$ and $D_{r,H} = D_{r,HH} + D_{r,HF} = L_{r,HH} + L_{r,HF}$. The lender then chooses $L_{r,HH}$ and $L_{r,HF}$ so as to maximize expected profit:

$$\begin{aligned}
\Pi_H &= p(r_H^I, a_H)\left(r_H^L\left(L_H^T\right)L_{r,HH} - r_H^D(D_H^T)L_{r,HH} - \xi L_{r,HH}\right) \\
&\quad + p(r_F^I, a_F)\left(r_F^L\left(L_F^T\right)L_{r,HF} - r_H^D(D_H^T)L_{r,HF} - \xi L_{r,HF} - \mu L_{r,HF}\right) \\
&\quad - \left(\kappa - \kappa^d\right).
\end{aligned}$$

The first order condition for profit maximization is:

$$\begin{aligned}
\frac{\partial \Pi_H}{\partial L_{r,HH}} &= p_1(r_H^I, a_H) r_H^{I\prime}\left(r_H^L\right) r_H^{L\prime}\left(L_H^T\right)\left(r_H^L\left(L_H^T\right)L_{r,HH} - r_H^D(D_H^T)L_{r,HH} - \xi L_{r,HH}\right) \\
&\quad + p(r_H^I, a_H)\left(r_H^{L\prime}\left(L_H^T\right)L_{r,HH} + r_H^L\left(L_H^T\right) - r_H^{D\prime}(D_H^T)L_{r,HH} - r_H^D(D_H^T) - \xi\right) \\
&\quad - p(r_F^I, a_F) r_H^{D\prime}(D_H^T) L_{r,HF} = 0.
\end{aligned} \tag{24}$$

Note that, as higher $L_{r,HH}$ increases interest payments also for deposits used for $L_{r,FH}$, the lender's first order condition can not be separated between markets as it was the case with multinational banks. This generates a novel trade-off. On the one hand, as $r_H^D(D_H^T)$ increases with D_H^T, being forced to tap a single market for deposits drives the deposit return up, which by itself would increase the loan rate. On the other hand, the lack of foreign competition for domestic deposits puts downward pressure on the deposit return, which by itself would decrease the loan rate. Hence, for the same number of banks, it is not obvious whether one should expect cross-border lending to lead to more or less risk taking than multinational banking.

For simplicity, we focus on the symmetric deterministic equilibrium with $\mu = 0$ and $a = 1$. In this case, symmetry implies that in equilibrium the total amount of loans offered by home and foreign banks in a market equals the total amount of deposits raised in the same market ($L^T = D^T$). This is due to the fact that home and foreign banks supply the same amounts of deposits rather than to the fact that banks can finance loans only with local deposits as in the case of multinational banks. Using our functional forms (14) and (15), the first order condition (24) becomes

$$L^T \left[\frac{1}{\alpha} - (\beta_1 + \gamma) L^T - \xi \right] + \left[\frac{1}{\alpha} - 2 (\beta_1 + \gamma) L^T - \xi \right] \ell - \gamma L^T \ell = 0.$$

Hence, after imposing $L^T = N^a \ell$, we can solve for the total amount of loans extended by cross-border lenders in each market:

$$L^T_{cbl} = N^a \ell = \frac{\frac{1}{\alpha} - \xi}{\beta_1 + \gamma} \frac{(N^a + 1) - \frac{1}{2}}{(N^a + 2) + \left(N^a + \frac{\gamma}{\beta_1 + \gamma} \right)}, \tag{25}$$

which shows that, also in the case of cross-border lending, a larger number of active banks raises the total amount of loans, thus reducing risk-taking. Expression (25) can be compared with its analogue (20) in the case of multinational banks:

$$L^T_{mnb} = N^a \ell = \frac{\frac{1}{\alpha} - \xi}{\beta_1 + \gamma} \frac{N^a + 1}{N^a + 2}.$$

Three comments are in order. First, for a given number of active banks N^a, cross-border lenders raise a smaller total amount of deposits and thus supply a smaller total amount of loans ($L^T_{cbl} < L^T_{mnb}$). Second, for a given initial number of active banks N^a, the increase in competition caused by the same increase in the number of active banks leads to a smaller increase in deposits and loans with cross-border lenders than with multinational banks ($dL^T_{cbl}/dN_a < dL^T_{mnb}/dN_a$). Hence, for given N^a, multinational banking generates less risk taking than cross-border lending ($p_{cbl} > p_{mnb}$) and more competition reduces risk by a larger extent ($dp_{cbl}/dN_a < dp_{mnb}/dN_a$). Third, when instead the number of active banks is endogenously determined by free entry, multinational banking still generates less risk than cross-border lending provided that the additional fixed cost of setting up a foreign subsidiary is not too large. Too see this, note that, for given N_a and net of the corresponding overall entry cost, the maximized profit of a cross-border lender evaluates to

$$\Pi_{cbl} = \frac{\alpha \beta_1 \left(\frac{1}{\alpha} - \xi \right)^3}{(\gamma + \beta_1)^2} \frac{(2N^a + 1)^2 \left(\frac{5\gamma + 3\beta_1}{\gamma + \beta_1} + 2N^a \right)}{8N^a \left(\frac{3\gamma + 2\beta_1}{\gamma + \beta_1} + 2N^a \right)^3} - [1 - \beta(1 - \varrho)] \left(\kappa - \kappa^d \right),$$

while, by (21), the profit of a multinational bank evaluates to

$$\Pi_{mnb} = \frac{\alpha \beta_1 \left(\frac{1}{\alpha} - \xi \right)^3}{(\beta_1 + \gamma)^2} \frac{(N^a + 1)^2}{N^a (N^a + 2)^3} - [1 - \beta(1 - \varrho)] \kappa.$$

Both Π_{cbl} and Π_{mnb} are decreasing in N^a and go to zero as N^a goes to infinity. However, it can be shown that the multinational bank's profit gross of the overall entry cost is larger than the cross-border lender's for any value of N^a. It then follows that for $\kappa^d = 0$ the multinational banking free entry condition $\Pi_{mnb} = 0$ holds for a value of N^a that is larger than the one at which the cross-border lending free entry $\Pi_{cbl} = 0$ holds. By continuity, this also holds for $\kappa^d > 0$ provided that κ^d is not too large. Otherwise, when κ^d is large enough, the reverse happens with $\Pi_{mnb} = 0$ holding for a value of N^a that is smaller than the one at which $\Pi_{cbl} = 0$ holds. Higher risk taking associated with cross-border lending is in line with evidence reported by the IMF [28] that the increase in cross-border lending prior to the 2007 produced larger default after the crisis erupted and this was followed by extensive re-trenchment (see also Milesi-Ferretti and Tille [35]).

5.2 Asymmetric Shocks

The modified equations of the model's extension allowing for asymmetric shocks across countries, and thus risk diversification, are detailed in Appendix C. Here we comment on the corresponding results. Figure 8 reports impulse response functions to foreign productivity shocks for different values of the cross-country shock correlation.

For given correlation, the qualitative reactions of variables discussed in Section 4 are confirmed also in this case. Again, a positive productivity shock increases the success probability and decreases projects' returns, thus lowering banks' margins. On balance, more banks enter since the total scale of future discounted profits rises due to the better project selection. Banks' profits and market shares fall, but the total number of active banks rises.

The amplitude of responses depends, however, on the degree of cross-country shock correlation. Comparing the impulse response functions under zero shock correlation and under positive shock correlation, two patterns stand out. First, with positive correlation aggregate profits, aggregate loans supplied and the number of active banks increase by less. This is due to the fact that with positive correlation the scope for risk diversification is smaller, hence fewer banks find profitable to enter. Second, with positive correlation risk-taking is more muted: as fewer banks are active, only the projects with the best risk-return profile are funded.

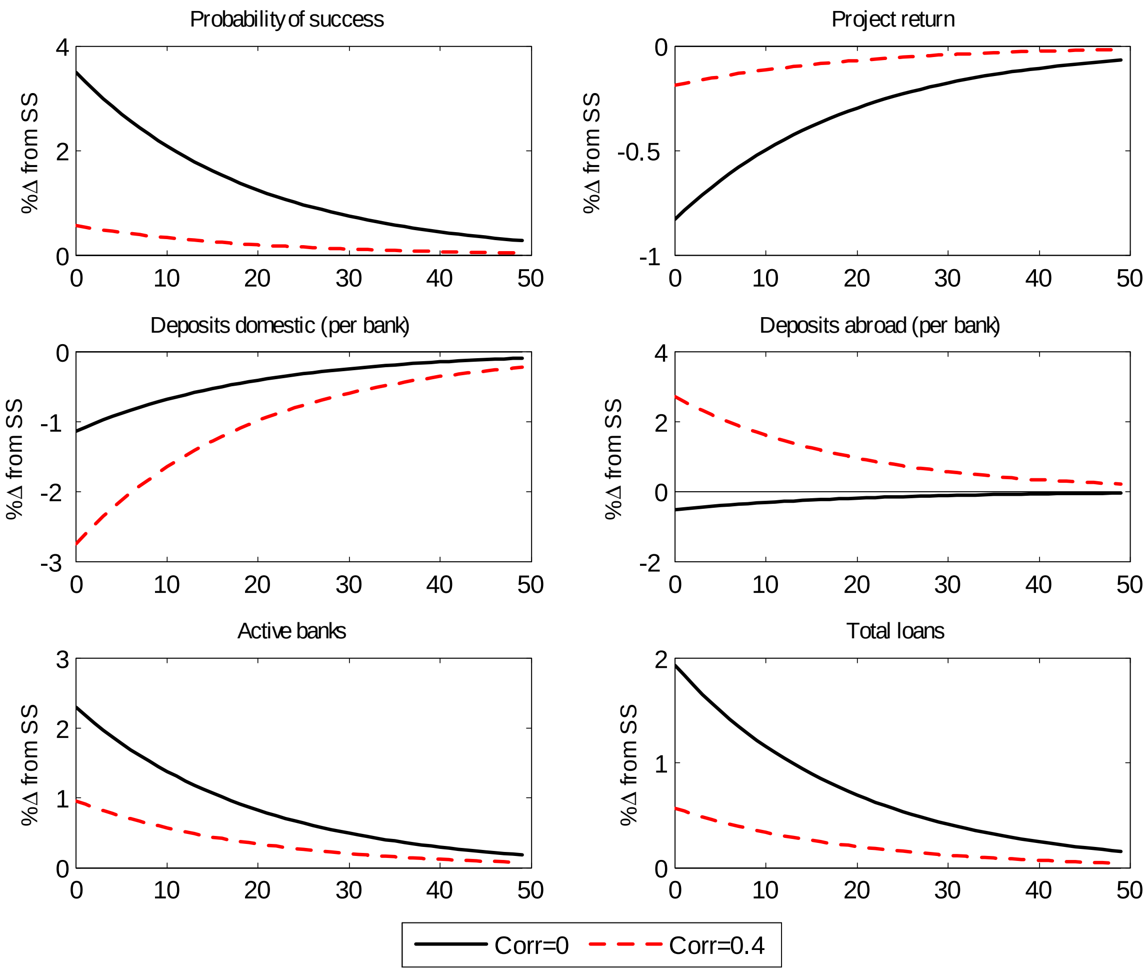

Figure 8: Impulse responses of selected variables to a positive foreign productivity for different values of the shock correlation across countries.

5.3 Endogenous Exit

To model endogenous exit we introduce a further selection effect that works through heterogeneous shocks to liquidity. This is a realistic feature of the banking system as banks might be subject to heterogeneous deposit withdrawals or other liability strains. In particular, we introduce a liquidity shock λ_t that is lognormally distributed according to the continuous cumulative density function Φ. The endogenous exit rate is modelled as the cumulative distribution of a liquidity shock, $1-\widetilde{\varrho}_t = 1-\Phi(\widetilde{\lambda}_t)$, whose threshold value $\widetilde{\varrho}_t$ is reached when the banks' future discounted profits equal the exit cost κ^{exit}. We can think of liquidity shocks as signals on deposits' withdrawals that might trigger a widespread run on deposits.[19] Signals are normally distributed with precision σ. The exit region is then given by:

$$\widetilde{V}_t = \widetilde{\Pi}_t + \widetilde{\Pi}_t^* + (1-\widetilde{\varrho}_t)\mathbb{E}_t\left\{\widetilde{V}_{t+1}\right\} = \kappa_t^{exit} \tag{26}$$

where $\widetilde{\Pi}_t = p(L_t^T, a_t)\left(r_t^L - r_t^D\widetilde{\lambda}_t - \xi\right)d_t$ and $\widetilde{\Pi}_t = p(L_t^T, a_t)\left(r_t^L - r_t^D\widetilde{\lambda}_t - \xi - \mu\right)d_t$. The exit cost, κ_t^{exit} is set to 25% of its entry counterpart, κ_t.[20]. Endogenizing the exit probability gives us the opportunity to match exit rate volatilities found in the data. We use again data from the bank ownership database of Claessens and Van Horen [13]. By calculating exit rate with the same procedure described above for the entry rate, we obtain a value of 2.1% for US banks over the same pre-crisis period. We therefore fit another exogenous process of the form:

$$\kappa_t^{exit} = (1-\rho_{\kappa^{exit}})\kappa^{exit} + \rho_{\kappa^{exit}}\kappa_{t-1}^{exit} + \varepsilon^{\kappa^{exit}} \tag{27}$$

employing the same grid search method outlined above. More precisely, we now loop through four parameters, namely ρ_κ, $\rho_{\kappa^{exit}}$, σ_κ, and $\sigma_{\kappa^{exit}}$, to hit both entry and exit rate volatilities as found in the data. The outcomes are shown in Table 4.[21] The remaining calibration remains the same as in Section (4).

[19] See Angeloni and Faia [3], Faia [20] and Rossi [39] for further details on macroeconomic models with banks' default that are induced by bank runs triggered by coordination problems on signals.

[20] Based on pre-crisis estimates of entry costs and scrap values, Temesvary [41] reports that banks could recover roughly 75% of their entry costs when closing their foreign offices.

[21] Since specifying a shock process for κ^{exit} does affect the dynamic behavior of both entry and exit rates, it is not possible to keep the calibration of the entry process obtained for the model with exogenous exit. Instead, one has to jointly re-optimize the calibration for both processes.

Table 4: **Entry rate moments in benchmark model.**

Variable	Model	Data
Entry rate mean	5.06	5.1
Entry rate std. dev.	3.13	3.3
Exit rate std. dev	2.01	2.1

Figure 9 shows the usual impulse response functions to a 1% increase in productivity, this time comparing the model with exogenous exit and the model with endogenous exit. The patterns described in Section (4) are confirmed in qualitative terms also under endogenous exit. Again, higher productivity fosters entry and increases the probability of success, implying lower project returns. The margins that banks extract decline since now more banks are active. However, the positive selection of projects induces a positive scale effect on all future discounted profits, which in turn implies that the number of active banks increases. Under endogenous entry changes in competition, and their benefits in terms of lower default rates, are smaller. Banks can now stay in business only if they are able to cope also with the additional liquidity shocks. This adds an extra selection mechanism, which reduces the number of active banks for given increase in the scale of aggregate discounted future profits. Overall, the number of active banks increases by less. Correspondingly the individual market shares of each bank fall by less. The risk-taking channel remains active, albeit more muted.

5.4 Systemic Risk

In extending our model to allow for imperfect correlation of projects' outcomes, we follow the established practice in the literature of conditioning those outcomes on common and idiosyncratic factors in the wake of Vasicek [42] as, for example, in Martinez-Miera and Repullo [31] and Bruno and Shin [7]. This allows us to capture possible interconnections, asset commonality or other features that make the probability of banks' portfolio failure different from the simple average of failure probability across projects. By checking the relation between entry and the resulting metric of systemic risk we can also check how competition and risk taking interact in presence of contagion effects. As we will see, our main result on the negative impact of entry on risk taking will stand,

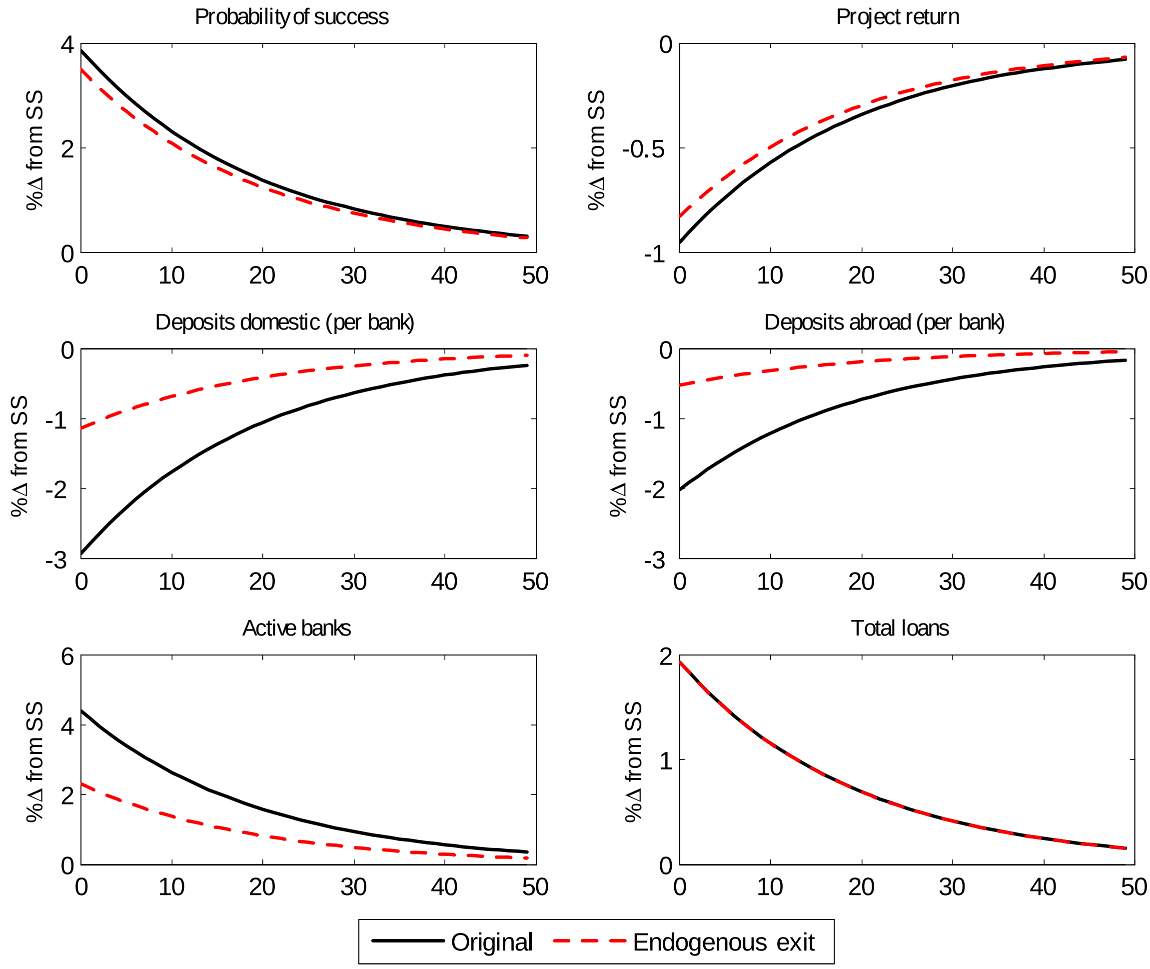

Figure 9: Impulse responses of selected variables to a positive productivity shock comparing the model with exogenous exit and with endogenous exit.

albeit with qualification.

We abstract from the aggregate productivity shock ($a_t = a = 1$) but, differently from the deterministic environment we analyzed in Section 4.2, we now allow projects to be subject to a risk of failure determined not only by firms' choices of the risk-return profile but also by the realizations of common and idiosyncratic factors. In particular, as in Martinez-Miera and Repullo [31], we assume that there is a continuum of firms indexed i and that the outcome of the project chosen by any given firm i is determined by the realizations of a random variable y^i defined as

$$y^i = -\Phi^{-1}(1-p^i) + \sqrt{\rho}z + \sqrt{1-\rho}\varepsilon^i, \tag{28}$$

where Φ is the cumulative density function of a standard normal distribution while z and ε^i are the common and idiosyncratic risk factors with distributions that are also independently standard normal. The project of firm i fails when the realization of y^i is negative. The parameter $\rho \in [0,1]$ measures the relative importance of the systematic risk factor with respect to the idiosyncratic one in determining the project's outcome, that is, the degree of risk correlation among projects. For $\rho = 0$ failures are statistically independent across firms; for $\rho = 1$ they are perfectly correlated; for $\rho \in (0,1)$ they are imperfectly correlated.

Given that both risk factors are generated by independent standard normal distributions, the probability of failure evaluates to $\Pr\left[y^i\right] = 1 - p^i$. Hence, given (4), firm i chooses its risk-return profile $(p^i, r^{I,i})$ to maximizes expected profit $p^i(r^{I,i} - r^L)$ subject to $r^{I,i} = (1-p^i)/\alpha$ as per (15). As all firms face the same loan return, the first order condition implies that they all choose the same success probability:

$$p = \frac{1-\alpha r^L}{2} \tag{29}$$

with the same associated return $r^I = (1+\alpha r^L)/2\alpha$. Once more, the fact that probability p is a decreasing function of r^L reveals the presence of a risk-shifting effect: faced with higher loan return, firms select projects with higher failure rate $1-p$.

As the (ex ante) risk-return profile chosen by firms before risk factors are realized is the same across firms and we have a continuum of firms, the Law of Large Numbers implies that (ex post) the share of projects that succeed (i.e. the aggregate success rate) depends only the realization of the common risk factor z and coincides with the probability of success of the representative firm conditional on the realization z:

$$\varsigma(z) = \Pr\left[-\Phi^{-1}(1-p) + \sqrt{\rho}z + \sqrt{1-\rho}\varepsilon^i \geq 0 \mid z\right] = 1 - \Phi\left(\frac{\Phi^{-1}(1-p) - \sqrt{\rho}z}{\sqrt{1-\rho}}\right),$$

where we have used the fact that ε^i follows a standard normal distribution. As also z follows a standard normal distribution, the cumulative density of the aggregate success rate $\varkappa$ is then given by:

$$G(\varkappa) = \Pr\left[\varsigma(z) \leq \varkappa\right] = \Phi\left(\frac{\Phi^{-1}(1-p) - \sqrt{1-\rho}\Phi^{-1}\left(1-\varkappa\right)}{\sqrt{\rho}}\right). \tag{30}$$

According to (30), the success rate has mean p while ρ regulates the dispersion around the mean with larger ρ associated with more dispersion. In the limit, for $\rho \to 0$, $G(\varkappa)$ becomes a Dirac delta function that is zero everywhere except at $\varkappa = p$: with independent failures a fraction p of projects succeed with probability 1. For $\rho \to 1$, $G(\varkappa)$ converges to p: with perfectly correlated failures all projects succeed with probability p and fail with probability $1-p$ as in our benchmark case.

Having characterized the underlying risk, we can now restate the banks' optimization problem, assuming for simplicity that there is no additional monitoring cost for foreign operations ($\mu = 0$) and that markets are characterized by their own uncorrelated common risk factors. A typical bank is active as long as the realized success rate is large enough to generate non-negative net cash flow:

$$2\varkappa m(L^T)\ell - \overline{\kappa} \geq 0,$$

where $m(L^T) = r^L\left(L^T\right) - r^D(L^T) - \xi$ is the lending-to-deposit rate spread (net of the insurance premium) and $\overline{\kappa} = \left[1 - \beta(1-\varrho)\right]\kappa$ is the annuity value of the overall fixed cost κ (which the bank finances in the capital market upon entry). This non-negativity condition generates a cutoff rule of survival: the bank will be active as long as the realized success rate $\varkappa$ does not fall short of the threshold:

$$\widetilde{\varkappa} = \frac{\overline{\kappa}}{2m(L^T)\ell}. \tag{31}$$

Note that in our benchmark case ($\rho = 1$) the cutoff would be immaterial ($\widetilde{\varkappa} = 1$). Totally differentiating (31) in the symmetric equilibrium ($\ell = L^T/N^a$) gives:

$$\frac{d\ln\widetilde{\varkappa}}{d\ln N^a} = 1 - \left[1 + \frac{d\ln m(L^T)}{d\ln L^T}\right]\frac{d\ln L^T}{d\ln N^a}, \tag{32}$$

which shows that the sign of the elasticity of the cutoff success rate $\widetilde{\varkappa}$ to changes in the number of active firms N^a is determined by the sign of the elasticity of the lending-to-deposit rate spread $m(L^T)$ to aggregate loans L^T and the sign of the elasticity of aggregate loans L^T to the the number of active firms N^a. With our functional forms (14), the sign of the former is negative as $m'(L^T) = -(\beta_1 + \gamma)$. To sign the latter we have, instead, to analyze the optimization problem of the typical bank. This maximizes profit

$$\Pi(\ell_-, \ell) = h(\ell_-, \ell)\ell - \overline{\kappa},$$

with:

$$h(\ell_-, \ell) = 2\left(1 - G(\widetilde{\varkappa}(\ell_-, \ell))\right)\; E_{\widetilde{\varkappa}(\ell_-,\ell)}(\varkappa)\; m((N^a - 1)\,\ell_- + \ell)$$

where ℓ_- refers to the vector of loans by the other $N^a - 1$ banks (hence $L^T = (N^a - 1)\,\ell_- + \ell$), the dependence of $\widetilde{\varkappa}$ on $\ell_-$and ℓ has been made explicit, and $E_{\widetilde{\varkappa}(\ell_-,\ell)}(\varkappa) = \int_{\widetilde{\varkappa}(\ell_-,\ell)}^{1} \varkappa dG(\varkappa)/\left(1 - G(\widetilde{\varkappa}(\ell_-, \ell))\right)$ is the conditional mean success rate. The function $h(\ell_-, \ell)$ is the 'generalized' residual demand in the sense of Martinez-Miera and Repullo [31]. Note however that, differently from their setup, here the bank affects the cutoff success rate $\widetilde{\varkappa}$ not only indirectly through its effect on total loans L^T but also directly through ℓ whereas the profit margin $m(L^T)$ does not depend on $\varkappa$. In the case of perfectly correlated project failures ($\rho = 1$), the bank's problem boils down to the one we already solved for the benchmark case as $\left(1 - G(\widetilde{\varkappa}(\ell_-, \ell))\right) E_{\widetilde{\varkappa}(\ell_-,\ell)}(\varkappa) = p$ with p given by (29).

The bank's maximization problem is well defined as long as $h(\ell_-, \ell)$ is decreasing and concave in ℓ (i.e. $h'(L^T) < 0$ and $h''(L^T) < 0$) as this ensures that the necessary and sufficient conditions for profit maximization are met. Henceforth, we assume that parameter values are such that those properties hold. The first order condition requires $h_2(\ell_-, \ell)\ell + h(\ell_-, \ell) = 0$, which in the symmetric equilibrium ($\ell_- = \ell = N^a/L^T$) implies:

$$h'(L^T)\frac{L^T}{N^a} + h(L^T) = 0.$$

Total differentiation then yields:

$$\frac{dL^T}{dN^a} = -\frac{h(L^T)}{h''(L^T)L^T + h'(L^T)\left(N^a + 1\right)} > 0, \tag{33}$$

with the sign granted by $h'(L^T) < 0$ and $h''(L^T) < 0$. Accordingly, given (14) and (29), we have $dr^L/dN^a < 0$ and $dp/dN^a > 0$ respectively. This shows that more competition (due to a larger number of active banks) lowers the probability of default of the loans in banks' portfolios $1-p$. However, as pointed out by Martinez-Miera and Repullo [31], that does not necessarily imply lower probability of failure $\Pr\left[\varkappa \leq \widetilde{\varkappa}\right]$.

Indeed, using the cumulative density function (30), the probability of failure can be written as:

$$G(\widetilde{\varkappa}) = \Phi\left(\frac{\Phi^{-1}(1-p) - \sqrt{1-\rho}\Phi^{-1}\left(1-\widetilde{\varkappa}\right)}{\sqrt{\rho}}\right),$$

which shows that, as N^a increases, the ensuing fall in $1-p$ may be contrasted by a parallel rise in $1-\widetilde{\varkappa}$. This requires $d\ln\widetilde{\varkappa}/d\ln N^a > 0$, which by (32) and (33) in turn requires the negative impact of a larger number of active banks on the lending-to-deposit rate spread to be strong enough relative to the parallel positive impact on the total provision of loans and deposits:

$$\frac{d\ln L^T}{d\ln N^a} + \frac{d\ln m(L^T)}{d\ln N^a} < 1. \tag{34}$$

This is a necessary condition for the probability of portfolio failure to rise despite lower probability of default of the loans in the portfolios. It would hold, for example, if aggregate bank profits fell with bank entry: $d\ln\left(m(L^T)L^T\right)/d\ln N^a = d\ln L^T/d\ln N^a + d\ln m(L^T)/d\ln N^a < 0$. Vice versa, the result of Section 4 that banks' competition reduces the risk would carry through to the case of imperfectly correlated projects' returns if condition (34) were violated as in such case we would have $d\ln\widetilde{\varkappa}/d\ln N^a < 0$. In other words, a sufficient condition for the result of Section 4 to extend to the more general setup is that the expansionary impact of competition on active banks' profits through total loans and deposits is strong enough to offset its parallel contractionary impact through the lending-to-deposit rate spread. ($d\ln\left(m(L^T)L^T\right)/d\ln N^a \geq 0$).

6 Conclusion

Venturing into foreign markets can enrich banks' opportunities, but can also have unintended consequences for risk-taking. It has, however, been argued that direct involvement in local retail activities promotes competition and, through this channel, reduces risk-taking. We have proposed a model in which imperfectly competitive banks are allowed to operate simultaneously in different national markets with direct involvement in local retail activities both on the deposit and the loan sides. Our banks make endogenous entry decisions (by comparing future discounted value of profits to entry costs) and select the risk-return profiles of their loan portfolios anticipating borrowers' risk-shifting due to limited liability. We have shown that, if borrowers' project success exhibits decreasing hazard rate, our model indeed predicts that direct involvement in retail activities reduces risk-taking provided that the expansionary impact of competition on multinational banks' aggregate profits through larger scale is strong enough to offset its parallel contractionary impact through lower loan-deposit return margin. This holds with both perfectly and imperfectly correlated loans' risk, whether there are cross-country symmetric or asymmetric shocks and whether exit is exogenous or endogenous. Finally, comparing a version of our model featuring cross-border lending with the benchmark one featuring multinational banks, we have found that also in the former case more competition can reduce risk-taking, but to a lesser extent than in the latter.

References

[1] Allen, F. and D. Gale, (2000). *Comparing Financial Systems.* MIT Press, Cambridge.

[2] Allen, F. and D. Gale, (2004). "Financial Intermediation and Markets." *Econometrica,* 72, 1023-1061.

[3] Angeloni, I. and E. Faia, (2013). "Capital Regulation and Monetary Policy with Fragile Banks." *Journal of Monetary Economics*, 60, 3, 311-382.

[4] Berger, A. N., Clarke, G., R. Cull, L. Klapper and G. Udell, (2005). "Corporate governance and bank performance: a joint analysis of the static, selection and dynamic effects of domestic, foreign and state ownership." *Journal of Banking and Finance,* 29, 2179-2221.

[5] Boyd, J. and G. De Nicolo, (2005). "The Theory of Bank Risk Taking and Competition Revisited." *Journal of Finance,* Vol. LX, 3.

[6] Brander, J. and Krugman, P., (1983). "A 'reciprocal dumping' model of international trade." *Journal of International Economics*, 15(3-4), pp. 313-321, November.

[7] Bruno, V. and H.-S. Shin, (2015a). "Cross-Border Banking and Global Liquidity." *Review of Economic Studies,* 82 (2), 535-564.

[8] Bruno, V. and H.S. Shin, (2015b). "Capital flows and risk-taking channels of monetary policy." *Journal of Monetary Economics,* 71, 119-132.

[9] Buch, C. and L. Goldberg, eds., (2015). "Mini-Symposium on International Banking." IMF Economic Review, 63, 2041-4161.

[10] Cetorelli, N. and L. Goldberg, (2012). "Banking Globalization and Monetary Transmission." *Journal of Finance,* vol. LXZ11, 5, 1811-1843.

[11] Cetorelli, N. and L. Goldberg, (2012). "Liquidity Management of U.S. global Banks: internal Capital Markets in the Great Recession." *Journal of International Economics,* 88, 2, 299-311.

[12] Claessens, S. and N. van Horen, (2015a). “Global Banking: Recent Developments and Insights from Research.”

[13] Claessens, S. and N. van Horen, (2015b). “The Impact of the Global Financial Crisis on Banking Globalization.” *IMF Economic Review,* 63(4).

[14] Claessens, S. and N. van Horen, (2012). “Being a Foreigner among Domestic Banks: Asset or Liability?” *Journal of Money, Credit and Banking,* 36(5).

[15] Claessens, S. Demigurc-Kunt, A. and H. Huinziga, (2001). “How does foreign entry affect the domestic banking market?” *Journal of Banking and Finance,* 25, 891-911.

[16] Corbae, d. and P. D’Erasmo, (2013). “A Quantitative Model of Banking Industry Dynamics.” Mimeo.

[17] De Blas, B. and K.N. Russ, (2013). “All banks great, small, and global: Loan pricing and foreign competition.” *International Review of Economics and Finance*, 26, 4-24.

[18] De Haan, J, Hale, G. and K.N. Russ, (2015). “Understanding Global Banking.” IMF Economic Review, 63, 693-1023.

[19] Dell’Ariccia, G., Laeven, L. and R. Marquez, (2014). “Real interest rates, leverage, and bank risk-taking.” *Journal of Economic Theory*, 149, 65-99.

[20] Faia, E. (2017). “Sovereign Risk, Bank Funding and Investors’ Pessimism.” Forthcoming *Journal of Economic Dynamics and Control.*

[21] Faia, E., Ottaviano, G. and I. Sanchez-Arjona, (2016). “International Expansion and Riskiness of Banks.” Mimeo, LSE and Goethe University Frankfurt.

[22] Giannetti, M. and S. Ongena, (2012). “Lending by example: direct and indirect effects of foreign banks in emerging markets.” *Journal of International Economics,* 86, 167-180.

[23] Goetz, M., Laeven, L. and R. Levine, (2013). "Identifying the Valuation Effects and Agency Costs of Corporate Diversification: Evidence from the Geographic Diversification of U.S. Banks." *Review of Financial Studies,* 26(7), 1787-1823.

[24] Hayashi, F., Grace Li, B. and Z. Wang, (2015). "Innovation, Deregulation and the Life-Cycle of a Financial Service Industry." IMF w.p. 15/192.

[25] Jensen, M. and W. Meckling, (1976). "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics,* 3, 4, 305-360.

[26] Judd, K. (1998). *Numerical Methods in Economics.* The MIT Press, Cambridge.

[27] Iacoviello, M. and Neri, S. (2010). "Housing Market Spillovers: Evidence from an Estimated DSGE Model." *American Economic Journal: Macroeconomics*, 2(2):125-164.

[28] IMF Financial Stability Report 2015.

[29] Keeley, M. (1990). "Deposit Insurance, Risk, and Market Power in Banking." *American Economic Review,* Vol. 80, No. 5, pp. 1183-1200.

[30] Levine, R., Lin, C. and W. Xie, (2015). "Geographic Diversification and Banks' Funding Costs." Mimeo.

[31] Martinez-Meira, D. and Repullo, R. (2010). "Does Competition Reduce the Risk of Bank Failure?" *Review of Financial Studies,* 23, 3638-3664.

[32] Melitz, M., and G. Ottaviano, (2008). "Market Size, Trade, and Productivity." *Review of Economic Studies* 75: 295-316.

[33] Melitz, M., (2003). "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71: 1695-1725.

[34] Merton, R. C., (1977). "An analytic derivation of the cost of deposit insurance and loan guarantees." *Journal of Banking and Finance,* 1, 3-11.

[35] Milesi-Ferretti, G.M. and C. Tille, (2011). "The Great Retrenchment: International Capital Flows During the Global Financial Crisis." *Economic Policy,* 26, 289-346.

[36] Niepmann, F., (2015). "Banking across borders." *Journal of International Economics*, 96, 244–265.

[37] Rajan, R. (2005). "Has Financial Development Made the World Riskier?" NBER Working Paper No. 11728

[38] Repullo, R. and Suarez, J. (2013). "The Procyclical Effects of Bank Capital Regulation." *Review of Financial Studies*, 26(2):452-490.

[39] Rossi, L. (2015). "Endogenous Firms' Exit, Inefficient Banks and Business Cycle Dynamics." DEM Working Papers Series 099, University of Pavia, Department of Economics and Management.

[40] Stiglitz, J. and A. Weiss, (1981). "Credit Rationing in Markets with Imperfect Information." *American Economic Review,* 71, 3, 393-410.

[41] Temesvary, J. (2014). "The determinants of U.S. banks international activities." *Journal of Banking and Finance*, 44(C),233-247.

[42] Vasicek, O. (2002). "Loan Portfolio Value." *Risk,* 15, 160-162.

[43] Vives, X. (2016). Competition and Stability in Banking. Princeton University Press.

7 Appendix A. Firms' Loan Demand

Firms get funds and can invest only in one national market. As markets are symmetric, we drop market indices. In each market there is continuum of firms with heterogenous outside options for investment. Firms' outside options h follow a continuous distribution with c.d.f. $G(h)$ for $h \geq 0$. Each firm can make only one unit investment yielding return

$$p(r^I, a)(ar^I - r^L). \tag{35}$$

The firm will make the investment as long as its expected profit does not fall short of its outside option. As a result investment is governed by a cutoff rule. Only firms with $p(r^I, a)(ar^I - r^L) \geq \overline{h}$ invest, where $\overline{h}$ corresponds to the outside option of marginal firms that are indifferent between investing or not: $\overline{h} \equiv p(r^I, a)(ar^I - r^L)$.

In this setup, the demand for loans is equal to the total number of entrepreneurs that invest

$$L^T = G(\bar{h}) = G(p(r^I, a)(ar^I - r^L)) \tag{36}$$

where r^I and r^L are linked by the firm's FOC:

$$\frac{d(p(r^I, a)(ar^I - r^L))}{dr^I} = p_1(r^I, a)(ar^I - r^L) + p(r^I, a)a = 0 \tag{37}$$

In order to find under which conditions $r^L(L)$ satisfies $r^{L\prime}(L) < 0$ and $r^{L\prime\prime}(L) \leq 0$, we can totally differentiate the second last equation and use (37) to obtain

$$\frac{dL}{dr^L} = -g(p(r^I, a)(ar^I - r^L))p(r^I, a) < 0 \tag{38}$$

and then

$$\frac{d^2L}{d\left(r^L\right)^2} = g'(p(r^I, a)(ar^I - r^L))\left(p(r^I, a)\right)^2 \geq 0. \tag{39}$$

Hence, $r^{L\prime}(L) < 0$ always holds and $r^{L\prime\prime}(L) \leq 0$ also holds as long as

$$g'(.) \geq 0. \tag{40}$$

8 Appendix B. Dynamic System of Equations

In this appendix we report the dynamic system of equations underlying our simulations in Section 4.3. In the presence of random shocks to the investment climate, return to investment (16) becomes

$$r_t^I = \frac{1}{\alpha} - \frac{\beta_1}{2a_t} L_t^T, \tag{41}$$

with associated success probability

$$p_t = \frac{\alpha\beta_1}{2} L_t^T. \tag{42}$$

As in the deterministic equilibrium, these expressions show that, for given a_t, more loans (and thus more deposits) make firms choose investments with lower return $a_t r_t^I$ and higher probability of success (i.e. a more cautious risk-return profile). On the other hand, for given L_t^T, an improvement in the investment climate (larger a_t) makes firm invest in projects with unchanged probability of success but higher return $a_t r_t^I$.

As (41) implies

$$r_t^{I\prime}\left(r_t^L\left(L_t^T\right)\right) r_t^{L\prime}\left(L_t^T\right) = -\frac{\beta_1}{2a_t}, \tag{43}$$

the banks' first order conditions (18) and (19) become respectively

$$L_t^T\left[\frac{a_t}{\alpha} - (\beta_1+\gamma) L_t^T - \xi\right] + \left[\frac{a_t}{\alpha} - 2(\beta_1+\gamma) L_t^T - \xi\right]\ell_t = 0 \tag{44}$$

and

$$L_t^T\left[\frac{a_t}{\alpha} - (\beta_1+\gamma) L_t^T - \xi - \mu\right] + \left[\frac{a_t}{\alpha} - 2(\beta_1+\gamma) L_t^T - \xi - \mu\right]\ell_t^* = 0, \tag{45}$$

where we again focus on the symmetric Cournot-Nash equilibrium in which all home banks choose the same amount of loans ℓ_t and all foreign banks choose the same amount of deposits ℓ_t^*.

The complete system of equations then consists of: (a) the banks' free entry condition (13)

$$V_t = \Pi_t + \Pi_t^* + \beta(1-\varrho)\mathbb{E}_t\{V_{t+1}\} = \kappa; \tag{46}$$

(b) the banks' first order conditions (18) and (19); (c) the definition of total loans (9); (d) the expression of banks' operating profits

$$\Pi_t + \Pi_t^* = \frac{\alpha\beta_1}{2} L_t^T\left[\frac{a_t}{\alpha} - (\beta_1+\gamma) L_t^T - \xi\right]\ell_t + \frac{\alpha\beta_1}{2} L_t^T\left[\frac{a_t}{\alpha} - (\beta_1+\gamma) L_t^T - \xi - \mu\right]\ell_t^*; \tag{47}$$

(e) the law of motion of the number of banks in each national market, which is given by

$$N_{t,H}^{a} = N_{t-1,H} + N_{t,H}^{e} = \frac{N_{t,H}}{1-\varrho} \tag{48}$$

for market H and its analogue for market F.

In the symmetry outcome we focus on, this is a system of five equations that can be solved numerically in the five unknowns, ℓ_t, ℓ_t^*, L_t^T, N_t and $\Pi_t + \Pi_t^*$. Equation (1) can then be used to find the corresponding number of entrants N_t^e.

9 Appendix C. Asymmetric Country Shocks

This appendix explains how we have adapted the dynamic system of Appendix 8 to generate the simulated results of Section 5.2. Specifically, we have allowed for a foreign-specific productivity shock a_t^*, distinct from its domestic counterpart a_t. As a consequence, we have introduced the following new expressions for the returns on projects, the success probability and the return on loans in foreign destination markets:

$$r_t^{I^*} = \frac{1}{\alpha} - \frac{\beta_1}{2a_t^*} L_t^T,\ p(L_t^T, a_t^*) = a_t^* - \alpha a_t^* r,\ r_t^{L^*} = \frac{a_t^*}{\alpha} - \beta_1 L^T \tag{49}$$

The foreign productivity shock follows the same type of AR(1) process as the domestic productivity shock, hence it takes the following form: $a_t^* = \rho_a a_{t-1}^* + \varepsilon_t^{a^*}$. The parameter of the foreign shock is assumed to be the same as the one of the domestic shock and is calibrated accordingly. We have further included some degree of correlation between domestic and foreign shocks, experimenting with different degrees of this correlation to check the sensitivity of results.

Finally, to take account for the asymmetry of the foreign shock, we have changed the following two equations for a bank's FOC and its profits in the foreign market to:

$$L_t^T \left[\frac{a_t^*}{\alpha} - (\beta_1 + \gamma) L_t^T - \xi \right] + \left[\frac{a_t^*}{\alpha} - 2(\beta_1 + \gamma) L_t^T - \xi - \mu \right] \ell_t^* = 0 \tag{50}$$

and

$$\Pi_t^* = p(L_t^T, a_t^*) \left(r_t^{L^*} - r_t^D - \xi - \mu \right) \ell_t^*. \tag{51}$$