The Joint Dynamics of the U.S. and Euro-area Inflation: Expectations and Time-varying Uncertainty*

Olesya Grishchenko
Federal Reserve Board

Sarah Mouabbi
Banque de France

Jean-Paul Renne
HEC Lausanne

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Abstract

We propose a dynamic factor model with time-varying uncertainty for the joint estimation of inflation expectations in the United States and the euro area. We exploit information in several U.S. and euro area surveys of professional forecasters to fit the first two moments of future inflation rates. Our model provides closed-form solutions for conditional expectations and variances of inflation at different horizons and is able to closely match their survey-based counterparts. Survey-consistent probabilities of future inflation falling within a given range of inflation outcomes are used to evaluate whether inflation expectations are anchored. We find that since 2010 inflation expectations decreased noticeably in both economies, and that over our sample period the U.S. displayed larger inflation uncertainty relative to the euro area. The correlation between future inflation rates in the two economies increased. This correlation and probability of deflation occurring jointly in both economies are related to economic policy uncertainty indices.

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1 Introduction

The Federal Reserve System and the European Central Bank are two of many central banks that have adopted a mandate of price stability devised to foster economic activity and employment. To meet their objective of price stability, central banks around the world rely on inflation forecasts and pay close attention to various measures of inflation expectations implied by both financial market data and various surveys to assess whether inflation expectations remain anchored. Surveys, in particular, have received a lot of attention from policymakers and academics. This notably reflects their documented success in forecasting inflation (Ang et al., 2007). Surveys are thus closely monitored and often mentioned in various monetary policy communications. For instance, the Federal Open Market Committee (FOMC) minutes for the 2015 October FOMC meeting state: *"Measures of expected longer-run inflation from a number of surveys, including the Michigan survey, the Blue Chip Economic Indicators, and the Desk's Survey of Primary Dealers, remained stable. However, market-based measures of inflation compensation moved a little lower."*[1] Similarly, the October 2015 European Central Bank's Governing Council's monetary policy meeting account states: *"It was noted that survey-based measures of longer-term inflation expectations had remained unchanged at 1.9 percent, according to the latest SPF, and that market-based measures of inflation expectations had broadly stabilized since late August."*[2]

Despite useful information surveys provide, they have various limitations: (i) surveys report only certain inflation forecast horizons, (ii) some surveys focus only on first-order moments and do not provide information about inflation uncertainty, (iii) surveys report specific inflation measures (such as year-on-year rates, average inflation rates over longer periods or even averages of year-on-year inflation rates) which makes their comparison across surveys ambiguous, (iv) surveys convey limited

[1]See the minutes of the October 27-28, 2015 FOMC meeting at http://www.federalreserve.gov/monetarypolicy/fomcminutes20151028.htm.

[2]See the account of the October 22, 2015 ECB Governing Council meeting at https://www.ecb.europa.eu/press/accounts/2015/html/mg151119.en.html.

information about the joint dynamics of inflation in different areas, and (v) surveys are released at a relatively low (and different) frequencies.

In this paper we exploit rich information contained in a handful of surveys of professional forecasters, including the Philadelphia Fed Survey of Professional Forecasters, the Federal Reserve Bank of New York Survey of Primary Dealers and the European Central Bank's Survey of Professional Forecasters, in order to gauge both inflation expectations and inflation uncertainty in the United States and the euro area. In particular, some surveys provide probability distributions of future inflation outcomes falling within given ranges. We make full use of these probability distributions to infer uncertainty measure surrounding the reported point forecasts.

We propose an approach that feeds in the model various available surveys in the U.S, and the euro area, in all their diversity, and produces survey-consistent distributions of inflation at any horizon. These model-implied probability distributions are directly comparable across US and euro area, unlike the raw probability distributions reported in surveys. This approach is based on a flexible time series model of inflation. This model notably features stochastic volatility, hence allowing for time-varying inflation uncertainty. The model is a factor model in which the same factors may drive the dynamics of inflation rates in different areas. As a result, the model is consistent with the existence of commonalities in price fluctuations across two economies, reflecting ever-increasing interconnectedness between developed economies and financial markets.

Designing a rich model of inflation is relatively straightforward. However, in our case, we not only need a flexible model, but further need one that can be fed with different types of survey data at the estimation stage. In other words, had we built a sophisticated model of inflation that lacked closed-form formulae to relate model-implied expectations with observed ones (actual survey data), making our model consistent with the observations would be infeasible. By contrast, our model is not only rich, it is also highly tractable. Specifically, it offers closed-form formulae for conditional first and second-order moments of different inflation measures used in the surveys. This key feature stems from the fact that the factors employed in our econometric model follow so-called affine processes. The affine property of the factors also implies that the model can easily be cast in the state-space form and then processed by Kalman filtering techniques. These techniques easily handle missing data issues, which is particularly useful in our case, where different surveys are released in different periods over time.

After having "digested" an anthology of survey-based information, our framework primarily allows us to compute the distribution of inflation, conditional on past and present information, for the US and the euro area, for any horizon. Importantly, even if the conditional moments reported in surveys and used in the estimation are not based on the same inflation measures (e.g., year-on-year growth rate of the price index, annualised growth rate over a given period or average of year-on-year growth rates) or do not coincide in terms of horizons, model-implied conditional distributions of inflation can be made perfectly comparable across areas. Typically, this means that we can appropriately compare the levels of inflation uncertainty, at any horizon, in the US and the euro area. This facilitates an investigation of the relative anchoring of inflation expectations, once we have properly defined one or several anchoring measures based on inflation distributions. Finally, our model allows us to study the probabilities of future joint inflation outcomes, including the probabilities of joint deflation.

Our empirical results are as follows.

First, our model is able to closely capture the first and second moments embedded in the surveys. Hence, to the extent that the model-implied expectations are rational (by construction), our results provide support to – or do not allow to reject – the idea that survey forecasts are rational.

Second, our estimation highlights the importance of the links existing between the dynamics of the U.S. and euro area inflation rates. Specifically, we find that the same (latent) factor is the most important one to account for the fluctuations of inflation levels in both economies. Interestingly, the same result holds for the conditional variances of the two inflation rates: Among latent factors driving the conditional volatilities of both inflation rates, the same factor is the most important one in both economies. We estimate conditional correlations between the two economies' inflation rates for one- and five-year horizons. According to our findings, these correlations trended up significantly since 2010 from roughly 0.65 to 0.80 in the end of 2015. These findings suggest that common fundamental shocks appear to drive inter-connected economies around the world and became stronger recently. Our results also indicate that the conditional covariances of future inflation rates in the two areas are related to the U.S. and euro area Economic Policy Uncertainty (EPU) indices (Baker et al., 2015), as well as to the European Commission and University of Michigan Economic Sentiment indices.

Third, we exploit our framework to compute measures of inflation expectations' anchoring. We compute probabilities of future inflation being in certain ranges, such as being between 1.5% and 2.5% or between 1% and 3%, over various horizons. Comparing these probabilities across the two economies, we find that, overall, inflation expectations in the euro area are more anchored than in the U.S. judjed by higher levels of these probabilities. However, the probability of longer-term (5-year 5 years forward) inflation in the U.S. increased notably since the financial crisis possibly reflecting explicit inflation targeting adopted by the Federal Reserve in September 2012.[3]

The remainder of the paper is organized as follows. Section 2 provides a brief literature review, Section 3 introduces surveys, Section 4 describes the estimation strategy, Section 5 presents empirical results, and Section 6 concludes. Appendix 7 gathers proofs, technical results and additional data descriptions.

2 Literature review

Surveys have become a popular tool in assessing expectations of inflation (and other macroeconomic variables). There is growing empirical evidence that surveys out-

[3]See press release at http://www.federalreserve.gov/newsevents/press/monetary/20120913a.htm.

perform numerous statistical forecasting methods. For example, surveys appear (1) to outperform simple time-series benchmarks in forecasting inflation (Grant and Thomas, 1999; Thomas, 1999; Mehra, 2002); (2) to outperform other forecasting methods such as term structure models and the Philips curve (Ang et al., 2007; Chun, 2012); (3) to beat other forecasts in real time (as opposed to ex-post revised data) (Faust and Wright, 2009; Croushore, 2010); (4) finally, to be consistent with inflation expectations embedded in Treasury yields (Chernov and Mueller, 2012). Faust and Wright (2013) provide a comprehensive overview of these inflation forecasting methods: they find that the Philadelphia Fed's Survey of Professional Forecasters (SPF), Blue Chip surveys, and the Fed staff's Greenbook forecasts outperform other numerous methods.[4]

As a result of this favourable attention, many researchers started using surveys in various settings: for example, as proxies for inflation expectations (Grishchenko and Huang, 2013; Grishchenko et al., 2016; Chun, 2014); and as inputs to constructing the term structure of inflation expectations (Haubrich et al., 2012; Aruoba, 2014). Surveys are becoming a popular input to term structure models of interest rates as well. As such, term structure researchers also use survey forecasts (of the three-month T-bill rate) to help pin some of the model parameters (Kim and Orphanides, 2012; D'Amico et al., 2016). Also, Volker (2016) used dispersion over the fed funds rate Blue Chip survey forecasts as a proxy for monetary policy uncertainty.

Despite the fact that surveys have been used a lot to extract consensus inflation forecasts, survey-based inflation uncertainty measures only recently attracted attention. Yet, inflation uncertainty appears to be an important risk factor in determining the nominal bond risk premium (Buraschi and Jiltsov, 2005; Piazzesi and Shneider, 2007; Rudebusch and Swanson, 2008; Campbell et al., 2013). So, it is important to measure this risk factor correctly. Surveys appear to be a natural data set to come up with an inflation uncertainty measure. Zarnowitz and Lambros (1987) discuss the inflation uncertainty concept as the second moment of the

[4]The sample period of SPF and Blue Chip forecasts covers 1985 — 2011 period, while Greenbook surveys end in 2006 in Faust and Wright's paper due to the Greenbook 5-year release embargo.

subjective inflation probability distribution functions in surveys. However, few surveys contain inflation probability distribution functions.[5] Thus, it became popular to substitute this measure with the disagreement over the point inflation forecasts because the two measures appear to be closely correlated (Zarnowitz and Lambros, 1987; Giordani and Soderlind, 2003; Wright, 2011). Yet, a growing strand of literature shows the differences between disagreement and uncertainty survey measures (Conflitti, 2010; Rich et al., 2012; Andrade and Bihan, 2013; Boero et al., 2014; D'Amico and Orphanides, 2014). Interestingly, Lahiri and Sheng (2010) decompose forecast errors into common and idiosyncratic shocks, and show that aggregate forecast uncertainty can be expressed as the sum of the disagreement among forecasters and the perceived variability of future aggregate shocks. This finding implies that the reliability of disagreement as a proxy for uncertainty depends primarily on the stability of the forecasting environment. It stands to reason that in an economic environment that appears to be quite volatile at least since the onset of the Great Recession, measures related to the diffuseness of inflation forecasts would better capture fundamental inflation uncertainty than inflation forecast disagreement.

Recent sharp declines in long-term inflation compensation measures raised concerns among market participants of whether such declines are attributable to declines in inflation expectations or declines in inflation risk.[6] Therefore, it seems reasonable to build inflation forecasting models where both first and second moments of inflation are time-varying. In our model, which features time-varying first and second moments of inflation, we make full use of the U.S. and euro surveys that contain subjective inflation probability distribution functions. We calibrate the model to various types of surveys, so that conditional moments of inflation are coherent across horizons. While this feature is already prevalent in some papers (Chernov and Mueller, 2012; Aruoba, 2014), our model is more flexible because it entails stochastic volatility, which constitutes a key ingredient to capturing time-

[5]In the Data section we describe in detail which U.S. and euro surveys contain inflation pdf's currently.

[6]TIPS-based measures of inflation compensation declined roughly 80 basis points mid-2014.

varying inflation uncertainty in the model. Another very important dimension of our contribution to the inflation forecast is that we model jointly U.S. and euro area inflation expectations. Indeed, recent debates on global inflation, (e.g. Ciccarelli and Mojon, 2010) have motivated the joint modelling of inflation expectations in several currency areas (Beechey et al., 2011; Ciccarelli and García, 2015). The literature has so far only focused on the analysis of first-order moments — or point estimates — of future inflation rates across different economies. By contrast, our approach makes it possible to study their joint distribution.

Our paper contributes to the literature that assesses the extent to which inflation expectations are anchored (Bernanke, 2007; Gurkaynak et al., 2007; Mertens, 2011; Mehrotra and Yetman, 2014; Nagel, 2015). Beechey, Johannsen, and Levin (2011) and Gurkaynak et al. (2007) find that inflation expectations are less anchored in the United States than in the euro area or in other countries such as Canada or Chile. They measure anchoring of inflation expectations as the sensitivity of the survey- and market-based inflation expectations to incoming macro-economic news.

In addition, our paper relates to a particular strand of the literature that augments models with variables that appear to have some predictive power for forecasting inflation. We do so by adding additional survey data and market-based variables, such as inflation swaps, risk-neutral volatility measures, and gold and oil prices, in the model estimation (e.g., Ghysels and Wright, 2009).[7]

3 Survey data

We construct a detailed database of inflation expectation surveys at various horizons for both the United States and the euro area. Details of this database can be found on Table 1. Several issues need to be accounted for when surveys are used in the

[7]This approach has grown increasingly common in the term structure literature and was first introduced by Kim and Orphanides (2012) in order to handle the persistence problem affecting the estimation of term structure models. Similar augmentations of state-space models can be found in Kozicki and Tinsley (2006) and Monfort, Pegoraro, Renne, and Roussellet (2015).

estimation of our model. First, different surveys use different definitions of inflation.[8] Second, different surveys provide inflation forecasts for different horizons. Third, surveys typically provide point estimates but may also provide information on the distribution of inflation as well as individual forecasters' estimates.[9] We summarize data from the various surveys we use and emphasize their differences in sections 7.4.1 and 7.4.2.

3.1 U.S. surveys

U.S. surveys used in our study include the Survey of Professional Forecasters (US-SPF), Blue Chip Financial Forecasts (BCFF), Blue Chip Economic Indicators (BCEI), the Desk's Survey of Primary Dealers (PDS), and Consensus Forecasts. Below we provide a brief description of each of them.

The Survey of Professional Forecasters (US-SPF) is conducted quarterly since 1969. For the purpose of this study, we use a few different inflation forecasts from the US-SPF. First, we use density forecasts for the price change in the GDP price deflator for the current and the following calendar year. US-SPF defines the price change as the annual-average over annual-average percent change. Second, we also use in our model estimation the point estimates of the CPI inflation forecasts for the five-year horizon in order to identify the more distant horizon inflation forecasts. Third, we obtain a survey-based inflation forecast uncertainty measure using the variance of the average forecast density function.[10] Our sample for the density functions starts in 1999:Q1 and extends to 2015:Q4. Our sample for the 5-year CPI inflation point estimates is from 2005:Q3 (the starting point of the forecast point estimates in the US-SPF) to 2015:Q4.

Blue Chip Financial Forecasts (BCFF) and Blue Chip Economic Indicators

[8] Specifically, there are three different definitions of inflation in the various surveys used and we account for all three in our model estimation.

[9] Our model assumes the existence of a representative forecaster. Hence, it does not account for the heterogeneity associated with the availability of individual estimates. Accordingly, in our estimation, we use the average of survey outputs (i.e. point estimates and/or distributions). See section 7.5 for further details.

[10] Ideally, these distributions ought to be smoothed. See 7.6 for more details.

(BCEI) surveys are published monthly. The structure of the BCFF and BCEI surveys is slightly different, however both Blue Chip surveys provide point estimates of inflation forecasts and disagreement. While Blue Chip inflation point forecasts have been used in the literature extensively (Chun, 2011; Grishchenko and Huang, 2013; D'Amico et al., 2016; Grishchenko et al., 2016), Blue Chip inflation disagreement measures have only recently become popular (Wright, 2011; Buraschi and Whelan, 2012; D'Amico and Orphanides, 2014). A peculiar feature of Blue Chip surveys is that they provide fixed event forecasts. Therefore, in order to obtain a constant forecast horizon we linearly interpolate available forecasts. Short-horizon inflation forecasts from one to five (or six) quarters out are available monthly and thus our sample for those forecasts starts from January 1999 and extends to December 2015. In addition, BCFF and BCEI surveys publish long-range forecasts twice a year. These long-range forecasts contain average annual forecasts usually five years out from the survey publication year, and the average forecast of the next five years afterwards. We also use the five-year five years ahead inflation forecasts in our set of observable inflation forecast variables.

The Federal Reserve of New York Survey of Primary Dealers (PDS hereafter) is relatively new (i.e. since 2004), and to the best of our knowledge, we are the first who use this survey in the academic literature. Prior to each FOMC meeting, the survey asks Federal Reserve primary dealers a number of questions including density forecasts for CPI inflation. In particular, starting March 2007 the survey primary dealers are asked to provide the percent chance attached to the annual average five-year CPI inflation five years ahead being below 1%, between 1.01% and 1.50%, between 1.51% and 2%, between 2.01% and 2.50%, between 2.51% and 3%, and above 3.01%.[11] Starting December 2014, primary dealers are also asked to provide the same inflation density forecasts over the next five years. Thus, the PDS survey nicely complements information from US-SPF surveys, which provide density inflation forecast functions for the shorter horizons (one and two years), with

[11] The bins did not change over the time of the survey.

density forecasts over the longer horizons, namely, five years out and five-year five years ahead. In addition, primary dealers are asked to provide the point estimates for the most likely inflation outcome for the same horizons.

3.2 European surveys

Euro area surveys include the European Central Bank's Survey of Professional Forecasters (EA-SPF), Consensus Forecasts (CF) and Blue Chip Economic Indicators (BCEI). We briefly describe each survey below.

The European Central Bank's Survey of Professional Forecasters (ECB-SPF) is a quarterly survey that was launched on the first quarter of 1999 and which has received a considerable amount of attention by academics and practitioners in recent years (see for instance Conflitti (2010); Rich et al. (2012); Andrade and Bihan (2013)). The survey provides GDP forecasts, inflation expectations, and unemployment forecasts. It also provides assumptions made by different forecasters. In our study we focus on inflation forecasts stemming from this survey. Participants are asked to provide point forecasts and probability distributions for rolling horizons (one and two years ahead year-on-year forecasts) and longer-term expectations (five years ahead).

Additional surveys for the euro area include the Consensus Forecasts (CF) and BCEI surveys. CF survey participants provide point estimates for the average annual per cent change of HICP (in the case of the euro area) relative to the previous calendar year. These projections are available for the current and the next calendar year, since January 1999 (in the case of the euro area). BCEI surveys also provide monthly euro area HICP projections for the current and next year point estimates, since December 2006. Specifically, published forecasts only include the average of point estimates across participants (also known as the consensus) as well as the average top and bottom 3 point forecasts.

4 Model and estimation strategy

4.1 Inflation and its driving factors

Let us denote by $\pi^{(i)}_{t,t+h}$ the annualized inflation rate in economy i between dates t and dates $t+h$, defined as the log difference in the price index $P^{(i)}_t$:

$$\pi^{(i)}_{t,t+h} = \frac{12}{h} \log\left(\frac{P^{(i)}_{t+h}}{P^{(i)}_t}\right). \tag{1}$$

We assume that the annual inflation rate, $\pi^{(i)}_{t-12,t}$, is a linear combination of factors gathered in the $n \times 1$ vector $Y_t = (Y_{1,t}, \dots, Y_{n,t})'$. As specified below, the dynamics of Y_t is such that the marginal mean of Y_t is zero. Importantly, $Y_{j,t}$ factors, where $j \in \{1, \dots, n\}$, may be common to different economies. Specifically:

$$\pi^{(i)}_{t-12,t} = \bar{\pi}^{(i)} + \delta^{(i)\prime} Y_t. \tag{2}$$

We assume that the distribution of Y_t is Gaussian conditional on its past realization $\underline{Y_{t-1}} = \{Y_{t-1}, Y_{t-2}, \dots\}$ and another $q \times 1$ exogenous vector $z_t = (z_{1,t}, \dots, z_{q,t})'$ that affects the variance of Y_t. In particular, we assume the following functional form for Y_t:

$$Y_t = \Phi_Y Y_{t-1} + \Theta(z_t - \bar{z}) + diag(\sqrt{\Gamma_{Y,0} + \Gamma'_{Y,1} z_t})\varepsilon_{Y,t}, \quad \varepsilon_{Y,t} \sim \mathcal{N}(0, I), \tag{3}$$

where $\bar{z}$ is the unconditional mean of z_t, $\Gamma_{Y,0}$ is an $n \times 1$ vector, and $\Gamma_{Y,1}$ is a $q \times n$ matrix. According to (3) z_t affects both the conditional expectation and variance of Y_t, so that the Y_t process features stochastic volatility. Similar modelling has been entertained in the literature, (see, e.g., Capistrán and Timmermann, 2009; Caporale et al., 2010; Andrade et al., 2014). Vector z_t is essential for modelling time-varying inflation variances so we refer to z_t as the uncertainty vector (and to $z_{j,t}$ as the uncertainty factors) hereinafter.

The specification of the conditional variance in (3) implies that the entries of

$\Gamma_{Y,0} + \Gamma'_{Y,1} z_t$ have to be non-negative for all t. To achieve this, we assume that all elements of Γ_Y vectors are non-negative and that z_t follows a multivariate auto-regressive gamma process (Appendix 7.2). As shown in the Appendix, the dynamics of z_t admits the following weak VAR representation:

$$z_t = \mu_z + \Phi_z z_{t-1} + diag(\sqrt{\Gamma_{z,0} + \Gamma'_{z,1} z_{t-1}})\varepsilon_{z,t}, \tag{4}$$

where, conditional on $\underline{z_{t-1}}$, $\varepsilon_{z,t}$ has a zero mean and a unit diagonal covariance matrix, and where $\Gamma_{z,0}$ is a $q \times 1$ vector and $\Gamma_{z,1}$ is a $q \times q$ matrix.

Given the dynamics for Y_t and z_t, the VAR form of the dynamics followed by $X_t = (Y'_t, z'_t)'$ is:

$$X_t = \begin{bmatrix} Y_t \\ z_t \end{bmatrix} = \mu_X + \Phi_X \begin{bmatrix} Y_{t-1} \\ z_{t-1} \end{bmatrix} + \Sigma_X(z_{t-1})\varepsilon_{X,t}, \tag{5}$$

where $\varepsilon_{X,t}$ is a $(n+q)$-dimensional unit-variance martingale difference sequence and where:

$$\mu_X = \begin{bmatrix} -\Theta\Phi_z(I-\Phi_z)^{-1}\mu_z \\ \mu_z \end{bmatrix}, \quad \Phi_X = \begin{bmatrix} \Phi_Y & \Theta\Phi_z \\ 0 & \Phi_z \end{bmatrix},$$

$$\Sigma_X(z_{t-1})\Sigma_X(z_{t-1})' = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma'_{12} & \Sigma_{22} \end{bmatrix}$$

$$\text{with} \quad \begin{cases} \Sigma_{11} = \Theta \times diag(\Gamma_{z,0} + \Gamma'_{z,1} z_{t-1}) \times \Theta' + diag(\Gamma_{Y,0} + \Gamma'_{Y,1}(\mu_z + \Phi_z z_{t-1})), \\ \Sigma_{22} = diag(\Gamma_{z,0} + \Gamma'_{z,1} z_{t-1}), \\ \Sigma_{12} = \Theta\Sigma_{22}. \end{cases}$$

An important property of X_t is that this process is affine (see Appendix 7.1.1). In particular, this implies that, at any date t, the first and second conditional moments of any linear combination of future X_t are affine functions of X_t. Since the realized log annual growth rate of the price index $\pi^{(i)}_{t-12,t}$ is an affine transformation of X_t

(see eq. (2)), its first and second moments can be written as an affine function of X_t factors as well:

$$\begin{aligned}
\mathbb{E}_t(\pi^{(i)}_{t+h-12,t+h}) &= \bar{\pi}^{(i)} + a^{(i)}_h + b^{(i)\prime}_h X_t \qquad (6)\\
\mathbb{V}ar_t(\pi^{(i)}_{t+h-12,t+h}) &= \alpha^{(i)}_h + \beta^{(i)\prime}_h X_t \qquad (7)
\end{aligned}$$

where $\mathbb{E}_t(\bullet)$ and $\mathbb{V}ar_t(\bullet)$ respectively denote the expectations and variances conditional on $\underline{X_t}$. In our empirical analysis we consider inflation forecast rates over different horizons because of the nature of the surveys we fit. In particular, we calculate the annualized h-period ahead inflation rates $\pi^{(i)}_{t,t+h} := (12/h)\log(P^{(i)}_{t+h}/P^{(i)}_t)$, which are also affine functions of X_t:

$$\pi^{(i)}_{t,t+h} = \frac{1}{k}\delta^{(i)\prime}(X_{t+12} + X_{t+24} + \cdots + X_{t+h}), \qquad (8)$$

where $h = 12 \times k$. Therefore, their first and second moments can be also written as affine functions of X_t:

$$\begin{aligned}
\mathbb{E}_t(\pi^{(i)}_{t,t+h}) &= \bar{\pi}^{(i)} + \bar{a}^{(i)}_h + \bar{b}^{(i)\prime}_h X_t \qquad (9)\\
\mathbb{V}ar_t(\pi^{(i)}_{t,t+h}) &= \bar{\alpha}^{(i)}_h + \bar{\beta}^{(i)\prime}_h X_t. \qquad (10)
\end{aligned}$$

Appendices 7.1.2 and 7.1.3 outline the recursive algorithms used to compute the parameters of the moments of the realized ($a^{(i)}_h$, $\alpha^{(i)}_h$, $b^{(i)}_h$, $\beta^{(i)}_h$) and forecast ($\bar{a}^{(i)}_h$, $\bar{\alpha}^{(i)}_h$, $\bar{b}^{(i)}_h$, $\bar{\beta}^{(i)}_h$) inflation rates.[12]

[12] $a^{(i)}_h$, $\alpha^{(i)}_h$, $b^{(i)}_h$ and $\beta^{(i)}_h$ are obtained by setting $\gamma_1 = \cdots = \gamma_{h-1} = \mathbf{0}$ and $\gamma_h = [\delta^{(i)\prime}, \mathbf{0}]'$ and $\bar{a}^{(i)}_h$, $\bar{\alpha}^{(i)}_h$, $\bar{b}^{(i)}_h$ and $\bar{\beta}^{(i)}_h$ are obtained by setting $\gamma_1 = \cdots = \gamma_{11} = \gamma_{13} = \cdots = \gamma_{12k-1} = \mathbf{0}$ and $\gamma_{12} = \gamma_{24} = \cdots = \gamma_h = [\delta^{(i)\prime}, \mathbf{0}]'$ in the recursive equations (20) and (21).

4.2 State-space model and Kalman-filter estimation

4.2.1 Objective and strategy

In addition to model parameters, we have to estimate the factors X_t that are not observed by the econometrician. We handle both estimations using Kalman filtering techniques. The affine property of the process X_t is key to the tractability of the estimation. Specifically, not only do we have closed-form formulae but the latter are also affine, allowing us to cast the model into the linear state-space form, which is the required form of the model for the Kalman filter algorithm. This is a fundamental difference between our approach and alternative inflation models exhibiting stochastic volatility (see, e.g., Stock and Watson, 2007; Mertens, 2016). Indeed, while the latter models entail closed-form expressions for the first two conditional moments of inflations, the second-order moments are non-linear in the unobserved factors, which substantially complicates the model estimation.

A state-space model consists of two types of equations: transition equations and measurement equations. Transition equations describe the dynamics of the latent factors, as in eq. (5). Measurement equations specify the relationship between the observed variables and the latent factors. A by-product of the Kalman filter algorithm is the likelihood function. Parameter estimates can therefore be obtained by maximising this function.

In our estimation we sequentially use two versions of our state-space model: the basic one and the augmented one.

The *basic* version of the model uses only realized inflation series and survey-based moments of inflation forecasts (eqs. (6) —(10)) to estimate the model parameters using the corresponding Kalman-filter-implied maximum likelihood function. The reason is that we would like to be confident that our estimated model provides a particularly good fit for the realized inflation series and associated survey-based conditional moments of inflation.

The *augmented* version of the model is set up to obtain more accurate estimates of the latent factors X_t. For this purpose we use the LASSO algorithm (see, e.g., Efron et al., 2004; Zou, 2006) that selects *the covariates*, that is, the most relevant additional macroeconomic and financial variables, that is, the variables that closely covary (i.e. displaying an R^2 greater of equal to 50%) with our fitted survey inflation variables.[13] Table 2 provides a detailed account of these variables. We then use the covariates to introduce additional measurement equations for the latent factors in the augmented version of our model. Heuristically, the idea is as follows: if some observed variables are closely related to the linear combinations of the latent factors then additional measurement equations would increase the accuracy of the latent factor estimates. The decline in uncertainty about the latent factors is expected to be stronger for the dates when no surveys are released. This is due to the fact that the information contained in the covariates becomes especially important on days where surveys are not available. We do not use the covariates in the basic version of the model because too many measurement equations that relate latent factors to additional variables (which are not directly related to inflation or inflation uncertainty) might cause over-fitting of these additional variables at the expense of the inflation-related observations.

In the next two sections, we specify the measurement equations for the basic and augmented models.

4.2.2 Measurement equations of the basic model

The basic state-space model involves three types of the measurement equations:

(a) The first set of equations states that, for each economy i, the realised inflation rate is equal to linear combination of factors Y_t, as stated by eq. (2), with area-specific loadings.

(b) The second set of equations states that, up to the measurement error, survey-based expectations of future inflation rates are equal to the model-implied

[13] The use of LASSO regressions is becoming popular among macro-finance researchers. For example, Huang and Shi (2014) use a similar technique to come up with the LASSO-based factor that appears to have a strong predictive power for time-varying Treasury excess bond returns.

ones, that is:

$$SPF_t = \bar{\boldsymbol{\pi}} + \mathbf{a} + \mathbf{b}'X_t + diag(\sigma^{avg})\eta_t^{avg} \tag{11}$$

where η_t^{avg} is a vector of *iid* Gaussian measurement errors, SPF_t gathers all survey-based expected inflations available at date t, and the entries of the vector $\bar{\boldsymbol{\pi}}$, the vector $\mathbf{a}$ and the matrix $\mathbf{b}$ are naturally based on the appropriate $\pi^{(i)}$s, $a_h^{(i)}$s, $b_h^{(i)}$s, $\bar{a}_h^{(i)}$s and $\bar{b}_h^{(i)}$s (see eqs. (6) and (9)).

(c) The third set of equations states that, up to the measurement error, survey-based variances are equal to the model-implied ones, i.e.:

$$VSPF_t = \boldsymbol{\alpha} + \boldsymbol{\beta}'X_t + diag(\sigma^{var})\eta_t^{var} \tag{12}$$

where η_t^{var} is a vector of *iid* Gaussian measurement errors, $VSPF_t$ gathers all survey-based conditional variances of inflation forecasts available at date t, and the entries of the vector $\boldsymbol{\alpha}$ and the matrix $\boldsymbol{\beta}$ are based on the appropriate $\alpha_h^{(i)}$s, $\beta_h^{(i)}$s, $\bar{\alpha}_h^{(i)}$s and $\bar{\beta}_h^{(i)}$s (see eqs. (7) and (10)).

4.2.3 Measurement equations of the augmented model

Measurement equations of the augmented version of our state-space model include all measurement equations of the basic model outlined in section 4.2.2 and measurement equations that specify the relationship between additional variables, stacked in the vector F_t, and the latent variables:

$$F_t = \boldsymbol{c} + \boldsymbol{d}'X_t + diag(\sigma^F)\eta_t^F, \tag{13}$$

where η_t^F is assumed to be a vector of Gaussian measurement errors. This set of equations augments the information set that we use to obtain estimates of the latent factors.

Let us denote by S_t the vector of observations used in the basic state-space model. Since the latter is based on equations of types (a), (b), and (c) in section 4.2.2, we have $S_t = [\pi_t^{(1)}, \dots, \pi_t^{(r)}, SPF_t', VSPF_t']'$. Using obvious notations, the measurement equations of the basic state-space model read:

$$S_t = A + B'X_t + diag(\sigma^S)\eta_t^S, \tag{14}$$

where $\mathbb{V}ar(\eta_t^S) = Id$. In the augmented state-space model, equations (13) are introduced in the set of measurement equations lending to the augmented vector of the measurement variables $S_t^a = [S_t', F_t']'$. The next section describes how we select observed F_t variables.

4.2.4 Selection of additional observed variables F_t

Additional F_t variables in the model estimation are meant to provide extra information relevant to the latent variables in each area, especially for months when survey forecasts are not available. We select these variables as follows.

First, we choose the set of financial and macroeconomic variables potentially relevant to inflation expectations and uncertainty. We report these variables in Table 2. Financial variables include U.S. and euro area 1-, 2-, 5-, and 10-year inflation swap rates for each area, respectively, VIX —implied volatility index for the S&P500 —for both areas, VSTOXX —implied volatility index of the euro STOXX50 index —for the euro zone only, and gold and oil prices for both areas. Macroeconomic variables include U.S. and euro area consensus and disagreement about one-year ahead inflation rates extracted from Consensus Forecasts surveys, for each area, respectively; U.S. consensus and disagreement about two-quarter and four-quarter ahead inflation rates from Blue Chip survey for U.S. area; and U.S. and euro-area economic uncertainty indices based on Baker, Bloom, and Davis (2015) for both areas.

Second, we regress each of the survey-based series used in the basic state-space model on these variables. For some survey-based series, only a small number of observations is available due to a relatively low frequency of surveys, therefore, we run LASSO regressions with at most two potential explanatory variables for the survey-based inflation expectations and uncertainty measures. The outcome of this second step is that we obtain a more contained set of covariates, F_t, that is, the set of variables that covary relatively closely with the survey-based series. A certain variable is not included into the set of covariates if the corresponding R^2 is lower than 50%. [14]

As an illustration, let us consider the j^{th} component of vector S_t. Let us denote by $\mathcal{F}_t$ the vector of all potential covariates we described above. So, the LASSO regression leads to:

$$S_{j,t} = \zeta_{0,j} + \zeta_{1,j}' \mathcal{F}_t + \sigma_j^e e_{j,t}, \quad \mathbb{V}ar(e_{j,t}) = 1, \tag{15}$$

where one or two entries of $\zeta_{1,j}$ are nonzero. Given eq. (14), we also have: $S_{j,t} = A_j + B_j' X_t + \sigma_j^S \eta_{j,t}^S$. Therefore, we obtain:

$$F_{j,t} = A_j + B_j' X_t + \sigma_j^S \eta_{j,t} - \sigma_j^e e_{j,t}. \tag{16}$$

where $F_{j,t}$ are the fitted values of the LASSO regression given by $\hat{\zeta}_{0,j} + \hat{\zeta}_{1,j}' \mathcal{F}_t$ and using up to two variables in $\mathcal{F}_t$. Of note, $S_{j,t}$ may often have missing observations, while $F_{j,t}$ is regularly observed. Thus, even if $S_{j,t}$ is not available, the filter can rely on $F_{j,t}$ to infer X_t. Finally, assuming orthogonality between the $e_{j,t}$ and the $\eta_{j,t}$, the variance of the errors of the selected variables, $(\sigma_j^F)^2$ is:

$$(\sigma_j^F)^2 = (\sigma_j^S)^2 + (\sigma_j^e)^2.$$

Note also that the measurement errors in the j^{th} equation of system (14) are correlated to those in eq. (16), and the covariance of the errors is equal to $(\sigma_j^S)^2$. The results of this procedure are summarised in Table 3.

[14] For the survey-based series with less than 30 observations, we use a single covariate. For references on LASSO regressions, see Efron, Hastie, Johnstone, and Tibshirani (2004) and Zou (2006).

4.2.5 Remarks about the model estimation.

At this stage, two remarks are in order. First, most survey forecasts are not released every month (with the exception of Blue Chip surveys), so SPF_t and $VSPF_t$ variables are not available every months and thus these series contain missing observations when measured at monthly frequency.[15] Fortunately, it is straightforward to adjust the Kalman filter in order to handle missing observations (for details see Harvey and Pierse, 1984; Harvey, 1989). For the months when no SPF_t and $VSPF_t$ variables are available, the filter is still able to produce estimates of all latent factors, though with lower precision. In those months additional information in factors F_t may be particularly useful.

The second remark regards the performance of the Kalman filter in the present case. While the affine form of the transition and measurement equations facilitates the implementation of the filter, the filter we eventually run is not optimal. It would be optimal had the conditional covariance matrix $\Sigma_X \Sigma_X'$ in eq. (5) not been dependent on X_{t-1}. However it is not the case since some entries of $\Gamma_{Y,1}$ are non-null.[16] Therefore, we estimate our model using a quasi-maximum-likelihood (QML) approach (see, e.g., Duan and Simonato, 1999; de Jong, 2000).

5 Results

5.1 Estimated model

Table 4 presents parameter estimates of the augmented model described in section 4.2.3. We assume that there are four Y_t factors that explain inflation variations

[15] An alternative, but equivalent, view would be that the vectors and matrices $\bar{\boldsymbol{\pi}}$, $\mathbf{a}$, $\mathbf{b}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ have time-varying sizes.

[16] Our filter algorithm makes use of the standard forecasting and updating steps of the Kalman filter except that, at iteration t, we replace the unobserved covariance matrix of the X_t innovations ($\Sigma_X(z_{t-1})\Sigma_X(z_{t-1})'$) by $\Sigma_X(z_{t-1|t-1})\Sigma_X(z_{t-1|t-1})'$, where $z_{t-1|t-1}$ denotes our filtered estimate of z_{t-1} (using the information up to date $t-1$). Another adjustment we have to make to the filter pertains to the fact that factors z_t are non-negative. For this purpose, after each updating step of the algorithm, negative entries in the z_t estimate are replaced by 0. Monte Carlo analyses by Duan and Simonato (1999) and Zhou (2001) suggest that in the case of linear but heteroskedastic models, that kind of approximation may be of limited importance in practice (see also Duffee and Stanton (2012)).

and two z_t volatility factors that explain inflation uncertainty. Most of the parameter estimates are highly statistically significant. We observe the autoregressive parameters pertaining to the first and fourth factors, namely, $\Phi_Y(1,1)$ and $\Phi_Y(4,4)$ are close to 1, thus these factors appear to be very persistent in our estimation.

Figure 1 displays the factor loadings of the estimated augmented model. The first factor appears to be the most important one both for the euro area (top left panel) and for the U.S. (top right panel). This factor has a similar loading for both economies. The second most important factor is the third one for the euro area and the fourth one for the U.S. economy. Two middle panels suggest that the first volatility factor is important for inflation expectations at shorter horizons (up to about five years) for both economies.

The fit of the surveys is illustrated in Figure 2. The fit is very satisfactory despite the fact that we are fitting the first two moments of many different types of inflation expectations across different economies.

Let us turn to the influence of adding covariates to the basic state-space model. Recall that some surveys are not released every month, thus some entries of vector S_t are not available. Let us introduce the vector $\widetilde{S}_t$, that is the vector of surveys that would prevail if surveys were conducted on a monthly basis. If the survey on which the j^{th} entry of the S_t vector is based on is released in month t, then we have $\widetilde{S}_{j,t} = S_{j,t}$. If this survey is not released in month t, one can get an estimate of it by computing $\mathbb{E}(\widetilde{S}_{j,t}|\underline{S_t})$. We call these estimates conditional expectations of the *pseudo surveys*. This latter expression is equal to $A_j + B_j'\mathbb{E}(X_t|\underline{S_t})$, where the expectation term represents the filtered estimate of X_t, which is an output of the Kalman algorithm. An additional output of the filter is the conditional variance $\mathbb{V}ar(X_t|\underline{S_t})$. Hence, it is straightforward to compute the variance associated with our *pseudo surveys*, given by:

$$\mathbb{V}ar(\widetilde{S}_{j,t} - \mathbb{E}(\widetilde{S}_{j,t}|\underline{S_t})|\underline{S_t}) = (\sigma_j^S)^2 + B_j'\mathbb{V}ar(X_t|\underline{S_t})B_j. \tag{17}$$

The variance in eq. (17) is obtained in the context of the basic state-space model, into which we do not incorporate information from additional variables F_t into the conditioning information set used in the previous conditional moments ($\underline{S_t}$).[17] Once the augmented state-space is settled, one can derive alternative *pseudo surveys* through the computation of $\mathbb{E}(\widetilde{S}_{j,t}|\underline{S^a_t})$. Equivalently, the variance associated with these *pseudo surveys* is:

$$\mathbb{V}ar(\widetilde{S}_{j,t} - \mathbb{E}(\widetilde{S}_{j,t}|\underline{S^a_t})|\underline{S^a_t}) = (\sigma^S_j)^2 + B'_j \mathbb{V}ar(X_t|\underline{S^a_t})B_j. \tag{18}$$

Note that conditional variances (17) and (18) are valid only if $S_{j,t}$ is not observed on date t. Figure 3 displays standard deviations implied by eqs. (17) and (18) for all survey-based components of S_t for the U.S. and the euro area. Standard deviations appear to be lower for the pseudo surveys implied by the augmented estimation, indicating that the F_t set informatively augments our estimation procedure in filling the gaps in survey data.

5.2 Model-implied conditional distributions

Figure 4 compares the one-year ahead survey-based inflation histograms to the one-year ahead model-implied distributions of inflation. For the model-implied distributions, two-standard-deviation confidence intervals are reported. These standard deviations reflect uncertainty associated with the estimation of the latent factors X_t and are obtained by applying the delta method on the function relating X_t factors to the conditional cumulative distribution function (c.d.f.) of future inflation.[18] As this figure shows, the fit of the model to the surveys is fairly good. It is also evident that both areas' distributions have shifted noticeably to the left from 2005 to 2014, suggesting a decline in inflation expectations, which is more pronounced in the euro

[17] Note that this estimation error reflects uncertainty about the estimation of the latent factors X_t but not the uncertainty about model parameters. Moreover, we consider here *filtering* errors, that are the differences between $\widetilde{S}_{j,t}$ and $\mathbb{E}(\widetilde{S}_{j,t}|\underline{S_t})$. We could have considered the *smoothing* errors, which are the differences between $\widetilde{S}_{j,t}$ and $\mathbb{E}(\widetilde{S}_{j,t}|\underline{S_T})$, where T is the size of the whole sample (in particular, $T \geq t$).

[18] The covariance matrix of the filtered values of X_t stems from the Kalman filter. Appendix 7.3 details the computation of the c.d.f. of future inflation rates.

area. In addition, the euro area's inflation distribution also flattened, somewhat indicating an increase in the variance of inflation expectations, and, thus, greater inflation uncertainty, as suggested by Figure 2.

Figure 5 displays model-implied conditional distributions of future inflation at 1-, 2- and 5-year maturities at three different dates, prior to, at the height of, and after the 2008 financial crisis. Figure 6 displays the term structure of model-implied expected inflation rates along with the 5th and 95th quantiles associated with the conditional distributions.[19] Both figures corroborate the fact that inflation expectations at shorter horizons (up to 5 years) have moved and are substantially lower than those at longer horizons. Euro area inflation uncertainty over expectations at short horizons increased relative to longer horizons. The low response of euro area long-run expectations to current events may be due to their high persistence, which might be an artefact of the explicit inflation target adopted by this economy. In the U.S., inflation uncertainty decreased at short horizons. Interestingly however, inflation uncertainty increased at long horizons.

Figure 7 shows model-implied probabilities of negative (top panels) and lower than 1 percent (bottom panels) future inflation rates for one- and three-year ahead horizons. The grey shaded areas are two-standard-deviation confidence intervals.[20] Unsurprisingly, low inflation probabilities are higher in the short run than in the long run. Both economies faced high low-inflation probabilities shortly after the Lehman Brothers collapse, but were more pronounced in the United States.

[Insert Figure 7 about here.]

Figure 8 compares model-implied (physical) probabilities of negative and lower than 1 percent future inflation rates to their risk-neutral counterparts. The risk-neutral probabilities are based on inflation derivatives, namely, zero-coupon inflation swaps and inflation floors.[21] As in Figure 7, low-inflation probabilities are higher

[19] The quantiles are derived from closed-form formulas given in Appendix 7.3.

[20] These standard deviations are obtained by applying the delta method on the function relating X_t factors to the conditional cumulative distribution function (c.d.f.) of future inflation.

[21] The risk-neutral distributions –more precisely the forward-neutral distributions– are assumed to be of the generalised Beta type (see Appendix 7.6). These distributions are specified by four

in the short run than in the long run. Importantly, risk-neutral probabilities are higher than their physical counterparts and their difference is substantial. This seems to suggest that the the current low-inflation environment is perceived as less persistent in survey-based model-implied probabilities than under their option-based counterparts.

[Insert Figure 8 about here.]

Panel (a), (b) and (c) in Figure 9 display the conditional standard deviations, covariances and correlations, respectively, of future inflation rates for the two areas. Panel (d) provides joint probabilities of deflation, i.e. $\mathbb{P}(\pi_{t+h}^{(E.A.)} \leq 0, \pi_{t+h}^{(U.S.)} \leq 0 | \underline{S_t^a})$. We find that the joint probability of deflation is not necessarily higher at shorter horizons. Nonetheless, at the height of the crisis in 2009 and in recent months the probability of the joint 12-month ahead deflation is higher than the joint probability of the 60-month ahead deflation. We also find that the correlation between euro area and U.S. inflation rates began increasing in 2010, reaching in late 2015 a correlation of roughly 70%. This finding supports the idea that joint inflation movements have grown in importance due to the intertwined nature of economies nowadays. However, it could also be an artefact of a common shock, such as the oil shock leading to substantial oil price declines (since mid-2014) and affecting both economies simultaneously.

[Insert Figure 9 about here.]

Figure 10 displays the contour plots of the bivariate conditional distribution of future one- and five-year inflation rates in the United States and the euro area. These distributions have been obtained by 10,000 simulations of the model, using smoothed estimates of X_t (for considered dates) as initial conditions. It appears that U.S. inflation expectations were worse-off in 2009, soon after the Lehman Brothers collapse, relative to the euro area. However, euro area inflation expectations have

parameters. For each area, each date, and each maturity, these four parameters are chosen so as to minimise the weighted sum of squared pricing errors.

been lower in the early-2015 period. The results are more pronounced for one-year rather than for five-year expectations.

[Insert Figure 10 about here.]

Table 5 reports regression results that relate deflation probabilities, inflation covariances and inflation variances to various explanatory variables. We find that Economic Policy Uncertainty (EPU), inflation risk premia and sentiment indices have a higher explanatory power relative to the stock markets' volatility when it comes to co-movement indicators such as joint deflation probabilities and covariances as well as individual deflation probabilities and variances. EPU indices are positively correlated and particularly interesting for the five-year (joint and area-specific) deflation probabilities, as well as for inflation covariances, euro area inflation variances, and U.S. five-year inflation variances. Both euro area and U.S. economic sentiment indices have low explanatory power for joint and euro area deflation probabilities, but high explanatory power for one-year ahead inflation covariances and U.S. variances. Moreover, economy-specific inflation risk premia are relevant for deflation probabilities and variances of that particular economy. Interestingly, we also find that euro area indicators (i.e. EPU and risk premia) seem to be more useful in explaining joint and U.S. deflation probabilities than their U.S. counterparts, suggesting interactions between the two economies.

Figure 11 displays the two areas' conditional standard deviations of annualised inflation over the next five years (Panel A), over the next ten years (Panel B), and for the five-year anuualized inflation betwen five and ten years ahead (Panel C). Our model reveals that there are differences in uncertainty measures.[22] Although we observe some convergence for the uncertainty measures over the next five years across the two areas, euro area standard deviations remain smaller. This may be explained by the average lower level of inflation in the euro area, and also by the absence of the explicit inflation target in the U.S, that has been adopted only in

[22]Raw data stemming from surveys do not allow to carry out comparisons because surveys are different in nature and cannot be directly compared.

September 2012.[23]

[Insert Figure 11 about here.]

In Figure 12 we propose thinking of the anchoring of inflation expectations in terms of conditional distributions. This novel approach inherently suggests that uncertainty around inflation expectations is key to gauging the extent to which those expectations are anchored. We observe that the probability of medium- and long-run inflation expectations in the euro area being within [1.5%,2.5%] and [1%,3%] has decreased by roughly 10% since late 2008. In the U.S. these probabilities have increased substantially in recent months and have reached pre-crisis levels. Nonetheless, these probabilities remain substantially higher in the euro area relative to the U.S. suggesting that inflation expectations in the euro area remain better anchored. These results are in line with the findings of Beechey, Johannsen, and Levin (2011). Finally, post-crisis increase in probabilities of the US future inflation rate being in the above-mentioned brackets may be due to the explicit inflation target announced by the Federal Open Market Committee in September 2012.

[Insert Figure 12 about here.]

6 Conclusion

We build a dynamic factor model with stochastic volatility for the joint estimation of inflation expectations in the United States the euro area. We use surveys of professional forecasters both in the U.S. and the euro area to fit the first and second moment of inflation expectations in the two currency areas. In our model, inflation rates and the first two moments of inflation distributions are affine combinations of the latent factors, thus preserving the sought-after affine property which guaranties closed-form solutions. In addition, the model benefits from the ability to account

[23]See the September 2012 FOMC statement http://www.federalreserve.gov/newsevents/press/monetary/20120913a.htm.

for different types of inflation expectations, as defined by the various available surveys. We find that since 2010 inflation expectations moved down noticeably in both economies and the correlation between future inflation rates in the two economies increased. This correlation and probability of deflation occurring jointly in both economies are related to economic policy uncertainty and economic sentiment indices. Moreover, we find that lately, inflation expectations have moved down in both economies. However, relative to the U.S., inflation expectations remain more anchored in the euro area despite the fact that their degree of anchoring has decreased during the financial crisis.

References

Andrade, P., Bihan, H. L., 2013. Inattentive professional forecasters. Journal of Monetary Economics 60, 967 – 982.

Andrade, P., Ghysels, E., Idier, J., 2014. Inflation risk measures and their informational content. Working paper, http://ssrn.com/abstract=2439607.

Ang, A., Bekaert, G., Wei, M., 2007. Do macro variables, asset markets, or surveys forecast inflation better? Journal of Monetary Economics 54, 1163–1212.

Aruoba, S. B., 2014. Term structures of inflation expectations and real interest rates: The effects of unconventional monetary policy. Staff Report 502, Federal Reserve Bank of Minneapolis.

Baker, S. R., Bloom, N., Davis, S. J., 2015. Measuring economic policy uncertainty. NBER Working Papers 21633, National Bureau of Economic Research, Inc.

Beechey, M. J., Johannsen, B. K., Levin, A. T., 2011. Are long-run inflation expectations anchored more firmly in the euro area than in the united states? American Economic Journal: Macroeconomics 3, 104–29.

Bernanke, B. S., 2007. Inflation expectations and inflation forecasting. Speech 306, Board of Governors of the Federal Reserve System (U.S.).

Boero, G., Smith, J., Wallis, K. F., 2014. The measurement and characteristics of professional forecasters' uncertainty. Journal of Applied Econometrics doi: 10.1002/jae.2400.

Buraschi, A., Jiltsov, A., 2005. Inflation risk premia and the expectation hypothesis. Journal of Financial Economics 75, 429–490.

Buraschi, A., Whelan, P., 2012. Term structure models with differences in beliefs. Working paper, Imperial College London.

Campbell, J. Y., Sunderam, A., Viceira, L., 2013. Inflation bets or deflation hedges? the changing risks of nominal bonds. Working Paper, Harvard University.

Capistrán, C., Timmermann, A., 2009. Disagreement and biases in inflation expectations. Journal of Money, Credit and Banking 41, 365–396.

Caporale, G. M., Paesani, P., Onorante, L., 2010. Inflation and inflation uncertainty in the euro area. Working Paper Series 1229, European Central Bank.

Chernov, M., Mueller, P., 2012. The term structure of inflation expectations. Journal of Financial Economics 106, 367–394.

Chun, A. L., 2011. Expectations, Bond yields, and Monetary Policy. Review of Financial Studies 24, 208–247.

Chun, A. L., 2012. Forecasting interets rates and inflation: Blue chip clairvoyants or econometrics? Working Paper.

Chun, A. L., 2014. A forward-looking model of the term structure of interest rates. Working Paper.

Ciccarelli, M., García, J. A., 2015. International spillovers in inflation expectations. Working Paper Series 1857, European Central Bank.

Ciccarelli, M., Mojon, B., 2010. Global inflation. The Review of Economics and Statistics 92, 524–535.

Clements, M. P., 2014. Probability distributions or point predictions? survey forecasts of us output growth and inflation. International Journal of Forecasting 30, 99–117.

Conflitti, C., 2010. Measuring uncertainty and disagreement in the european survey and professional forecasters. Working Papers ECARES ECARES 2010-034, ULB – Universite Libre de Bruxelles.

Cox, J. C., Ingersoll, J. E., Ross, S. A., 1985. An intertemporal general equilibrium model of asset prices. Econometrica 53, 363–84.

Croushore, D., 2010. An evaluation of inflation forecasts using real-time data. BE Journal of Macroeconomics, Contributions 10.

D'Amico, S., Kim, D. H., Wei, M., 2016. Tips from tips: The informational content of treasury inflation-protected security prices. Journal of Financial and Quantitative Analysis, forthcoming .

D'Amico, S., Orphanides, A., 2014. Inflation uncertainty and disagreement in bond risk premia. Tech. rep., Working paper 2014-24, Federal Reserve Bank of Chicago.

de Jong, F., 2000. Time series and cross-section information in affine term-structure models. Journal of Business & Economic Statistics 18, 300–314.

Duan, J.-C., Simonato, J. G., 1999. Estimating exponential-affine term structure models by Kalman filter. Review of Quantitative Finance and Accounting 13, 111–135.

Duffee, G. R., Stanton, R. H., 2012. Estimation of dynamic term structure models. Quarterly Journal of Finance 2.

Duffie, D., Pan, J., Singleton, K., 2000. Transform analysis and asset pricing for affine jump-diffusions. Econometrica 68, 1343–1376.

Efron, B., Hastie, T., Johnstone, I., Tibshirani, R., 2004. Least angle regression. The Annals of Statistics 32, 407–499.

Engelberg, J., Manski, C. F., Williams, J., 2009. Comparing the point predictions and subjective probability distributions of professional forecasters. Journal of Business & Economic Statistics 27, 30–41.

Faust, J., Wright, J., 2009. Comparing greenbook and reduced form foreacasts using a large real-time data set. Journal of Business and Economic Statistics 27, 468 – 479.

Faust, J., Wright, J. H., 2013. Forecasting Inflation, Elsevier, vol. 2 of *Handbook of Economic Forecasting*, chap. 0, pp. 2–56.

Ghysels, E., Wright, J. H., 2009. Forecasting professional forecasters. Journal of Business and Economic Statistics 27, 504–516.

Giordani, P., Soderlind, P., 2003. Inflation forecast uncertainty. European Economic Review 47, 1037–1059.

Gouriéroux, C., Jasiak, J., 2006. Autoregressive gamma processes. Journal of Forecasting 25, 129–152.

Grant, A. P., Thomas, L. B., 1999. Inflation expectations and rationality revisited. Economic Letters 62, 331–338.

Grishchenko, O. V., Huang, J.-Z., 2013. Inflation risk premium: Evidence from the tips market. Journal of Fixed Income 22, 5–30.

Grishchenko, O. V., Vanden, J. M., Zhang, J., 2016. The informational content of the embedded deflation option in tips. Journal of Banking and Finance 65, 1–26.

Gurkaynak, R., Levin, A. T., Marder, A. N., Swanson, E. T., 2007. Inflation targeting and the anchoring of inflation expectations in the western hemisphere. Economic Review pp. 25–47.

Harvey, A. C., 1989. Forecasting, Structural Time Series Models and the Kalman Filter. Cambridge University press, cambridge ed.

Harvey, A. C., Pierse, R. G., 1984. Estimating missing observations in economic time series. Journal of the American Statistical Association 79, 1–36.

Haubrich, J., Pennacchi, G., Ritchken, P., 2012. Inflation expectations, real rates, and risk premia: Evidence from inflation swaps. Review of Financial Studies 25, 1588–1629.

Huang, J.-Z., Shi, Z., 2014. Determinants of bond risk premia. Working paper, Penn State University.

Kim, D. H., Orphanides, A., 2012. Term structure estimation with survey data on interest rate forecasts. Journal of Financial and Quantitative Analysis 47, 241–272.

Kozicki, S., Tinsley, P., 2006. Survey-based estimates of the term structure of expected u.s. inflation. Tech. rep., Bank of Canada.

Lahiri, K., Sheng, X., 2010. Measuring forecast uncertainty by disagreement: The missing link. Journal of Applied Econometrics 25, 514–538.

Mehra, Y. P., 2002. Survey measures of expected inflation: revisiting the issues of predictive content and rationality. Economic Quarterly pp. 17–36.

Mehrotra, A., Yetman, J., 2014. Decaying expectations: what inflation forecasts tell us about the anchoring of inflation expectations. BIS Working Papers 464, Bank for International Settlements.

Mertens, E., 2011. Measuring the level and uncertainty of trend inflation. Finance and Economics Discussion Series 42, Board of Governors of the Federal Reserve System (U.S.).

Mertens, E., 2016. Measuring the level and uncertainty of trend inflation. Review of Economic and Statistics forthcoming.

Monfort, A., Pegoraro, F., Renne, J.-P., Roussellet, G., 2015. Staying at zero with affine processes: An application to term-structure modelling. Tech. rep., Paris December 2014 Finance Meeting EUROFIDAI - AFFI Paper.

Nagel, S., 2015. Long-run inflation uncertainty. Tech. Rep. December, Comment for the International Journal of Central Banking Annual Conference issue.

Piazzesi, M., Shneider, M., 2007. Equilibrium yield curves. In: *NBER Macroeconomics Annual*, MIT Press, Cambridge, vol. 21, pp. 389–472.

Rich, R., Song, J., Tracy, J., 2012. The measurement and behavior of uncertainty: evidence from the ecb survey of professional forecasters. Staff Report 588, Federal Reserve Bank of New York.

Rudebusch, G., Swanson, E., 2008. Examining the bond premium puzzle with a dsge model. Journal of Monetary Economics 55, S111–S126.

Stock, J. H., Watson, M. W., 2007. Why Has U.S. Inflation Become Harder to Forecast? Journal of Money, Credit and Banking 39, 3–33.

Thomas, L. B., 1999. Survey measures of expected u.s. inflation. Journal of Economic Perspectives 13, 125–144.

Volker, D., 2016. Monetary policy uncertainty and interest rates. Working paper, Copenhagen Business School.

Wright, J. H., 2011. Term premia and inflation uncertainty: Empirical evidence from an international panel dataset. American Economic Review 101, 1514–34.

Zarnowitz, V., Lambros, L. A., 1987. Consensus and uncertainty in economic prediction. Journal of Political Economy 95, 591–621.

Zhou, H., 2001. A Study of the Finite Sample Properties of EMM, GMM, QMLE, and MLE for a Square-Root Interest Rate Diffusion Model. Journal of Computational Finance 5, 99–122.

Zou, H., 2006. The Adaptive Lasso and Its Oracle Properties. Journal of the American Statistical Association 101, 1418–1429.

7 Appendix

7.1 Conditional means and variances of X_t

In this appendix we compute conditional expectations and variances of linear combinations of future X_ts. Formally, we consider the first two moments of the random variable $\Sigma_{i=1}^{h}\gamma_i' X_{t+i}$ conditionally on the information available as of date t (i.e. $\underline{X_t}$).

Appendix 7.1.1 shows that X_t is an affine process. This property implies that the first two conditional moments of X_t are affine in X_t. That is, there exist functions a_h, b_h, α_h and β_h that are such that, for any set of γ_is:

$$
\begin{aligned}
\mathbb{E}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= a_h(\gamma_1,\dots,\gamma_h) + b_h(\gamma_1,\dots,\gamma_h)' X_t \\
\mathbb{V}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \alpha_h(\gamma_1,\dots,\gamma_h) + \beta_h(\gamma_1,\dots,\gamma_h)' X_t.
\end{aligned}
$$

Appendix 7.1.2 (Appendix 7.1.3) provide the recursive formulas that can be used to compute a_h and b_h (α_h and β_h).

7.1.1 Affine property of X_t

Showing that X_t has an affine dynamics amounts to showing that the Laplace transform of X_{t+1}, conditional on $\underline{X_t}$, is exponential affine in X_t.

Lemma 7.1 *The Laplace transform of X_{t+1}, conditional on $\underline{X_t}$, is given by:*

$$
\begin{aligned}
& \mathbb{E}(\exp(u' X_{t+1})|\underline{X_t}) \\
= \; & \exp(u_Y' \Phi_Y Y_t + b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2)' z_t + \\
& a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2) - u_Y' \Theta \bar{z} + 0.5\Gamma_{Y,0}' u_Y^2),
\end{aligned} \tag{19}
$$

where $u = (u_Y', u_z')'$, $u_Y^2 = u_Y \odot u_Y$ (by abuse of notation), Γ_Y is a $q \times n$ matrix and where the functions a_z and b_z define the conditional Laplace transform of z_t (see Appendix 7.2, eq. (22) and (23)).

Proof We have:

$$
\begin{aligned}
& \mathbb{E}(\exp(u'X_{t+1})|\underline{X_t}) \\
= \; & \mathbb{E}(\exp(u_Y'Y_{t+1} + u_z'z_{t+1})|\underline{X_t}) \\
= \; & \mathbb{E}(\mathbb{E}[\exp(u_Y'Y_{t+1} + u_z'z_{t+1})|\underline{X_t}, z_{t+1}]|\underline{X_t}) \\
= \; & \exp(u_Y'\{\Phi_Y Y_t - \Theta\bar{z}\})\mathbb{E}\{\exp((u_z + \Theta' u_Y)'z_{t+1}) \times \\
& \mathbb{E}[\exp(u_Y' diag(\sqrt{\Gamma_{Y,0} + \Gamma_{Y,1}'z_{t+1}})\varepsilon_{Y,t+1})|\underline{X_t}, z_{t+1}]|\underline{X_t}\} \\
= \; & \exp(u_Y'\{\Phi_Y Y_t - \Theta\bar{z}\})\mathbb{E}(\exp((u_z + \Theta' u_Y)'z_{t+1} + 0.5u_Y' diag(\Gamma_{Y,0} + \Gamma_{Y,1}'z_{t+1})u_Y)|\underline{X_t}) \\
= \; & \exp(u_Y'\{\Phi_Y Y_t - \Theta\bar{z}\} + 0.5\Gamma_{Y,0}'u_Y^2)\mathbb{E}(\exp((u_z + \Theta' u_Y + 0.5\Gamma_{Y,1}u_Y^2)'z_{t+1})|\underline{X_t}) \\
= \; & \exp(u_Y'\Phi_Y Y_t + b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1}u_Y^2)'z_t + \\
& a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1}u_Y^2) - u_Y'\Theta\bar{z} + 0.5\Gamma_{Y,0}'u_Y^2),
\end{aligned}
$$

which leads to the result. ■

The fact that X_t follows an affine process implies the following result.

Lemma 7.2 *The multi-horizon Laplace transforms of X_t, conditional on $\underline{X_t}$, are exponential affine in X_t. Specifically, for any set of vectors u_i, $i \in [1, h]$, we have:*

$$
\mathbb{E}(\exp(u_1'X_{t+1} + \dots + u_h'X_{t+h})|\underline{X_t}) = \exp(A_h(u_1, \dots, u_h) + B_h(u_1, \dots, u_h)'X_t),
$$

where the functions A_i and B_i are given by:

$$
\left\{
\begin{array}{l}
A_h([u_Y', u_z']') = a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1}u_Y^2) - u_Y'\Theta\bar{z} + 0.5\Gamma_{Y,0}'u_Y^2 \\
B_h([u_Y', u_z']') = [u_Y'\Phi_Y, b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1}u_Y^2)']'
\end{array}
\right. \quad \textit{if } h = 1,
$$

and

$$
\left\{
\begin{array}{l}
A_h(u_1, \dots, u_h) = A_{h-1}(u_2, \dots, u_h) + A_1(u_1 + B_{h-1}(u_2, \dots, u_h)) \\
B_h(u_1, \dots, u_h) = B_1(u_1 + B_{h-1}(u_2, \dots, u_h))
\end{array}
\right. \quad \textit{otherwise.}
$$

Proof eq. (19) proves that Lemma 7.2 is valid for $h = 1$. Assume Lemma 7.2 is valid for a given $h \geq 1$, we have:

$$
\begin{aligned}
& \mathbb{E}(\exp(u_1'X_{t+1} + \dots + u_{h+1}'X_{t+h+1})|\underline{X_t}) \\
= \; & \mathbb{E}\{\exp(u_1'X_{t+1})\mathbb{E}[\exp(u_2'X_{t+2} + \dots + u_{h+1}'X_{t+h+1})|\underline{X_{t+1}}]|\underline{X_t}\} \\
= \; & \mathbb{E}\{\exp(u_1'X_{t+1})\exp(A_h(u_2, \dots, u_{h+1}) + B_h(u_2, \dots, u_{h+1})'X_{t+1})|\underline{X_t}\} \\
= \; & \exp(A_h(u_2, \dots, u_{h+1}) + A_1(u_1 + B_h(u_2, \dots, u_{h+1})) + B_1(u_1 + B_h(u_2, \dots, u_{h+1})'X_t)),
\end{aligned}
$$

which leads to the result. ■

7.1.2 Computation of a_h and b_h

We have:

$$\begin{aligned}
\mathbb{E}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \mathbb{E}_t\left(\mathbb{E}_{t+1}\sum_{i=1}^{h}\gamma_i' X_{t+i}\right)\\
&= \mathbb{E}_t\left(\gamma_1' X_{t+1} + a_{h-1}(\gamma_2,\dots,\gamma_h) + b_{h-1}(\gamma_2,\dots,\gamma_h)' X_{t+1}\right)\\
&= a_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h)) +\\
&\quad b_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h))' X_t,
\end{aligned}$$

which implies that:

$$\begin{cases} a_h(\gamma_1,\dots,\gamma_h) = a_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h)) \\ b_h(\gamma_1,\dots,\gamma_h) = b_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h)), \end{cases} \tag{20}$$

with $a_1(\gamma) := \gamma'\mu_X$ and $b_1(\gamma) := \Phi_X'\gamma$.

7.1.3 Computation of α_h and β_h

We have:

$$\begin{aligned}
\mathbb{V}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \mathbb{V}_t\left(\mathbb{E}_{t+1}\left[\sum_{i=1}^{h}\gamma_i' X_{t+i}\right]\right) + \mathbb{E}_t\left(\mathbb{V}_{t+1}\left[\sum_{i=1}^{h}\gamma_i' X_{t+i}\right]\right)\\
&= \mathbb{V}_t\left(\gamma_1' X_{t+1} + \mathbb{E}_{t+1}\left[\sum_{i=2}^{h}\gamma_i' X_{t+i}\right]\right) + \mathbb{E}_t\left(\mathbb{V}_{t+1}\left[\sum_{i=2}^{h}\gamma_i' X_{t+i}\right]\right)\\
&= \mathbb{V}_t\left(a_{h-1}(\gamma_2,\dots,\gamma_h) + (b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1)' X_{t+1}\right) +\\
&\quad \mathbb{E}_t\left(\alpha_{h-1}(\gamma_2,\dots,\gamma_h) + \beta_{h-1}(\gamma_2,\dots,\gamma_h)' X_{t+1}\right)\\
&= \alpha_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + \beta_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1)' X_t +\\
&\quad \alpha_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h)) + b_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h))' X_t.
\end{aligned}$$

Therefore:

$$\begin{cases} \alpha_h(\gamma_1,\dots,\gamma_h) = \alpha_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + \alpha_{h-1}(\gamma_2,\dots,\gamma_h) + \\ \quad a_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h)) \\ \beta_h(\gamma_1,\dots,\gamma_h) = \beta_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + b_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h)), \end{cases} \tag{21}$$

where, with $S_p = \sum_{i=1}^{p} [e_i^{(p)} \otimes e_i^{(p)}] e_i^{(p)\prime}$:

$$\begin{cases} \alpha_1(\gamma) = (\gamma_Y \otimes \gamma_Y)'[(\Theta \otimes \Theta) S_q \Gamma_{z,0} + S_n \Gamma_{Y,0} + S_n \Gamma'_{Y,1} \mu_z] + (\gamma_z \otimes \gamma_z)' S_q \Gamma_{z,0} \\ \quad +2(\gamma_z \otimes \gamma_Y)'(I_q \otimes \Theta) S_q \Gamma_{z,0}, \\ \beta_1(\gamma)' = (\gamma_Y \otimes \gamma_Y)'[(\Theta \otimes \Theta) S_q \Gamma'_{z,1} + S_n \Gamma'_{Y,1} \Phi_z] + (\gamma_z \otimes \gamma_z)' S_q \Gamma'_{z,1} \\ \quad +2(\gamma_z \otimes \gamma_Y)'(I_q \otimes \Theta) S_q \Gamma'_{z,1}. \end{cases}$$

7.2 Auto-regressive Gamma processes

The vector z_t follows a multivariate $\text{ARG}_\nu(\varphi, \mu)$ process. This process, introduced by Gouriéroux and Jasiak (2006), is the time-discretized Cox, Ingersoll, and Ross (1985) process (see also Monfort, Pegoraro, Renne, and Roussellet (2015)).

Conditionally on $\underline{z_{t-1}} = \{z_{t-1}, z_{t-2}, \dots\}$, the distributions of the different components of z_t, denoted by $z_{i,t}$, are independent and drawn from non-centered Gamma distributions, i.e.:

$$z_{i,t} | \underline{z_{t-1}} \sim \gamma_{\nu_i}(\varphi_i' z_{t-1}, \mu_i),$$

where ν, μ, φ_1, ..., φ_{q-1} and φ_q are q-dimensional vectors. (Recall that W is drawn from a non-centered Gamma distribution $\gamma_\nu(\varphi, \mu)$, iif there exists an exogenous $\mathcal{P}(\varphi)$-distributed variable Z such that $W|Z \sim \gamma(\nu + Z, \mu)$ where $\nu + Z$ and μ are, respectively, the shape and scale parameters of the gamma distribution.)

Importantly, it can be shown that this process is affine, in the sense that its conditional Laplace transform is exponential affine. Formally, the conditional log-Laplace transform of z_{t+1}, denoted by ψ_t, is given by:

$$\psi_t(w) := \log(E_t[\exp(w' z_{t+1})]) = a_z(w) + b_z(w)' z_t,$$

with

$$a_z(w) = -\nu' \log(1 - \mu \odot w) \tag{22}$$

$$b_z(w) = \varphi \left(\frac{w \odot \mu}{1 - w \odot \mu} \right), \tag{23}$$

where φ is the $q \times q$ matrix equal to $[\varphi_1, \dots, \varphi_q]$, where $\odot$ is the element-by-element (Hadamard) product and where, by abuse of notations, the log and division operator are applied element-by-element wise.

The weak vector auto-regressive form of process z_t is given by:

$$z_t = \mu_z + \Phi_z z_{t-1} + diag(\sqrt{\Gamma_{z,0} + \Gamma'_{z,1} z_{t-1}}) \varepsilon_{z,t},$$

where, conditionally on $\underline{z_{t-1}}$, $\varepsilon_{z,t}$ is of mean zero and has a covariance matrix equal to the identity matrix and where:

$$\mu_z = \mu \odot \nu, \quad \Phi_z = (\mu \mathbf{1}'_{q\times 1}) \odot (\varphi'), \quad \Gamma_{z,0} = \mu \odot \mu \odot \nu \quad \text{and} \quad \Gamma'_{z,1} = 2[(\mu \odot \mu)\mathbf{1}'_{q\times 1}] \odot (\varphi').$$

This last formula notably implies that, assuming that the eigenvalues of Φ_z lie (strictly) within the unit circle, the unconditional mean of z_t is equal to $(I_q - \Phi_z)^{-1}\mu_z$ whilst z_t's unconditional variance is equal to $(I_{q^2} - \Phi_z \otimes \Phi_z)^{-1} S_q(\Gamma_{z,0} + \Gamma'_{z,1}\bar{z})$.

7.3 Computation of model-implied conditional distributions

In the model, inflation rates of different areas are equal to the linear combinations of the affine process X_t. This implies the existence of closed-form formulas to derive the conditional distribution functions of future inflation rates for any maturity (see Duffie, Pan, and Singleton (2000)). Specifically, we have:

$$\mathbb{P}(\gamma'_1 X_{t+1} + \dots + \gamma'_h X_{t+h} < y | \underline{X_t}) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[\Psi_h(iv\boldsymbol{\gamma}, X_t)]e^{-ivy}}{v} dv,$$

where $Im(c)$ denotes the imaginary part of $c \in \mathbb{C}$ and where Ψ_h is the multi-horizon Laplace transform of X_t, defined by:

$$\Psi_h(\mathbf{u}, X_t) = \mathbb{E}(\exp(u'_1 X_{t+1} + \dots + u'_h X_{t+h}) | \underline{X_t}), .$$

with $\mathbf{u} = [u_1, \dots, u_h]$. A simple computation of the latter Laplace transform is provided by Lemma 7.2 in Appendix 7.1.1.

7.4 Survey data

We construct a detailed database of inflation expectation surveys at various horizons for both the United States and the euro area. We summarize data from various surveys and emphasize differences among them below.

7.4.1 U.S. surveys

U.S. surveys used in our study include the Survey of Professional Forecasters (US-SPF) maintained by the Federal Reserve Bank of Philadelphia, Blue Chip Financial Forecasts (BCFF), Blue Chip Economic Indicators (BCEI) published by Aspen Publishers, Inc., the Desk's Survey of Primary Dealers (PDS) maintained by the Federal Reserve Bank of New York, and Consensus Forecasts published by Consensus Economics, Inc. Below we provide details of each of them.

1. Philadelphia Fed Survey of Professional Forecasters

The Survey of Professional Forecasters (US-SPF) is conducted quarterly and provides forecasts on a wide range of macroeconomic and financial variables. The American Statistical Association (ASA) started the US-SPF in 1969; The Philadelphia Fed took over the survey in 1990 and conducts it since then. For the purpose of this study, we use a few different inflation forecasts from the US-SPF.

First, we use density forecasts for the price change in the GDP price deflator for the current and the following calendar year. US-SPF defines the price change as the annual-average over annual-average percent change.[24] The US-SPF inflation measure is thus consistent with the following inflation target: $\frac{1}{4}(\pi_{t+h-21,t+h-9} + \pi_{t+h-18,t+h-6} + \pi_{t+h-15,t+h-3} + \pi_{t+h-12,t+h})$. The density functions are available both on an individual and aggregate basis but we use the information from the aggregated (averaged) forecast density functions. Second, we also use in our model estimation the point estimates of the CPI inflation forecasts for the five-year horizon in order to identify the more distant horizon inflation forecasts. Third, we obtain a survey-based inflation forecast uncertainty measure using the variance of the average forecast density function.[25]

Since the forecast density functions are available for the current and the following calendar year, and thus the forecast horizon changes relatively to the survey's timing, we are able to construct the first and the second moments of the density functions four, five, six, seven, and eight quarters out. Our sample for the density functions starts in 1999:Q1 and extends to 2015:Q4. While US-SPF survey data is available starting 1968, the beginning of our sample is motivated by the onset of the euro-zone and availability of the euro area surveys. Our sample for the 5-year CPI inflation point estimates is from 2005:Q3 (the starting point of the forecast point estimates in the US-SPF) to 2015:Q4. Long-term CPI inflation point estimates (i.e. typically 5- and 10-year ahead inflation forecasts) are defined as the annual average h-year CPI inflation, $\frac{1}{h}(\pi_{t+1} + \cdots + \pi_{t+h})$, where h is the forecast horizon.

2. Blue Chip Financial Forecasts and Economic Indicators

Blue Chip Financial Forecasts (BCFF) and Blue Chip Economic Indicators (BCEI) surveys are published monthly by Aspen Publishers, Inc. These surveys represent a reasonably stable panel of about 50 top professional analysts who forecast financial (in the case of the BCFF) and macroeconomic (in the case of the BCEI) variables. The panels of the BCFF and BCEI analysts are different yet the overlap is sig-

[24] We are mindful of different definitions of inflation in various surveys and account for this in our model estimation.

[25] Information about the structure of the survey and definitions of the variables can be obtained in the spf-documentation.pdf in https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters. We obtain aggregate histograms, which are averages of the corresponding individual forecasters' histograms.

nificant.[26] The structure of the BCFF and BCEI surveys is also slightly different. Every month BCFF forecasters provide average forecasts of the U.S. and foreign interest rates, currency values, and the factors that influence them (such as real GDP and inflation). They forecast these variables for the current quarter, next quarter, and so on until five (or, in some cases, six) quarters out. At the same time BCEI analysts provide forecasts of the U.S. economic outlook for the current year and the year ahead. Therefore, both of these surveys provide inflation forecasts, the object of our study.

Both Blue Chip surveys provide only point estimates of inflation forecasts, thus these surveys differ from the US-SPF survey that provides the density of inflation forecasts.[27] Nevertheless, Blue Chip surveys provide useful information on consensus inflation point estimates and on inflation disagreement. While Blue Chip inflation point forecasts have been used in the literature extensively (Chun, 2011; Grishchenko and Huang, 2013; D'Amico et al., 2016; Grishchenko et al., 2016), Blue Chip inflation disagreement measures have only recently become popular (Wright, 2011; Buraschi and Whelan, 2012; D'Amico and Orphanides, 2014). We also use Blue Chip inflation disagreement measures when we need the second moment of the inflation and when inflation uncertainty is not available. Inflation disagreement has received some support earlier in the literature in the context of the US-SPF, as Giordani and Soderlind (2003) point out that disagreement might be a better proxy of inflation uncertainty than what the previous literature has indicated.

A peculiar feature of Blue Chip surveys is that, unlike other surveys, they do not provide constant-horizon forecasts for any of the variables. For example, the BCEI survey provides point forecasts for the current calendar year and the next calendar year. Thus, the actual forecast horizon that pertains to a particular calendar year (current or next) would diminish from January to February to March surveys and so on. Therefore, in order to obtain a constant forecast horizon (that is, horizon two, three, four quarters out (in the case of BCFF surveys) or one year out (in the case of BCEI surveys)) we linearly interpolate available forecasts. In doing so, we assume that the point forecasts correspond to the mid-quarter (mid-year) in the case of BCFF (BCEI).[28]

Short-horizon inflation forecasts from one to five (or six) quarters out are available monthly and thus our sample for those forecasts starts from January 1999 and

[26] Out of 47 and 53 participating analysts in the December 2015 BCFF and November 2015 BCEI surveys, respectively, 35 analysts were participating in both surveys.

[27] Since we study inflation dynamics, we concentrate our discussion on inflation forecasts from now on.

[28] A similar assumption has been made in the literature, see, for example, Kim and Orphanides (2012). For a more detailed treatment of the interpolations used to obtain constant-horizon forecasts, see Chun (2011) and Grishchenko and Huang (2013).

extends to December 2015. In addition, BCFF and BCEI surveys publish long-range forecasts twice a year, BCFF — in December and June and BCEI — in March and October. Thus, there are four long-range forecasts equally spaced throughout a year. These long-range forecasts contain average annual forecasts usually five years out from the survey publication year, and the average forecast of the next five years afterwards. We also use the five-year five years ahead inflation forecasts in our set of observable inflation forecast variables. Thus, our sample period for long-horizon inflation forecasts starts in March 1999 (the first available long-range forecast in a given year is always in March) and extends to December 2015.

3. The Federal Reserve of New York Survey of Primary Dealers

This survey is relatively new, and to the best of our knowledge, we are the first who use this survey in the academic literature. The Markets Group of the Federal Reserve Bank of New York launched the survey of primary dealers (PDS hereafter) in 2004. Prior to each FOMC meeting, the survey asks Federal Reserve primary dealers a number of questions related to monetary policy expectations and the U.S. economic outlook. Thus the survey is published with the FOMC frequency. The survey questions sometimes vary depending on the economic environment but are posted in advance on the New York Federal Reserve website.[29] Nonetheless, certain questions such as the density forecasts for CPI inflation are routinely asked.

In particular, starting March 2007 the survey primary dealers are asked to provide the percent chance attached to the annual average five-year CPI inflation five years ahead being below 1%, between 1.01% and 1.50%, between 1.51% and 2%, between 2.01% and 2.50%, between 2.51% and 3%, and above 3.01%.[30] Starting December 2014, primary dealers are also asked to provide the same inflation density forecasts over the next five years. PDS forecasts are hence consistent with the inflation measure $\pi_{t,t+h}$ defined in eq. (8).[31] Thus, the PDS survey nicely complements information from US-SPF surveys, which provide density inflation forecast functions for the shorter horizons (one and two years), with density forecasts over the longer horizons, namely, five years out and five-year five years ahead. In addition, primary dealers are asked to provide the point estimates for the most likely inflation outcome for the same horizons.

7.4.2 European surveys

Euro area surveys include the European Central Bank's Survey of Professional Forecasters (EA-SPF), Consensus Forecasts (CF) and Blue Chip Economic Indicators

[29]See https://www.newyorkfed.org/markets/primarydealer_survey_questions.html.

[30]The bins did not change over the time of the survey.

[31]For the annual average five-year CPI inflation five years ahead, this inflation measure is adjusted in order to get conditional moments of $\frac{1}{60}(\pi_{t+61} + \cdots + \pi_{t+120})$.

(BCEI). We briefly describe each survey below.

1. European Central Bank's Survey of Professional Forecasters (ECB-SPF)

The EA-SPF quarterly survey was launched on the first quarter of 1999 and has received a considerable amount of attention by academics and practitioners in recent years (see for instance Conflitti (2010); Rich et al. (2012); Andrade and Bihan (2013)). The survey provides GDP forecasts, inflation expectations, and unemployment forecasts. It also provides assumptions made by different forecasters. In our study we focus on inflation forecasts stemming from this survey. The panel of forecasters includes 79 listed international and European institutions as well as a number of other participants who chose to remain anonymous. More than half of the participants involved in the first survey remain in the pool of participants today. The number of participants has nonetheless not decreased given that more than 20 participants have been added throughout the years. The panel of veteran participants is thus relatively stable and the average number of participants who answer all inflation-related questions across surveys remains stable and is on average equal to 34. Participants are asked to provide point forecasts and probability distributions for rolling horizons (one and two years ahead year-on-year forecasts) and longer-term expectations (five years ahead). EA-SPF inflation measures are defined as $\pi_{t+h-12,t+h}$, where h is the forecast horizon.[32]

2. Other surveys for European inflation forecasts

Additional surveys for the euro area include the CF and BCEI surveys.

CF survey participants provide point estimates for the average annual per cent change of HICP (in the case of the euro area) relative to the previous calendar year. These projections are available for the current and the next calendar year, since January 1999 (in the case of the euro area). These surveys are published on a monthly basis, usually in the second week of the month, and cover many other macroeconomic and financial variables which we do not exploit in this paper. There are roughly 20 institutions participating in the euro area survey; less than half of which coincide with disclosed EA-SPF participants. Moreover, most panellists are domestically located, providing thus an economy-specific expertise.

BCEI surveys also provide monthly euro area HICP projections for the current and next year point estimates, since December 2006. Specifically, published forecasts

[32] The survey also provides fixed calendar year horizons (current year, next year and year after next) but we do not include this information due to the fact that only point forecasts are supplied. Moreover, the nature of the fixed horizons forecasts may not allow for consistent comparisons of uncertainty across time given that we ought to see a decrease in uncertainty at every survey round in which more information becomes available.

only include the average of point estimates across participants (also known as the consensus) as well as the average top and bottom 3 point forecasts.

7.5 Survey-based uncertainty

Measuring uncertainty has gained a lot of attention in recent years. Several proxies for uncertainty including stock market volatility, conditional volatility of series, cross-sectional dispersions and keyword counts in newspapers have been used. However, survey-based measures of uncertainty, also known as subjective measures of uncertainty, have become increasingly popular due to their model-free nature.

Three survey-based uncertainty measures have dominated the literature: i) the disagreement among forecasters (*ex-ante* measure), ii) the variance of the surveys' aggregate probability distribution (*ex-ante* measure), and iii) the average individual forecast error variance (*ex-post* measure).

For obvious reasons, *ex-ante* measures of uncertainty have been found to be more adequate representations of uncertainty in real time. Moreover, the variance of the surveys' aggregate probability distribution is the proxy that seems to converge the most towards the notion of Knightian uncertainty (i.e. risk that is immeasurable and for which no probabilities can be assigned). In our paper, we thus draw attention to this particular survey-based uncertainty measure. Nonetheless, in this subsection we shall provide a brief account of the survey-based uncertainty literature.

Due to its simplicity and availability, one of the most common survey-based uncertainty proxies in the literature is the disagreement of forecasters defined as follows:

$$d_{th} = \frac{1}{N}\sum_{i=1}^{N}(f_{ith} - f_{.th})^2, \tag{24}$$

where N is the number of forecasters, f_{ith} is the forecast at time t, for horizon h of individual i and $f_{.th}$ is the mean forecast (i.e. consensus). This proxy of uncertainty, though easily computable, becomes irrelevant if heterogeneity in forecasters vanishes.

In recent years, survey designs have caught-up with the increasingly popular concept of uncertainty and often provide individual probability distributions. The observed heterogeneity of forecasters is tackled using the average of all individual distributions. This aggregation inherently implies the assumption of a representative forecaster. Uncertainty can now be defined as the variance of the aggregate probability distribution. One important implication is that the conditional variance of the aggregate distribution becomes the sum of disagreement and of the average of individual variances. Thus, denoting by $\sigma^2_{agg,th}$ and σ^2_{ith} the conditional variance of the aggregate distribution and the individual variances, respectively, the proxy for uncertainty is given by:

$$\sigma^2_{agg,th} = d_{th} + \frac{1}{N}\sum_{i=1}^{N}\sigma^2_{ith}. \tag{25}$$

This measure of uncertainty captures both forecasters' heterogeneity via the cross-sectional variance of individual means (i.e. disagreement) and the uncertainty of individual forecasters.

An important strand of the literature focuses on the differences between disagreement and uncertainty survey measures (Conflitti, 2010; Rich et al., 2012; Andrade and Bihan, 2013; Boero et al., 2014; D'Amico and Orphanides, 2014). Notably, Giordani and Soderlind (2003) find that disagreement is a fairly good proxy for other measures of uncertainty that are more theoretically appealing, but less easily available.[33] Lahiri and Sheng (2010) decompose forecast errors into common and idiosyncratic shocks, and show that aggregate forecast uncertainty can be expressed as the sum of the disagreement among forecasters and the perceived variability of future aggregate shocks. This finding implies that the reliability of disagreement as a proxy for uncertainty depends primarily on the stability of the forecasting environment.

7.6 Smoothing survey-based and risk-neutral distributions

7.6.1 Overview

Our analysis makes use of the generalised beta distribution twice. First, we use it in order to convert the forecasters views regarding the probabilities of future inflation outcomes into smoothed distributions. Second, the generalised beta distribution is used to convert inflation option prices into risk-neutral distributions. While the former distributions are essential in the estimation of our model, the latter are used after the estimation, when we study our model outputs.

In both cases, the spirit of the smoothing methodology, that broadly builds on Engelberg, Manski, and Williams (2009) (see also Boero et al., 2014; Clements, 2014), is the same. We consider the data associated with a specific inflation distribution, as defined by: (a) one area, (b) one measure of inflation (year-on-year growth rate of the price index, annualised growth rate over a given period or average of year-on-year growth rates), (c) one horizon and (c) a given type of probability measure (historical in the first case, risk-neutral in the second case). Then, we as-

[33] According to their paper, previous research on SPF data implies a weak correlation between disagreement and other measures of uncertainty possibly due to: the failure in using a long enough sample and the failure in fitting a normal distribution to each histogram for obtaining a robust measure of disagreement.

sume that these data are coherent with a generalised Beta distribution and look for the parameterisation of this distribution that provides the closest fit to the considered data (minimising a sum of weighted squared deviations between the data and its theoretical counterpart).

In the first case, the data consists of survey-based probabilities of future inflation outcomes falling within given ranges (see 7.4.1.1, 7.4.1.3, 7.4.2.1). These survey-based data provide us with evaluations of the cumulative distribution function (c.d.f.) of the associated distribution at the bounds of the bins. Let us stress that these smoothed distributions are fundamentally different from those resulting from the approach developed in the present paper. Indeed, the latter are coherent across time and horizons, which is not the case of the former. Heuristically, the smoothing approach presented in this appendix constitutes a preliminary processing of the data before using them in the model estimation.

In the second case, the data consists of market quotes of inflation derivatives, namely inflation floors and swaps. As explained in Subsection 7.6.3, these market quotes closely relate to the forward-neutral distribution of inflation, which is a probability measure that is equivalent to the physical one. As soon as one observes a sufficiently large number of inflation derivatives' quotes, one can estimate the generalised Beta distribution that provides the closest set of "theoretical" quotes. For each considered horizon and date, we use six market quotes to estimate the forward-neutral distribution: five prices of inflation floors (with strikes of -2%, -1%, 0%, 1% and -2%) and the inflation swap rate.

7.6.2 Generalised Beta distribution

X is distributed as a generalised Beta distribution of parameters (a,b,c,d) if $(X-c)/(d-c)$ is distributed as $B(a,b)$. In that case, we use the following notation: $X \sim \mathcal{B}(a,b,c,d)$.

If $X \sim \mathcal{B}(a,b,c,d)$, we have $\mathbb{P}(X<x) = \mathbb{P}(Y<(x-c)/(d-c))$, where Y is distributed as $B(a,b)$. Therefore, the c.d.f. of X is:

$$F(x) = \frac{Beta((x-c)/(d-c);a,b)}{B(a,b)},$$

where $Beta(x;a,b)$ is the incomplete Beta function, defined by:

$$Beta(x;a,b) := \int_0^x t^{a-1}(1-t)^{b-1}dt.$$

The distribution function of X then is:

$$f(x;a,b,c,d) := \mathbb{I}_{\{x\in[c,d]\}}\frac{1}{(d-c)B(a,b)}\left(\frac{x-c}{d-c}\right)^{a-1}\left(\frac{d-x}{d-c}\right)^{b-1}.$$

7.6.3 T-forward-neutral distribution of inflation

Denoting by f_t^{T-t} the T-forward-neutral distribution of the inflation rate $\pi_{t,T}$, the price of a zero-coupon inflation floor with an exercise rate of S and an expiry date $t+h$ is, as of date t:

$$\begin{aligned} floor_{t,h}(S) &= e^{-hr_{t,h}}\int_{-\infty}^{S}\left\{(1+x)^h-(1+S)^h\right\}f_t^h(x)dx \\ &\approx he^{-hr_{t,h}}\int_{-\infty}^{S}(S-x)f_t^h(x;a,b,c,d)dx, \end{aligned} \tag{26}$$

where $r_{t,h}$ is the risk-free interest rate between dates t and $t+h$ (known at date t).

Let us assume that the T-forward neutral distribution of $\pi_{t,t+h}$ is $\mathcal{B}(a,b,c,d)$. In that case, the price of the previous floor is approximately equal to:

$$\begin{aligned} & e^{-hr_{t,h}}h\int_c^S(S-x)\frac{1}{(d-c)B(a,b)}\left(\frac{x-c}{d-c}\right)^{a-1}\left(\frac{d-x}{d-c}\right)^{b-1}dx \\ = \; & e^{-hr_{t,h}}h(d-c)\int_0^{\frac{S-c}{d-c}}\left(\frac{S-c}{d-c}-y\right)\frac{1}{B(a,b)}y^{a-1}(1-y)^{b-1}dy \\ = \; & e^{-hr_{t,h}}h(d-c)\frac{S-c}{d-c}\int_0^{\frac{S-c}{d-c}}\frac{1}{B(a,b)}y^{a-1}(1-y)^{b-1}dy - \\ & e^{-hr_{t,h}}h(d-c)\int_0^{\frac{S-c}{d-c}}\frac{1}{B(a,b)}y^{a}(1-y)^{b-1}dy \\ = \; & \frac{e^{-hr_{t,h}}h}{B(a,b)}\times \\ & \left\{(S-c)Beta\left(\frac{S-c}{d-c};a,b\right)-(d-c)Beta\left(\frac{S-c}{d-c};a+1,b\right)\right\} \end{aligned} \tag{27}$$

Moreover, in that context, the inflation swap rate of maturity h, denoted by $s_{t,t+h}$ is such that

$$\int_{-\infty}^{+\infty}\left\{(1+x)^h-(1+S)^h\right\}f_t^h(x)dx = 0,$$

which implies that:

$$s_{t,t+h} \approx \int_c^d x f_t^h(x)dx = \frac{bc+ad}{a+b}.$$

Table 1: Summary of the U.S. and euro area survey data

Variable	Horizon	Frequency	Description	Sample	Estimation
US survey data					
Panel A: Philadelphia Fed Survey of Professional Forecasters					
GDP defl.	Current calendar year	Quarterly	Hist.	1999:Q1 - 2015:Q4	
GDP defl.	Next calendar year	Quarterly	Hist.	1999:Q1 - 2015:Q4	Est.
CPI	Five-year	Quarterly	P.e., Dis.	2005:Q3 - 2015:Q3	Est.
Panel B: Blue Chip Survey of Financial Forecasts and Economic Indicators					
CPI	Zero to five quarters ahead	Monthly	P.e., Dis.	1/1999 - 12/2015	
CPI	Six-to-ten year average	Four times a year	P.e., Dis.	3/1999 - 12/2015	Est.
Panel C: Federal Reserve Bank of New York Primary Dealer Survey					
CPI	One-year	FOMC Freq.	P.e., Dis.	1/2005 - 12/2015	
CPI	Two-year	FOMC Freq.	P.e., Dis.	1/2005 - 12/2015	
CPI	Five-year	FOMC Freq.	Hist.	3/2007 - 12/2015	Est.
CPI	Five-year five years ahead	FOMC Freq.	Hist.	3/2007 - 12/2015	Est.
Panel D: Consensus Forecasts Survey					
CPI	Current calendar year	Monthly	P.e., Dis.	1/1999 - 10/2015	Est.
CPI	Next calendar year	Monthly	P.e., Dis.	1/1999 - 10/2015	Est.
Euro area survey data					
Panel A: European Central Bank's Survey of Professional Forecasters					
HICP	One-year	Quarterly	Hist.	1/1999 - 10/2015	Est.
HICP	One-year one year ahead	Quarterly	Hist.	1/1999 - 10/2015	Est.
HICP	One-year four years ahead	Quarterly	Hist.	1/1999 - 10/2015	Est.
Panel B: Consensus Forecasts Survey					
HICP	Current calendar year	Monthly	P.e., Dis.	1/1999 - 10/2015	
HICP	Next calendar year	Monthly	P.e., Dis.	1/1999 - 10/2015	
HICP	One-year one year ahead	Semiannually	P.e.	4/2003 - 10/2015	
HICP	One-year two years ahead	Semiannually	P.e.	4/2003 - 10/2015	
HICP	One-year three years ahead	Semiannually	P.e.	4/2003 - 10/2015	
HICP	One-year four years ahead	Semiannually	P.e.	4/2003 - 10/2015	
HICP	One-year five years ahead	Semiannually	P.e.	4/2003 - 10/2015	
HICP	Five-to-ten year average	Semiannually	P.e.	4/2003 - 10/2015	Est.
Panel C: Blue Chip Economic Indicators Survey					
HICP	Current calendar year	Monthly	P.e., Dis.	12/2006 - 5/2015	
HICP	Next calendar year	Monthly	P.e., Dis.	12/2006 - 5/2015	

This table summarizes survey variables from the U.S. and euro area surveys used in the study. Source: Federal Reserve Bank of Philadelphia, Federal reserve Board, Federal Reserve Bank of New York, European Central Bank, Consensus Forecasts, and Blue Chip Economic Indicators. Samples for the surveys used in the study are based on availability. "Four times a year" frequency of the Blue Chip surveys refers to those forecasts obtained from long-range inflation forecasts from Blue Chip Financial Forecasts survey in June and December and from Blue Chip Economic Indicators in March and October. Thus, this frequency slightly deviates from quarterly frequency. CPI is the Consumer Price Index, HICP is the Harmonized Index of Consumer Prices in the euro area, P.e., Dis., Hist., and Freq. stand for point estimates, disagreement, histogram and frequency, respectively. The Est. in the last column indicates the inclusion of this variable in the basic state-space model. Note, in our estimation we only use information on point estimates and aggregate variances.

Table 2: Candidate variables for an F_t covariates' set

Variable	Starting date
Panel A: Euro area	
EA infl. swap, 1-y	2004-06-15
EA infl. swap, 2-y	2003-11-15
EA infl. swap, 5-y	2004-04-15
EA infl. swap, 10-y	2004-04-15
VIX	1998-12-15
VSTOXX	1998-12-15
EA, Cons. Fcst, 1-y, disagr.	1998-12-15
EA, Cons. Fcst, 1-y, cons.	1998-12-15
Europ. Comm. Econ. Sent.	1998-12-15
US EPU	1998-12-15
EA EPU	1998-12-15
Gold price in USD	1998-12-15
Gold price, in EUR	1998-12-15
Oil prices (WTI)	1998-12-15
Oil prices (Brent)	1998-12-15
Panel B: U.S.	
US infl. swap, 1-y	2004-07-15
US infl. swap, 2-y	2004-07-15
US infl. swap, 5-y	2004-07-15
US infl. swap, 10-y	2004-07-15
VIX	1998-12-15
US, Cons. Fcst, 1-y, disagr.	1998-12-15
US, Cons. Fcst, 1-y, cons.	1998-12-15
US Blue Chip, 4-q, cons.	1998-12-15
US Blue Chip, 4-q, disagr.	1998-12-15
US Blue Chip, 2-q, cons.	1998-12-15
US Blue Chip, 2-q, disagr.	1998-12-15
Uni. of Michigan Consum. Sent.	1998-12-15
US EPU	1998-12-15
EA EPU	1998-12-15
Gold price in USD	1998-12-15
Oil prices (WTI)	1998-12-15
Oil prices (Brent)	1998-12-15

Panel A lists the covariates that we use in the univariate regressions for the European inflation surveys data (namely, ECB's SPF survey) used in the basic state-space model. Panel B lists the covariates that we use in the univariate regressions of the U.S. inflation survey data (namely, Philadelphia Fed's SPF) in the basic state-space model. VIX is the implied volatility index on the S&P500 index, VSTOXX is the implied volatility index on the euro STOXX50 index. Economic Policy Uncertainty indices are based on Baker, Bloom, and Davis (2015). All the variables are measured at the mid-month. The outputs of these regressions are used to construct new measurement equations in the augmented state-space model (see Sections 4.2.2 and 4.2.4).

Table 3: LASSO regressions results

Survey series	1st covariate	2nd covariate	Obs.	R^2
Panel A: Euro area covariates				
EA SPF, 1-y horiz., avg.	(+) EA, Cons. Fcst, 1-y, cons.	(+) EA infl. swap, 5-y	46	0.93
EA SPF, 2-y horiz., avg.	(+) EA, Cons. Fcst, 1-y, cons.	(+) EA infl. swap, 5-y	46	0.87
EA SPF, 5-y horiz., avg.	(+) EA infl. swap, 10-y	(+) EA, Cons. Fcst, 1-y, cons.	46	0.53
EA SPF, 1-y horiz., var.	(+) EA EPU	(+) EA infl. swap, 2-y	46	0.54
EA SPF, 2-y horiz., var.	(+) EA EPU	(+) EA, Cons. Fcst, 1-y, cons.	46	0.69
EA SPF, 5-y horiz., var.	(+) EA EPU	(+) EA infl. swap, 5-y	46	0.67
Panel B: U.S. covariates				
US SPF, 5-y horiz., avg	(+) US, Cons. Fcst, 1-y, cons.	(+) US Blue Chip, 2-q, cons.	40	0.66
US PDS, 5y-in-5y, var	(+) EA EPU	(+) US Blue Chip, 2-q, cons.	65	0.55
US SPF, 4-q horiz., avg.	(+) US Blue Chip, 4-q, cons.		11	0.64
US SPF, 5-q horiz., avg.	(+) US, Cons. Fcst, 1-y, cons.		11	0.83
US SPF, 6-q horiz., avg.	(+) US Blue Chip, 2-q, cons.		10	0.87
US SPF, 7-q horiz., avg.	(+) US infl. swap, 5-y		11	0.78
US SPF, 8-q horiz., avg.	(+) US Blue Chip, 4-q, cons.		11	0.64
US SPF, 7-q horiz., var.	(+) US, Cons. Fcst, 1-y, disagr.		11	0.86

This table reports at most two covariates selected by the LASSO procedure among the set of potential regressors for each survey-based variable reported in the first column. Survey-based variables are monthly smoothed series obtained from the first round of the Kalman smoother (see Section 4.2.1). The last two columns indicate the number of observations and R^2 implied by each regression of the survey-based variables on the potential covariates. The regression coefficient sign of a covariate is indicated in parentheses. When the dependent variable has more (less) than 30 observations, we use two (one) covariate(s) in the regressions. Surveys' abbreviations are as follows: EA —euro area; SPF - Survey of Professional Forecasters; CF - Consensus Forecasts; PDS - Primary Dealer Survey; BCFF - Blue Chip Financial Forecasts. Variables' abbreviations are as follows: avg - average; var - variance; y - year; q - quarter; cons. - consensus; disagr. - disagreement, CES - consumer economic sentiment; EPU - economic policy uncertainty.

Table 4: Parameter estimates

	Adjust.	Value	St.dev.		Adjust.	Value	St.dev.
$\bar{\pi}^{(1)}$		1.796	–	$\Theta_{1,1}$	$(\times 10^3)$	0.336	*0.308*
$\bar{\pi}^{(2)}$		2.402	–	$\Theta_{2,1}$	$(\times 10^3)$	−1.060	*1.243*
				$\Theta_{3,1}$	$(\times 10^3)$	−1.342	*0.594*
$\delta_1^{(1)}$		1.000	–	$\Theta_{4,1}$	$(\times 10^3)$	−0.779	*0.437*
$\delta_2^{(1)}$		0.311	*0.060*	$\Theta_{1,2}$	$(\times 10^3)$	0.102	*0.102*
$\delta_3^{(1)}$		1.000	–	$\Theta_{2,2}$	$(\times 10^3)$	−2.665	*1.240*
$\delta_4^{(1)}$		0.147	*0.069*	$\Theta_{3,2}$	$(\times 10^3)$	−0.168	*0.347*
$\delta_1^{(2)}$		1.199	*1.186*	$\Theta_{4,2}$	$(\times 10^3)$	−0.498	*0.303*
$\delta_2^{(2)}$		1.000	–				
$\delta_3^{(2)}$		0.610	*0.135*	$\Gamma_{Y,0[1]}$	$(\times 10^3)$	2.883	*4.527*
$\delta_4^{(2)}$		1.000	–	$\Gamma_{Y,0[2]}$	$(\times 10^3)$	4.662	*58.689*
				$\Gamma_{Y,0[3]}$	$(\times 10^3)$	6.614	*12.745*
$\Phi_{Y[1,1]}$		0.982	*0.004*	$\Gamma_{Y,0[4]}$	$(\times 10^3)$	0.008	*26.715*
$\Phi_{Y[2,1]}$		0.014	*0.377*				
$\Phi_{Y[3,1]}$		0.013	*0.089*	$\Gamma_{Y,1[1,1]}$	$(\times 10^3)$	0.000	*0.041*
$\Phi_{Y[4,1]}$		−0.034	*0.120*	$\Gamma_{Y,1[2,1]}$	$(\times 10^3)$	0.000	*0.019*
$\Phi_{Y[2,2]}$		0.706	*0.045*	$\Gamma_{Y,1[3,1]}$	$(\times 10^3)$	0.491	*1.001*
$\Phi_{Y[3,2]}$		0.004	*0.030*	$\Gamma_{Y,1[4,1]}$	$(\times 10^3)$	1.310	*0.734*
$\Phi_{Y[4,2]}$		−0.092	*0.034*	$\Gamma_{Y,1[1,2]}$	$(\times 10^3)$	0.968	*0.253*
$\Phi_{Y[3,3]}$		0.894	*0.012*	$\Gamma_{Y,1[2,2]}$	$(\times 10^3)$	0.256	*0.108*
$\Phi_{Y[4,3]}$		0.005	*0.014*	$\Gamma_{Y,1[3,2]}$	$(\times 10^3)$	0.464	*0.374*
$\Phi_{Y[4,4]}$		0.928	*0.006*	$\Gamma_{Y,1[4,2]}$	$(\times 10^3)$	0.994	*0.374*
ν_1		0.100	*0.068*				
ν_2		0.331	*0.315*				
$\Phi_{z[1,1]}$		0.992	*0.002*				
$\Phi_{z[2,1]}$	$(\times 10^2)$	0.030	*0.028*				
$\Phi_{z[2,2]}$		0.983	*0.002*				

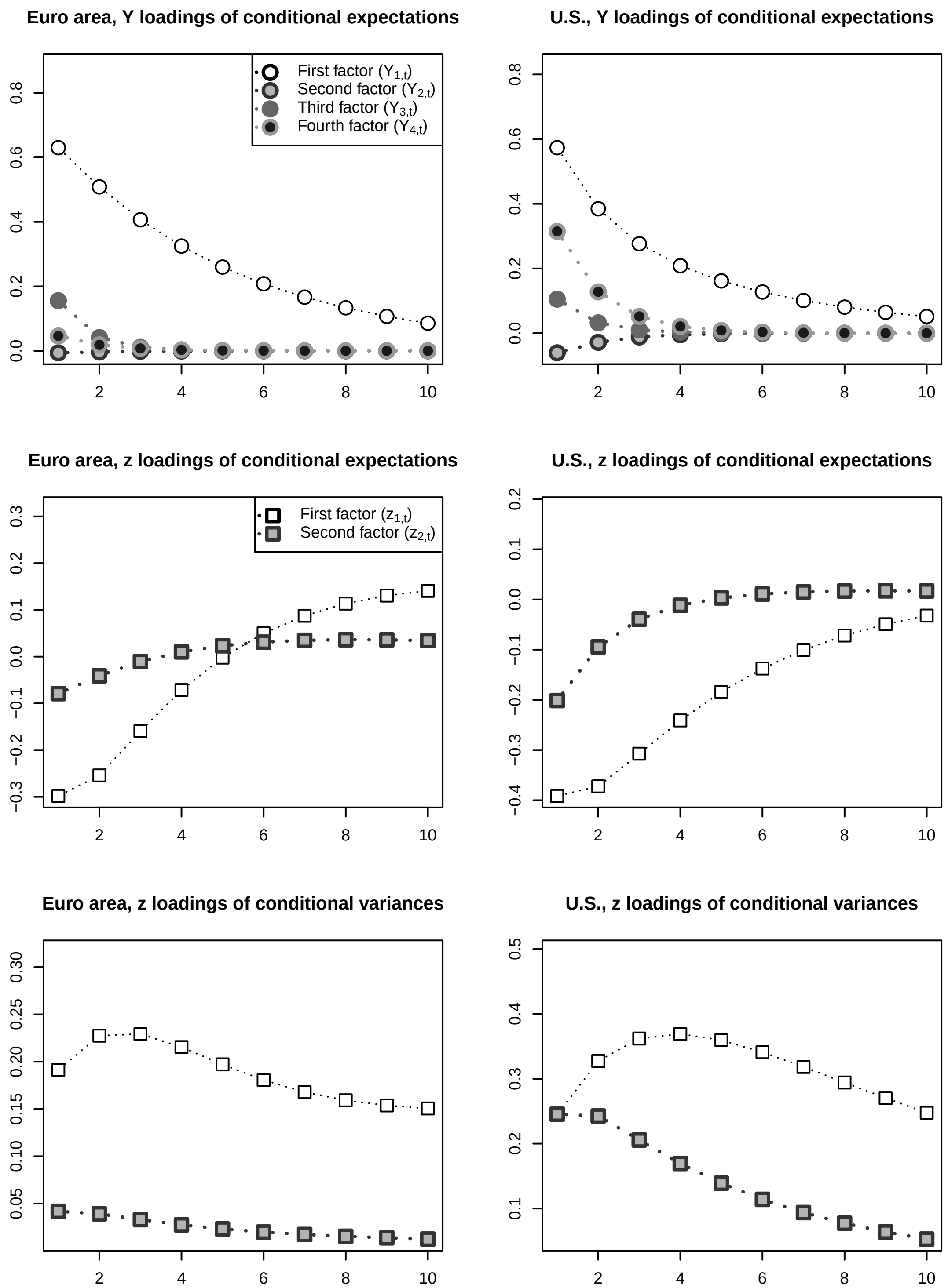
The model is estimated by maximizing the quasi-likelihood stemming from a modified Kalman filter. Standard deviations (in italics) are calculated from the outer product of the log-likelihood gradient, evaluated at the estimated parameter values. For the sake of identification, different elements of δ are set to 1. Superscripts in parentheses indicate the currency areas: 1 for the euro area and 2 for the US.

Table 5: **Euro area and U.S. inflation: deflation probabilities, comovements, and risk measures**

	VIX	*VSTOXX*	*US EPU*	*EA EPU*	*EC ES*	*UM SC*	*EA 1-y IRP*	*EA 5-y IRP*	*US 1-y IRP*	*US 5-y IRP*
Panel A: Comovements										
Joint Deflation Proba., 1-y	(+) 3	(+) 2	(+) 7	(+) 2	(−) 20*	(−) 11	(−) 9**	(−) 7	(−) 5**	(−) 9*
Joint Deflation Proba., 5-y	(−) 3	(−) 1	(+) 17**	(+) 38**	(−) 5	(−) 8	(−) 23*	(−) 55**	(−) 1	(−) 1
Inflation Covariance, 1-y	(+) 9**	(+) 9*	(+) 50**	(+) 25**	(−) 37**	(−) 54**	(−) 1	(−) 0	(−) 17**	(−) 12**
Inflation Covariance, 5-y	(−) 0	(+) 0	(+) 39**	(+) 48**	(−) 19**	(−) 30**	(−) 16*	(−) 34**	(−) 7	(−) 6
Panel B: U.S.										
U.S. Deflation proba., 1-y	(+) 18*	(+) 13	(+) 11	(+) 1	(−) 39*	(−) 21	(−) 4	(−) 0	(−) 18**	(−) 17**
U.S. Deflation proba., 5-y	(+) 0	(+) 1	(+) 37**	(+) 42**	(−) 20**	(−) 28**	(−) 15*	(−) 29**	(−) 7	(−) 7
U.S. Inflation variance, 1-y	(+) 25**	(+) 17**	(+) 10*	(−) 1	(−) 18	(−) 22**	(+) 8	(+) 32**	(−) 11**	(−) 6
U.S. Inflation variance, 5-y	(+) 9*	(+) 9*	(+) 49**	(+) 27**	(−) 38**	(−) 55**	(−) 1	(−) 0	(−) 20**	(−) 14**
Panel C: Euro area										
E.A. Deflation proba., 1-y	(+) 0	(+) 0	(+) 3	(+) 5	(−) 8	(−) 4	(−) 26*	(−) 31**	(−) 4	(−) 8
E.A. Deflation proba., 5-y	(−) 3	(−) 1	(+) 14**	(+) 36**	(−) 4	(−) 7	(−) 24*	(−) 58**	(−) 1	(−) 1
E.A. Inflation variance, 1-y	(+) 0	(+) 1	(+) 44**	(+) 47**	(−) 22**	(−) 35**	(−) 14*	(−) 27**	(−) 8	(−) 7
E.A. Inflation variance, 5-y	(−) 1	(−) 0	(+) 32**	(+) 48**	(−) 13*	(−) 21**	(−) 20*	(−) 44**	(−) 4	(−) 3

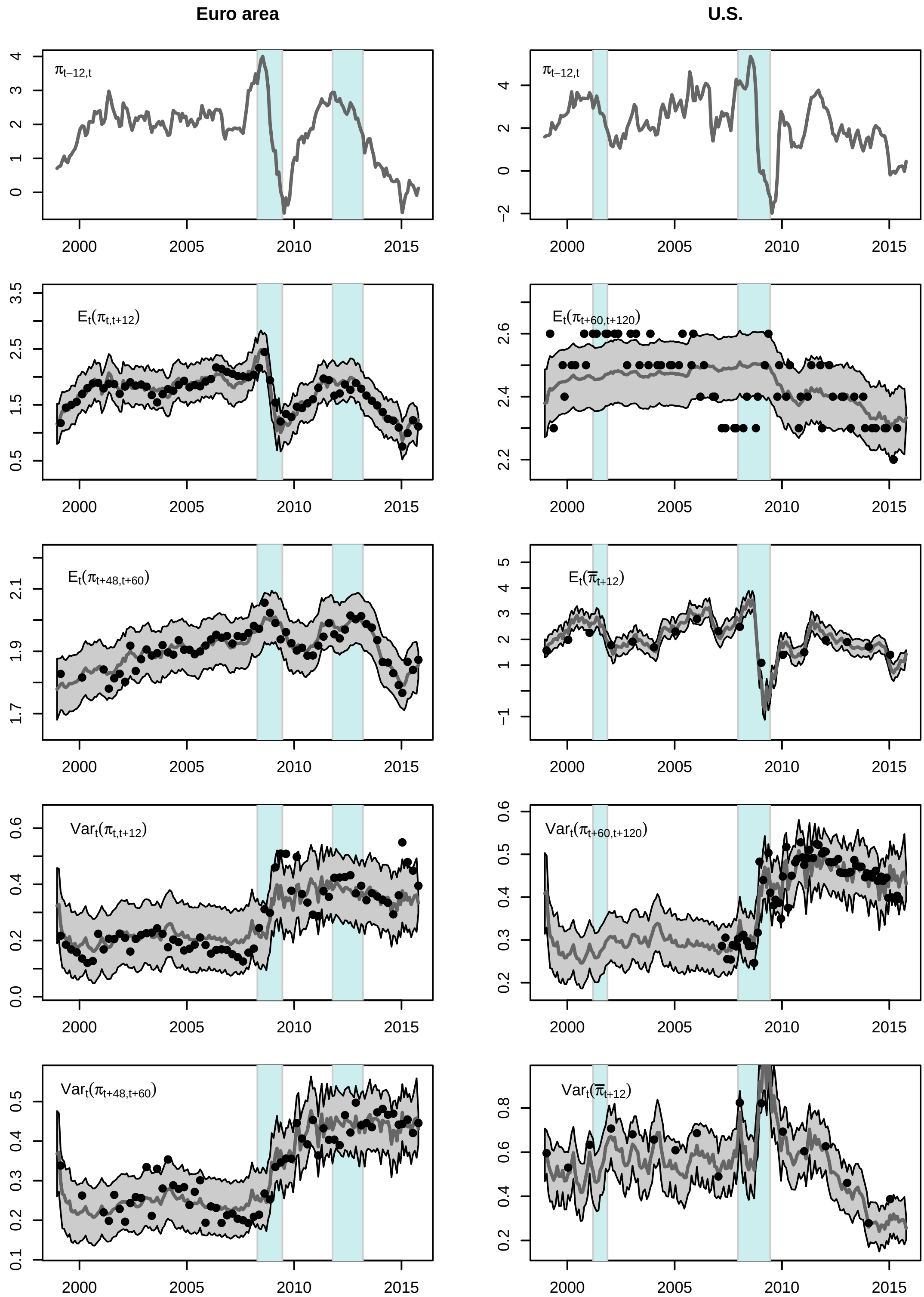
This table reports synthetic results of bivariate regressions of the variables appearing on the first column of the table on those reported in the first row. The sign of the slope is in parentheses. Reported figures are regression R^2s, expressed in percentage points. *, **, and *** denote statistical significance of the slope coefficient at the 10%, 5%, and 1% level, respectively. Standard errors are HAC Newey-West (12 lag) corrected. The number of observations is 203. Variables' abbreviations are as follows: Defl. - deflation; y - year; EC ES - European commission economic sentiment index; UM CS - University of Michigan survey of consumers; EPU - economic policy uncertainty; IRP - inflation risk premium.

Figure 1: Factor loadings of expectations and variances of future inflation rates



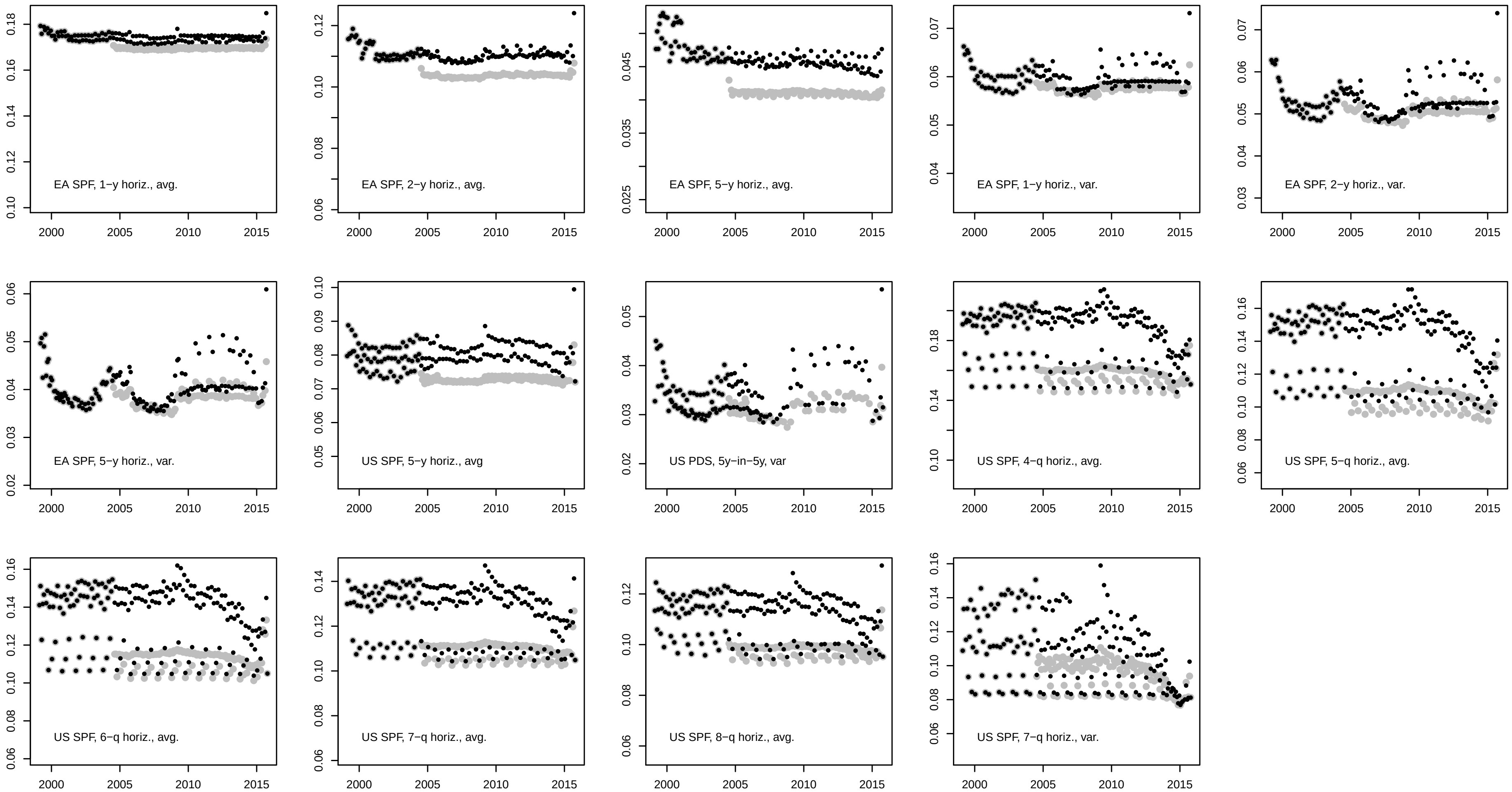
This figure displays, for different horizons h, the entries of vectors $b_h^{(i)}$ and $\beta_h^{(i)}$ appearing in eqs. (9) and (10). In order to facilitate interpretation, these loadings have been multiplied by the marginal standard deviations of the associated factors. That is, the y-coordinates correspond to the effect of a one-standard deviation change in the factors on the conditional level of inflation expectations (or variances for the bottom charts). $x-$axis are measured in years.

Figure 2: Fit of inflation and survey data



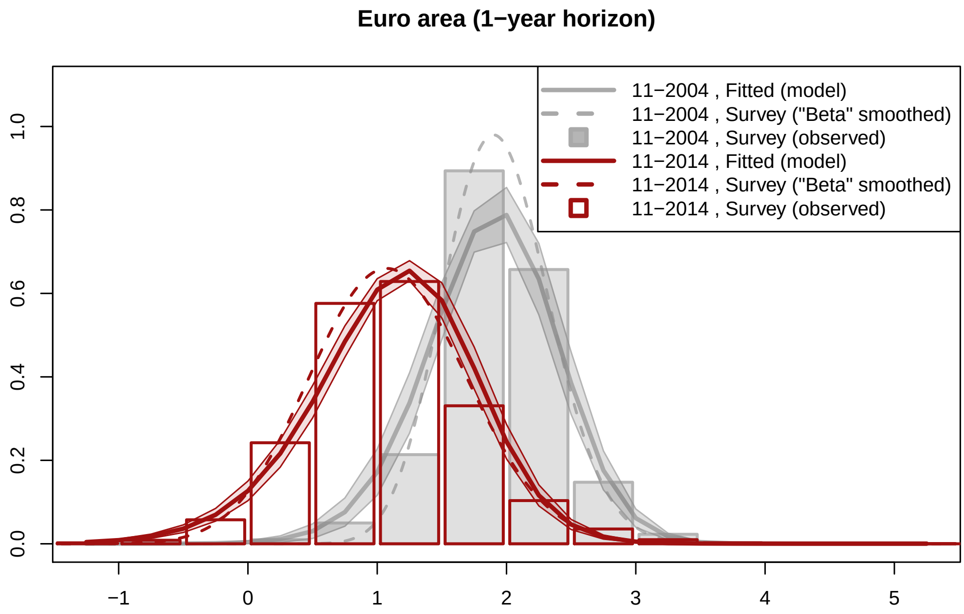
This figure illustrates the fitting properties of the model. The dots correspond to the observed surveys. The grey-shaded areas are the 2-standard-deviation confidence intervals. For the sake of readability, this figure does not show the fit of all observed surveys. For the U.S., the notation $\bar{\pi}_{t+h}$ refers to the measure of inflation used in the Philadelphia Fed Survey of Professional Forecasters, which is the annual-average over annual-average percent change in prices (see appendix 7.4.1).

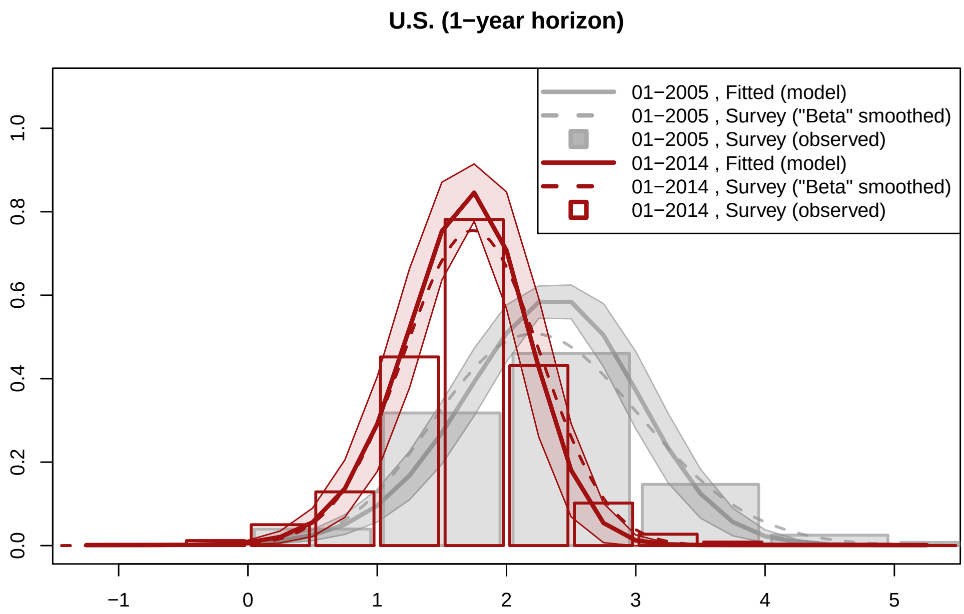
Figure 3: Standard deviations associated to *pseudo surveys*, with (grey) and without (black) additional covariates



On these plots, each dot corresponds to a date on which the survey considered by the given chart is not available. Consider the j^{th} plot. $\widetilde{S}_{j,t}$ denotes the survey that would prevail if the survey on which is based the j^{th} component of S_t had been conducted that month. Applications of the Kalman filter on the estimated state-space models yield estimates of $\widetilde{S}_{j,t}$. In the basic state-space model, one could for instance obtain $\mathbb{E}(\widetilde{S}_{j,t}|\underline{S_t})$, where $\underline{S_t}$ contains the inflation rates and the surveys that have been released up to date t. The black dots report the standard deviation associated to the difference between this estimate (see eq. (17)) and $\widetilde{S}_{j,t}$. The grey dots correspond to the standard deviations of $\widetilde{S}_{j,t} - \mathbb{E}(\widetilde{S}_{j,t}|\underline{S_t^a})$ where $\underline{S_t^a}$ is a wider information set including, in addition to $\underline{S_t}$, the current and past observations of additional variables F_t that are correlated to the latent factors (see Section 4.2.4).

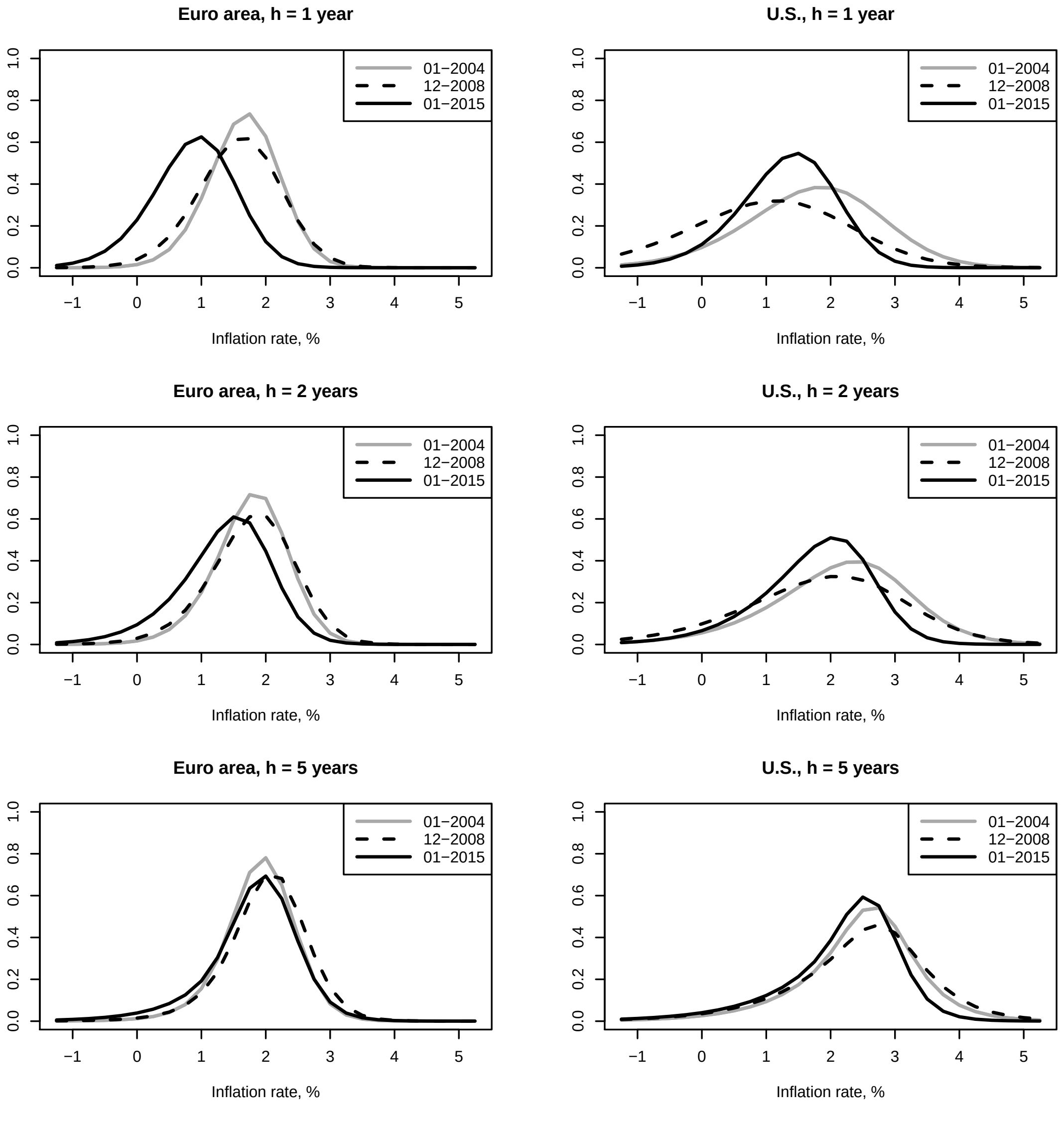
Figure 4: Fit of survey distributions





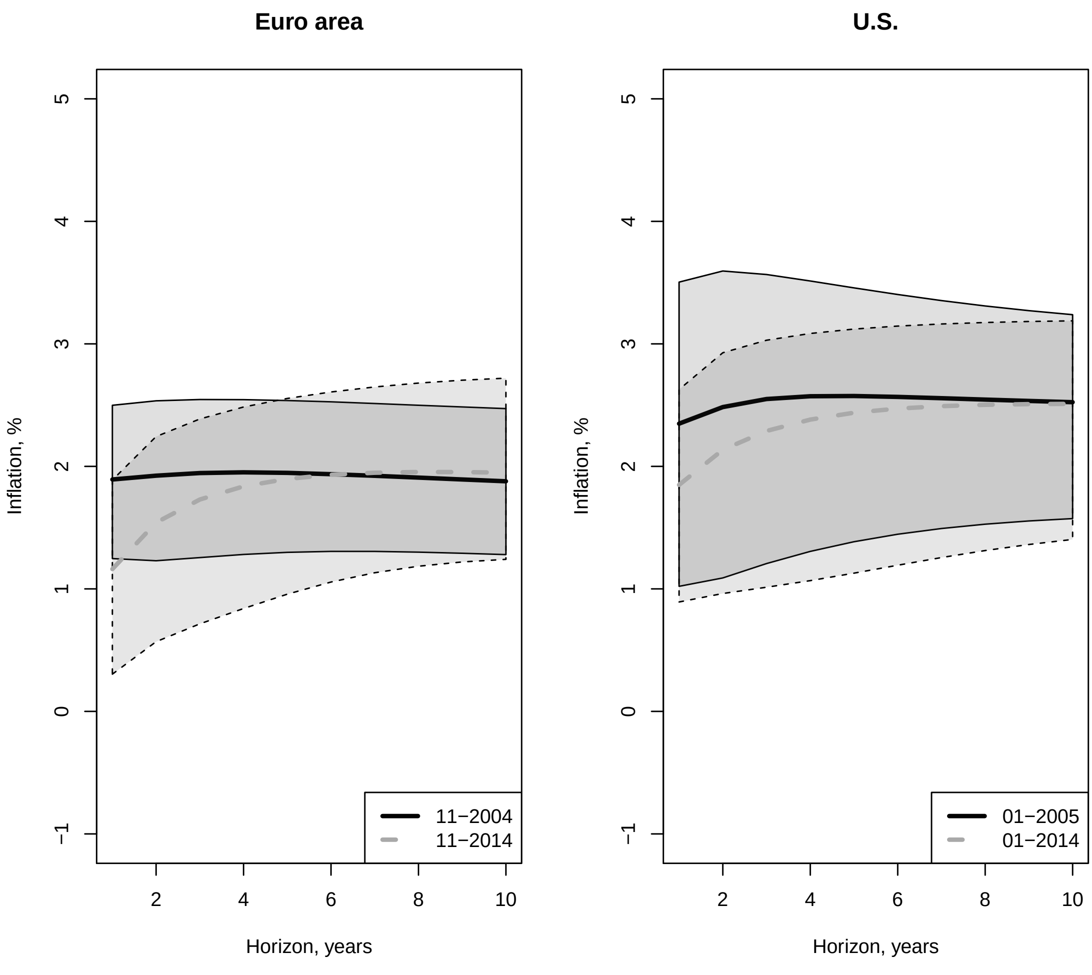
This figure compares the one-year ahead survey-based histograms to the one-year ahead model-implied distributions. For the model-implied distributions, two-standard-deviation confidence intervals are reported. These standard deviations reflect the uncertainty associated to the estimation of the latent factors X_t. These standard deviations are obtained by applying the delta method on the function relating factors X_t to the conditional cumulative distribution function (c.d.f.) of future inflation. (The covariance matrix of the filtered values of X_t stems from the Kalman filter; appendix 7.3 details the computation of the c.d.f. of future inflation rates.)

Figure 5: Model-implied conditional distributions of future inflation



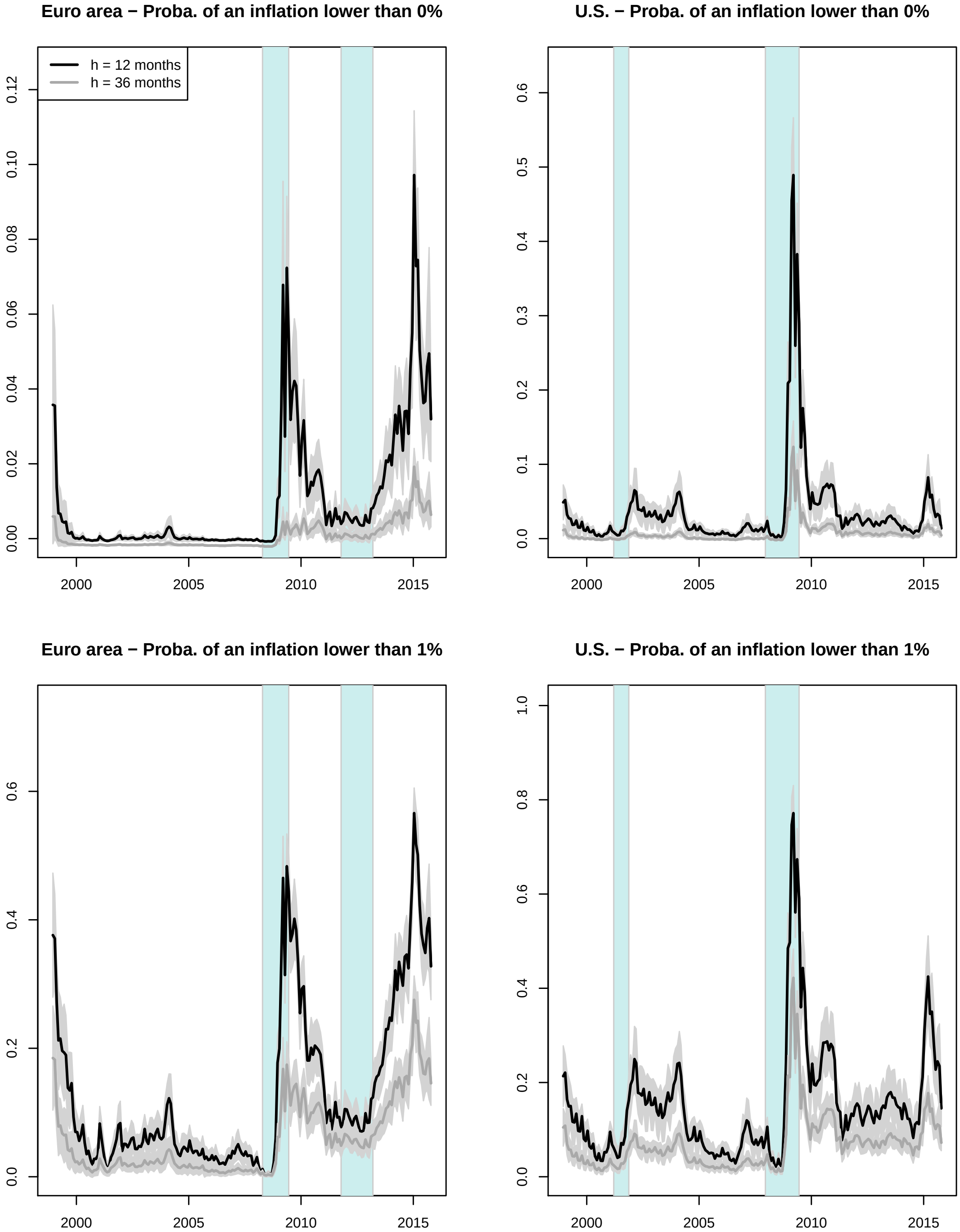
This figure displays model-implied distributions of future year-on-year inflation rates (i.e. $\pi^{(i)}_{t+h-12,t+h}$ for each area i) conditional on current and past (filtered) values of X_t. These distributions can be seen as estimates of the distributions that would have been revealed in surveys that would have taken place on these selected dates.

Figure 6: Term structure of expected future inflation



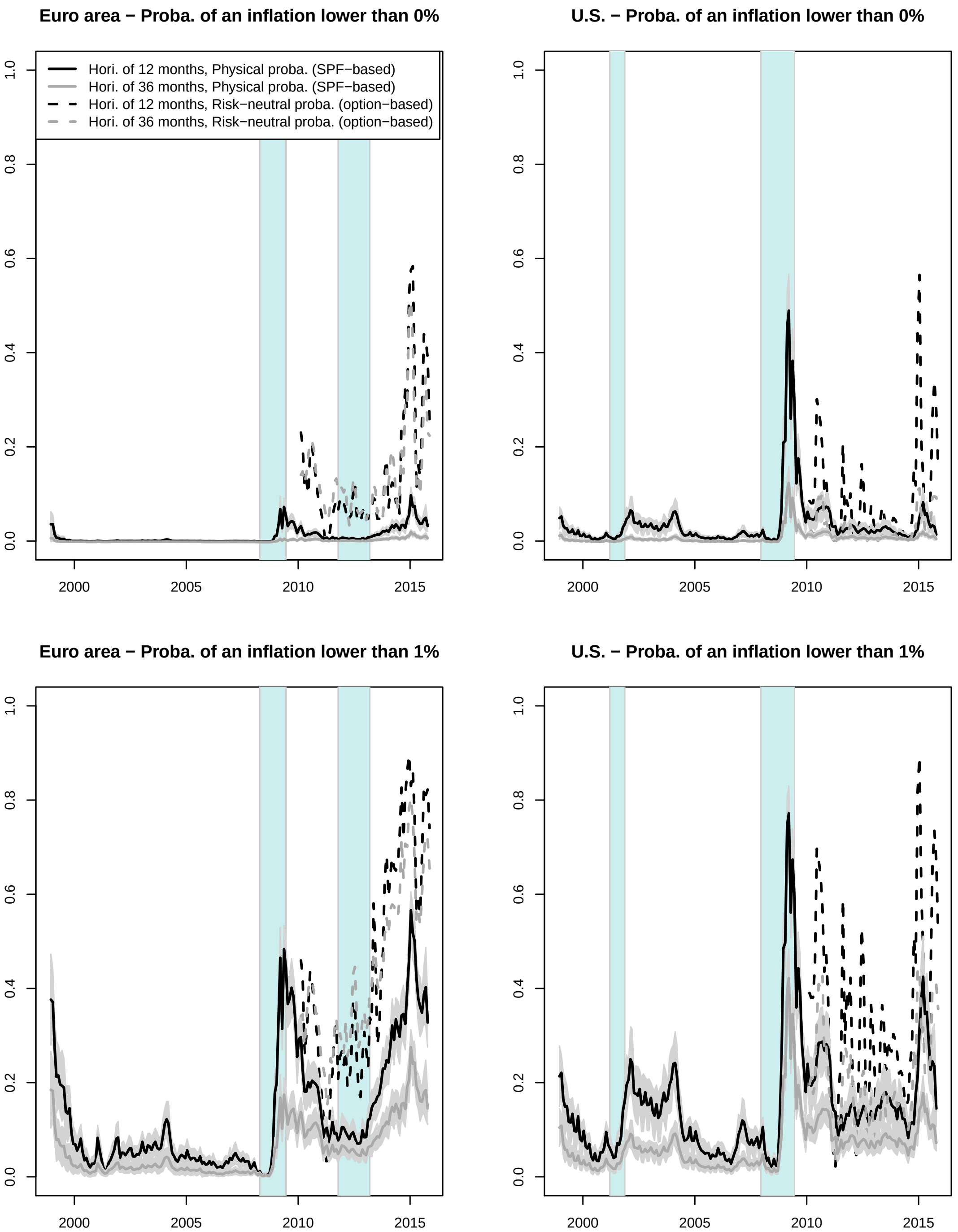
This figure displays the term structure of model-implied expected inflation rates along with the 5th and 95th quantiles associated with the respective conditional distributions. The quantiles are derived from closed-form formulas given in Appendix 7.3.

Figure 7: Model-implied probabilities of low inflation



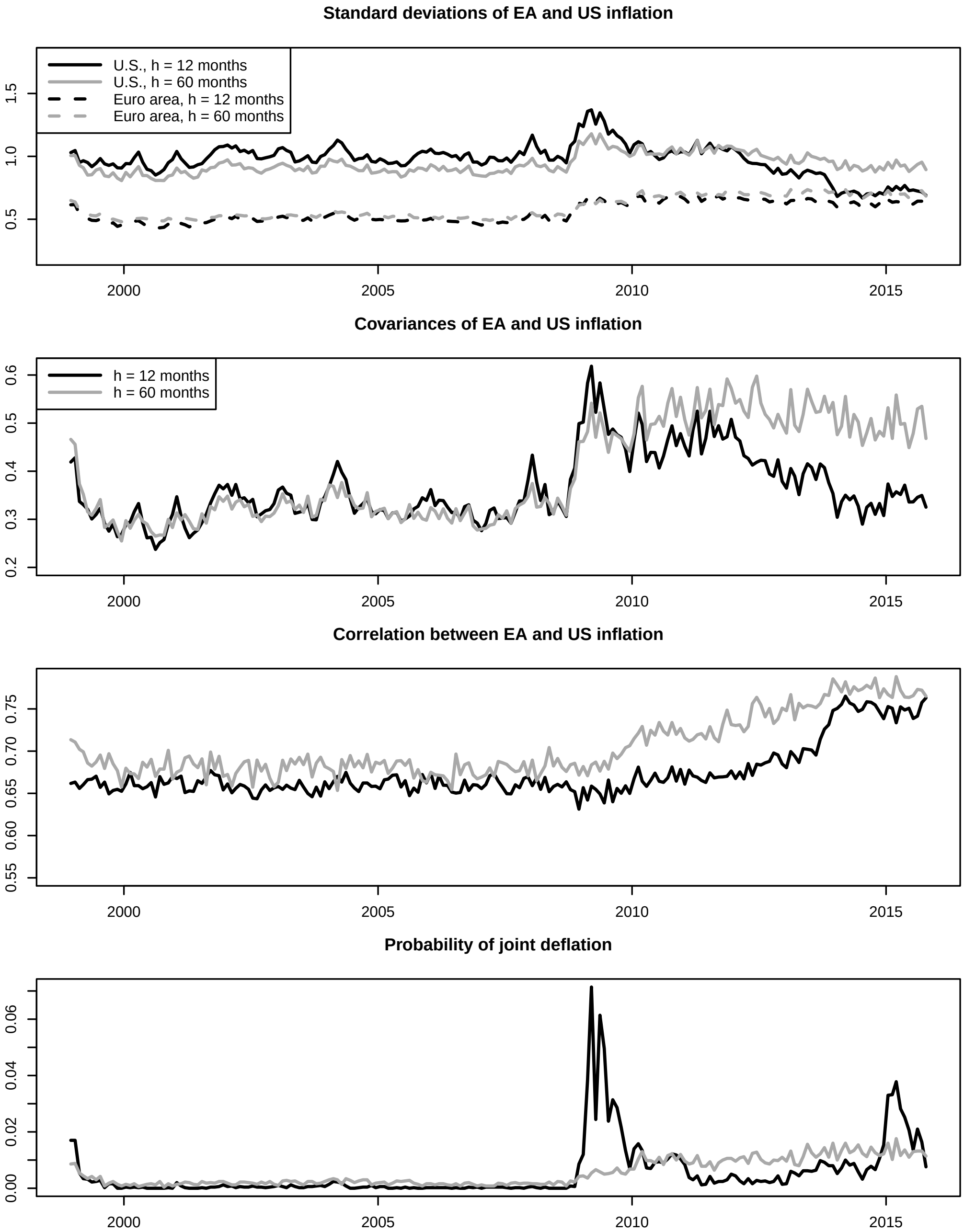
This figure displays model-implied probabilities of very-low future inflation rates. The grey shaded areas are two-standard-deviation confidence intervals. These standard deviations are obtained by applying the delta method on the function relating factors X_t to the conditional cumulative distribution function (c.d.f.) of future inflation.

Figure 8: Option-based and model-implied probabilities of low inflation



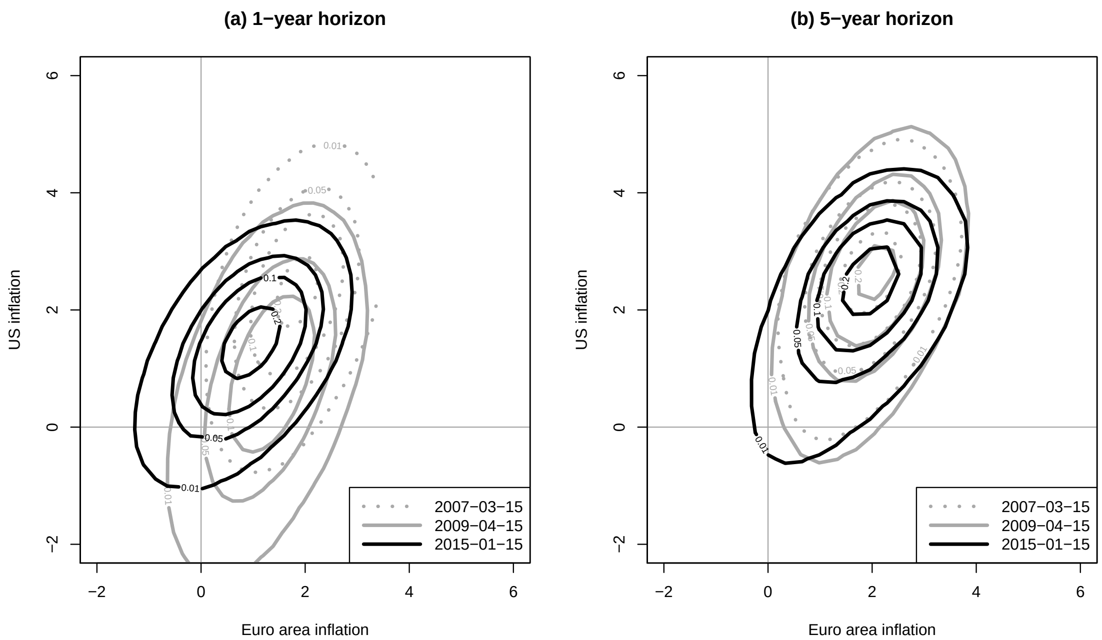
This figure compares model-implied (physical) probabilities of very low future inflation rates to their risk-neutral counterparts. The risk-neutral probabilities are based on inflation derivatives, namely zero-coupon inflation swaps and inflation floors.

Figure 9: Expected joint movements of euro area and U.S. inflation rates



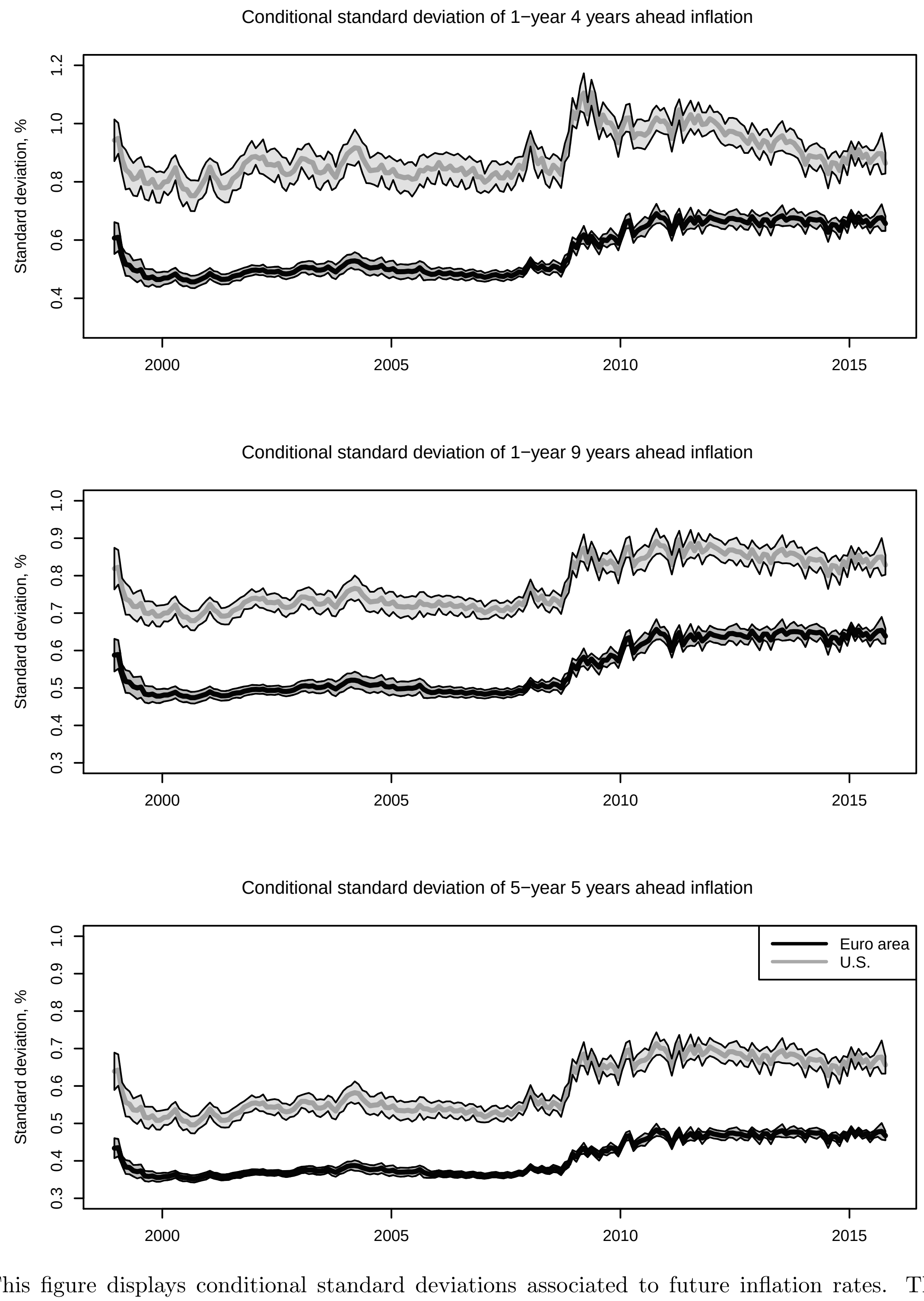
Panel (a), (b) and (c) respectively display the U.S. and euro area standard deviations, conditional covariances and correlations of future inflation rates for two distinct horizons. Panel (d) shows the joint probabilities of deflation, i.e. $\mathbb{P}(\pi_{t+h}^{(E.A.)} \leq 0, \pi_{t+h}^{(U.S.)} \leq 0 | \underline{S_t^a})$.

Figure 10: Joint conditional distributions of inflation



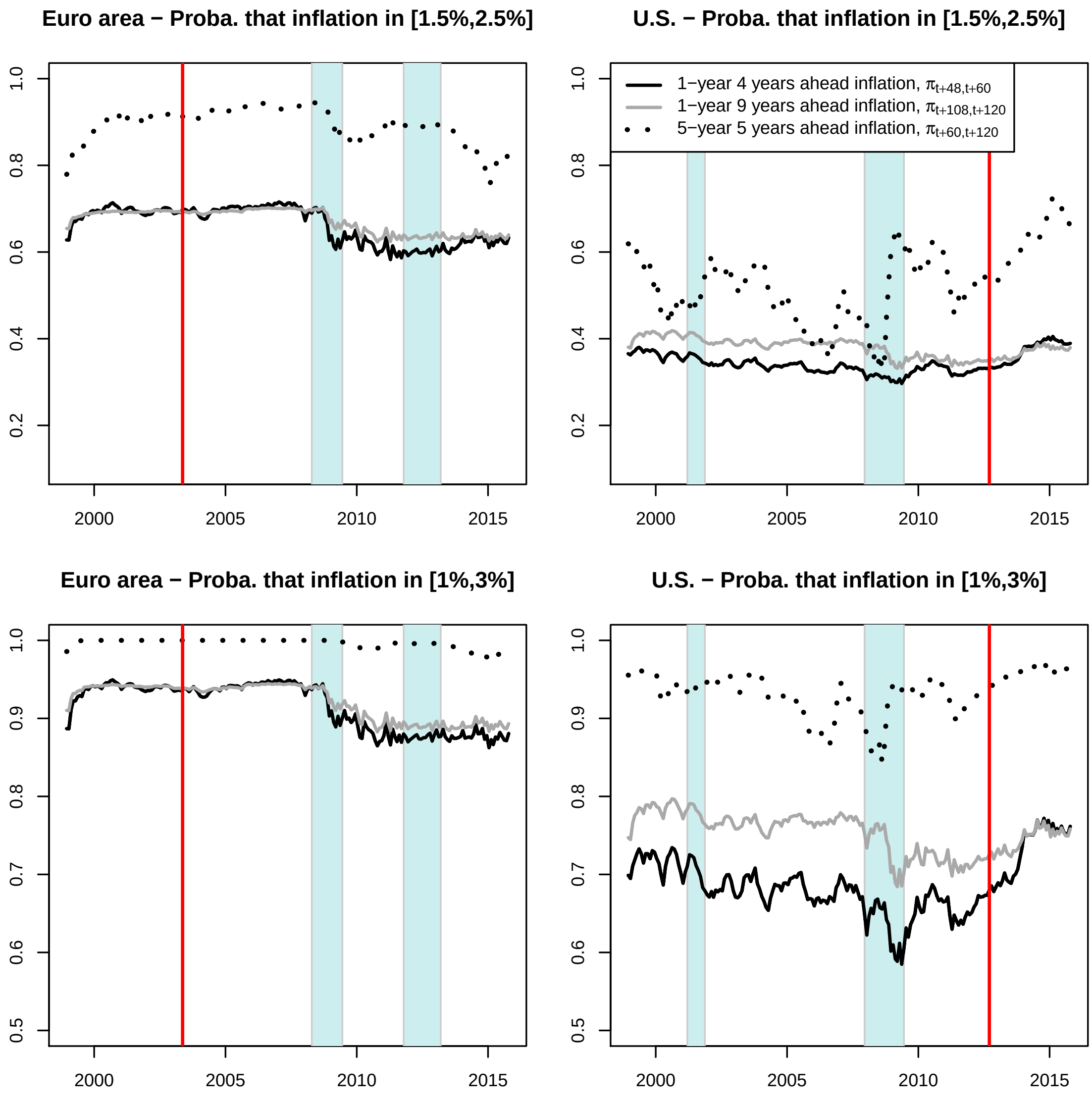
This figure displays the contour plots of the bivariate conditional distribution of future inflation rates in the euro area and in the U.S.. These distributions have been obtained by 10,000 simulations of the model, using smoothed estimates of X_t (for the considered dates) as initial conditions.

Figure 11



This figure displays conditional standard deviations associated to future inflation rates. These standard deviations can be seen as measures of inflation uncertainty. Specifically, the panel (a) plots $\sqrt{\mathbb{V}ar(\pi^{(i)}_{t+48,t+60}|\underline{X_t})}$, panel (b) plots $\sqrt{\mathbb{V}ar(\pi^{(i)}_{t+108,t+120}|\underline{X_t})}$, and panel (c) plots $\sqrt{\mathbb{V}ar(\pi^{(i)}_{t+60,t+120}|\underline{X_t})}$.

Figure 12: Measuring the anchoring of inflation expectations



This figure displays probabilities that future inflation rates will fall in the two intervals: $I_1 = [1.5\%, 2.5\%]$ (upper plots) and $I_2 = [1\%, 3\%]$ (lower plots). Formally, for an interval I_j, $j \in \{1, 2\}$, they show the time series of the conditional probabilities $\mathbb{P}(\pi^{(i)}_{t+h-12,t+h} \in I_j|\underline{X_t})$. On each plot, two horizons are considered: $h = 60$ months and $h = 120$ months. The red vertical bars indicate the months when the ECB governing council and the FOMC announced their medium-run inflation objectives of 2%, in May 2003 and September 2012, respectively.