Central Counterparty Capitalization and Misaligned Incentives [1]

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Abstract

Central Counterparties (CCPs) are systemic nodes in financial markets. Incentive regulation on CCPs becomes crucial for financial stability. I investigate incentives and optimal regulation of a profit-driven CCP with limited liability. Conditional on available capital, the CCP fine-tunes collateral requirements to balance fee incomes against counterparty risk. High collateral reduces potential default losses, but leads to foregone profitable trades. Limited liability creates a wedge between the CCP's collateral policy and the socially optimal solution to this trade-off. However, regulators can use capital requirements to close the wedge, unless clearing fee exceeds a threshold.

Keywords: Central Counterparties (CCPs), Capital Requirement, Collateral
JEL Codes: G01, G12, G21, G22

1 Introduction

After the 2007-2008 global financial crisis, central clearing is introduced for OTC derivatives to enhance financial stability. Central Counterparties (CCPs) stand between buyers and sellers, effectively providing insurance against counterparty risk (Singh, 2010). CCPs have become systemic nodes in financial markets and CCP insolvency could be a catastrophe(see, e.g., Singh, 2014; Duffie, 2015; Wendt, 2015).

CCPs have high priority on regulation agenda. The Committee on Payments and Market Infrastructure (CPMI), the International Organization of Securities Commissions (IOSCO), the Financial Stability Board (FSB), and the Basel Committee on Banking Supervision (BCBS) have agreed on a joint work plan to improve CCP resilience. As the international standard for CCPs, the Principles for Financial Market Infrastructure (PFMI) requires CCPs to "*provide a viable capital plan for maintaining an appropriate level of equity*" (CPMI-IOSCO, 2012). The CPMI-IOSCO and the FSB further stress the importance for CCPs to have sufficient capital to "*absorb losses resulting from a participant default and the custody and investment of participant assets*" (see, e.g., CPMI-IOSCO, 2014; FSB, 2014; CPMI-IOSCO, 2016; FSB, 2016).

Although CCPs are systemically important, many CCPs operate as profit-driven public companies, such as CME in the U.S. and Eurex in Europe. The potential conflict of interest between CCPs' systemic role and profit-driven character sparks the public debate over the capitalization problem of CCPs: Do CCPs have enough skin-in-the-game (SITG) to align proper incentives? The fact that CCPs are not infallible is highlighted by the failure of Korean exchange clearinghouse (KRX) in 2014.[1] Clearing members who are exposed to CCP risk call for more CCP SITG to safeguard financial stability, arguing that CCPs are not properly incentivized to manage risk (see, e.g., Albuquerque, Perkins, and Rafi, 2016).

This paper investigates the capitalization problem and the potential misaligned incentives of CCPs. I consider a static partial equilibrium model with two dates ($t = 0, 1$), a mass-one continuum

[1] There are more clearinghouse failures in recent decades: the French Caisse de Liquidation (1973), the Kuala Lumpur Commodities Clearing House (1983), the Hong Kong Futures Exchange (1987), and the New Zealand Futures and Options Exchange (1989).(see, e.g., Hills et al., 1999; Buding, Cox, and Murphy, 2016)

of risk-averse protection buyers, a mass-one continuum of heterogeneous risk-neutral protection sellers and a risk-neutral CCP. Buyers and sellers are randomly matched and trade a standardized protection contract.[2] The CCP determines a "universal" collateral requirement for the sellers who may have strategic defaults. Furthermore, the sellers need to contribute to the default fund. A seller's contribution to the default fund is proportional to his collateral. In case of some sellers' defaults, the losses will first be covered by the collateralized financial resources, then by the CCP's SITG, and finally by the default fund contributed by other sellers that do not default. Without regulations on capital requirement, the CCP can choose the size of her capital, which is exposed to potential default losses. For my model, I use SITG and capital interchangeably. When the CCP exhausts all available financial resources, the CCP becomes insolvent.[3]

The main frictions in the model are profit maximization and limited liability of a CCP. To maximize profit, the CCP is inclined to lower collateral requirement when there is little SITG.[4] Such misaligned incentives give rise to a trade-off between the increase of systemic risk and the increase of trading volume.

The trading environment in the model is similar to the settings of Biais, Heider, and Hoerova (2016). But there are several key features that distinguish my paper from theirs. First, I focus on CCP's incentives and model CCP insolvency explicitly, while they stress the traders' incentives and model the interaction between central clearing and risk-taking behaviors. Second, in order to focus on CCP's misaligned incentives, I assume that sellers exert effort and there is no moral hazard between buyers and sellers. This is a simplification of their setup. However, I do model the heterogeneity of protection sellers by introducing heterogeneous hedging capability as Perez Saiz,

[2] In reality, such a contract can be viewed as a Credit Default Swap (CDS).

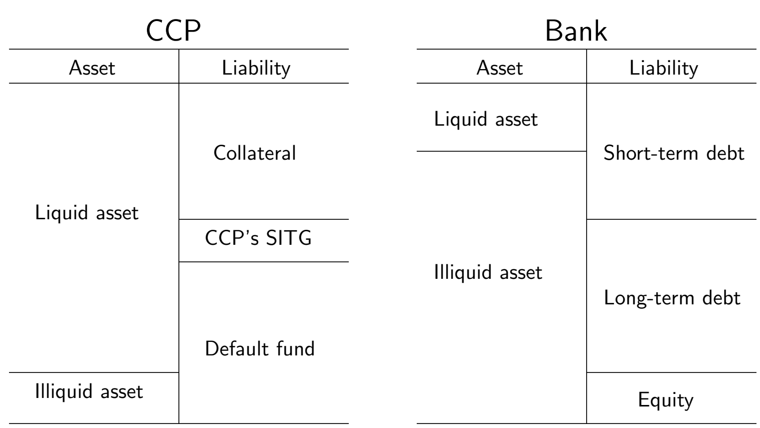
[3] Normally, CCPs have recovery plan when their pre-funded financial resources drain out. It includes Variation Margin Gains Haircut (VMGH), cash call, and other assessment power. Strictly speaking, a CCP does not become insolvent when she exhausts all available pre-funded financial resources mentioned in the text. But since the analysis of losses allocation at the end of the default waterfall is not the focus of the current model, I simplify the real operations and assume the CCP becomes insolvent as long as the available financial resources are exhausted.

[4] In the model, I assume that a CCP has full power in determining the risk management requirement. But in real operations, the change of risk models normally need to be stress tested and to be approved by regulators, which is different from the model setup. However, as long as there is asymmetric information between regulators and the CCP, there is room for the CCP to manipulate the risk management requirement. In particular, when the CCP introduces clearing service for new products, it might be tricky to assess the riskiness of the new products. Furthermore, although setting a high collateral helps safeguarding CCP resilience, high collateral requirement might impair market efficiency by making trades too expensive.

Fontaine, and Slive (2013) do. The benefit of doing so is to have a clean threshold between sellers who default (default sellers) and those who do not default (non-default sellers) in equilibrium. It enables me to study cases when different layers of the default waterfall are used to cover the default losses.

To the best of my knowledge, this paper is the first in the literature that models CCP insolvency from the perspective of CCP's misaligned incentives. There is a large banking literature that studies capital requirement (see, e.g., Dewatripont, Tirole et al., 1994; Hellmann, Murdock, and Stiglitz, 2000). But CCPs are different from banks in several ways (see, e.g., Manning and Hughes, 2016). Figure 1 shows simplified balance sheets for a CCP and a Bank. One key factor is that CCPs have an additional layer for loss-absorbing: default fund contributed by clearing members. To build the connection between a bank and a CCP, one can view clearing members' collateral as *debt* and a CCP's SITG as *equity*. Apart from debt- and equity-type liabilities on the balance sheet, a CCP also has default fund which is a type of mutualized financial resources. Since clearing members contribute to default fund, in the case of large default losses, the default fund contributed by non-default sellers will be used to cover the losses from default sellers. When the CCP becomes insolvent, the counterparties of the default sellers will bear the remaining losses. Hence, the financial resources of a CCP create different types of incentives.

Figure 1: Simplified balance sheets



There are several results from the model. First, when there is no capital requirement for CCPs, a profit-driven CCP chooses the minimum capital, whereas a benevolent CCP will favor high capital when capital cost is low. A profit-driven CCP here refers to a CCP that maximizes her own value and has limited liability. On the contrary, a benevolent CCP is a CCP that maximizes total welfare surplus, including the utility improvement of traders and the CCP value, and will not default.[5] These two types of CCPs reflect the reality of non-user-owned CCPs and user-owned CCPs (see, e.g., Cox and Steigerwald, 2016). My model suggests different capital regulations for different types of CCPs.

Second, without capital regulation, the low SITG chosen by a profit-driven CCP leads to insolvency problem. When the CCP only exposes little amount of her own capital to potential default losses, the CCP has strong incentive to lower risk management standard to attract higher trading volume. The traders are aware of the riskiness of CCP insolvency and will become reluctant to trade, which requires the CCP to further lower the collateral requirement to increase trading volume. Thus, the default losses in this case will be even larger than those in the case of high collateral requirement.

Third, a higher SITG gives rise to a higher collateral requirement monotonically. In other words, the higher the CCP capital is, the higher collateral cost will be paid by the traders. This is the argument used by CCPs against high SITG (see, e.g., LCH, 2014). However, the welfare effects of increasing SITG depend on the situation. On one hand, a higher SITG makes trading more expensive because of the higher collateral cost. On the other hand, a higher SITG also makes trading less expensive by providing a safer CCP.

Fourth, the optimal capital requirement for a profit-driven CCP depends on the profitability of the volume-based fee charged by the CCP. Although the commission fee of clearing is normally not a policy instrument for regulators, it could be an indicative policy variable for regulators. When the fee level is high, the "temptation" for a profit-driven CCP to increase trading volume is high. Even with a high capital requirement, the CCP will still go for a relatively low collateral level to increase trading volume. There will be defaults in equilibrium anyway. Hence, the optimal capital requirement for a profit-driven CCP should retain market discipline. In other words, it is

[5] Since the focus of the paper is the misaligned incentives of profit-driven CCPs, without mentioning specifically, the CCP in the paper is a profit-driven CCP.

not optimal to use CCP's SITG to absorb all the default losses in this case. It is better to allocate the default losses between CCP's SITG and clearing members' default fund contribution. To some extent, CCP's capital is more of an incentive instrument than loss-absorbing capacity. When the fee level is low, the CCP is less incentivized to increase trading volume. In this case, the optimal capital requirement for a profit-driven CCP should be high enough so that there is no default in equilibrium.

Last but not least, my model also captures the impact of default fund breach and partial insurance on trading volume. Default fund breach refers to the case that the default fund contributed by traders who do not default is used to cover the losses resulted from other traders' defaults. When traders observe low collateral and low capital, they can "foresee" potential default fund breach, which in turn will disincentivize them to trade *ex ante*. Hence, the CCP needs to further decrease collateral requirement to increase trading volume. Similarly, when traders observe even lower collateral and lower capital, they can expect that they will not be fully insured when the CCP becomes insolvent. The losses from partial insurance will also lower their utility improvement from trading, which means the CCP will lower collateral requirement to maximize trading volume. It is worthwhile to point out that, in the literature, default fund breach and partial insurance will reduce trading volume (see, e.g., Murphy, 2016; Raykov et al., 2016). But in my model, since the CCP can choose the collateral level, default fund breach and partial insurance do not necessarily reduce trading volume. Instead, the CCP can still maximize trading volume at the cost of larger potential default losses.

My paper contributes to a rapidly growing literature on CCPs. Carter and Garner (2016) discuss CCP's role in risk management and the incentives created by CCPs' SITG. They point out that the size of a CCP's SITG should be substantial for the CCP but not necessarily proportional to the default fund size, since the CCP has very different risk profile from the clearing members. Their view on SITG is more of an incentive scheme than loss absorbing capability. Albuquerque, Perkins, and Rafi (2016) propose a risk-based quantitative method to calculate CCPs' SITG, based on expected shortfall over and above the largest two clearing members' initial margin and default fund contributions. Their calibration shows that a long-term average SITG is 8.1% for an interest rate derivative CCP and 11.4% for a credit derivative CCP. Murphy (2016) analyzes the composition of CCPs' financial resources and finds that profit-driven CCPs will always minimize their SITG. But since clearing members may trade less in the case of large probability of default fund breach, CCPs have incentives to increase initial margin and to reduce default fund contribution, with the assumption that the total financial resources need to satisfy the "Cover 2" principle. I take a different approach and model CCPs' insolvency explicitly. My model suggests different capital requirements for CCPs with different ownership structures. In particular, my model takes into account the profitability of clearing industry, which alters the optimal capital requirement for profit-driven CCPs.

My model is also related to the branch of literature that studies CCPs' total financial resources. Elliott (2013) proposes principles to guide the design of loss allocation rules for CCPs. Loss allocation rules that are intended to maintain the continuity of clearing services should incentivize traders to participate competitively and should not compromise the CCP's risk management of open positions. Cumming and Noss (2013) use daily data on a CCP's member exposures to assess the CCP's total default resources and to quantify the trade-off that occurs in the balance of resources between initial margin and default funds. Singh (2014) argues that CCPs have become "too important to fail" and suggests Variation Margin Gains Haircut (VMGH) as a loss-sharing tool in case of insolvent CCP. Murphy and Nahai-Williamson (2014) model the likelihood of CCP failure and study how the distribution of risk among clearing members affects the prudence of the cover 2 standard. Their findings suggest that CCPs meeting the cover 2 standard are not highly risky provided that tail risks are not distributed too uniformly amongst CCP members. Ghamami (2014) proposes a risk-sensitive measurement for the default waterfall of derivative CCPs based on the static copula threshold portfolio credit risk approach. Duffie (2015) has a thorough review on possible recovery and resolution plans for insolvent CCPs. Raykov et al. (2016) models a CCP as a long-term break even institute and studies two loss-sharing tools for CCPs: VMGH and cash call. The model suggests that VMGH leads to a trade-off between *ex ante* trading volume and *ex post* loss-sharing, while cash call gives rise to non-performance risk that clearing members may default on the cash calls. These are critical features of central clearing, but they all assume CCPs are benevolent organizations and overlook the misaligned incentives of profit-driven CCPs. My model shows that a profit-driven CCP with thin SITG has strong incentives to lower risk management standard. In other words, CCP failure could be an endogenous risk instead of an exogenous risk.

Another relevant stand of literature is about systemic risk in central clearing. Amini, Filipovic, and Minca (2014) study the impact of central clearing on systemic risk in financial network. Their model suggests, although CCPs mitigate aggregate bank liquidation risk, they introduce tail risk to the financial system. Menkveld (2016b) models CCP systemic risk from the perspective of crowded trades. Large default losses from speculative traders that have (same) directional positions may lead to CCP failure, which constitutes a systemic risk for financial system. Menkveld (2016a) investigates CCP exposure as a Value-at-Risk (VaR) measure of a CCP's aggregate loss exposure to her clearing members, accounting for the crowdedness of trades by incorporating the correlations between portfolio returns. Cruz Lopez et al. (2016) propose a quantitative risk measure for CCP risk management based on CoVaR (Adrian and Brunnermeier, 2011). Their measure takes into account the bilateral interdependence between traders' portfolio return.

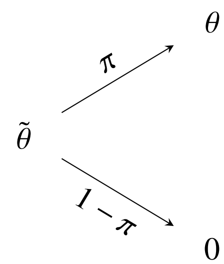
The remainder of this paper is as follows. Section 2 introduces the model primitives and discusses the first best allocation. Section 3 studies the case of a benevolent CCP that maximizes the total social welfare surplus. Section 4 focuses on the misaligned incentives of a profit-driven CCP. Section 5 analyses the optimal capital requirement of a profit-driven CCP. Section 6 concludes.

2 Model and first best allocation

2.1 Model primitives

I consider a static model with two dates ($t = 0, 1$), a mass-one continuum of protection buyers, a mass-one continuum of protection sellers and a CCP. At $t = 0$, the CCP, the buyers and sellers design and participate in the trading and clearing system. The trading environment is a modified version of Biais, Heider, and Hoerova (2016). But the focus of my model is CCP's misaligned incentives instead of protection sellers' risk-taking incentives. At $t = 1$, payoffs of risky assets are realized and some traders may default, which could potentially wind down the CCP.

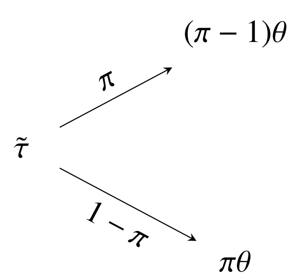
Protection buyers. There is one unit mass of *homogeneous* protection buyers. They are endowed with one unit risky asset at $t = 0$. The asset has random return $\tilde{\theta}$ at $t = 1$. $\tilde{\theta}$ can take on two values: θ with probability π and 0 with probability $(1 - \pi)$.



Protection buyers are also endowed with cash m at $t = 0$. Protection buyers are risk averse with mean-variance utility.[6]

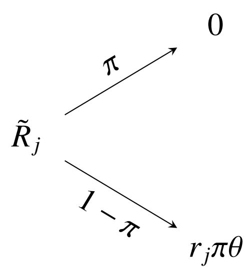
$$U^b(m, \tilde{\theta}) = m + \mathrm{E}(\tilde{\theta}) - \frac{\gamma}{2}\mathrm{var}(\tilde{\theta})$$

Protection sellers. There is one unit mass of *heterogeneous* protection sellers who are risk neutral and have limited liability. A protection seller enters into a contract with a protection buyer that the seller will have the following payment $\tilde{\tau}$ to the buyer at $t = 1$: $\tilde{\tau}$ is $(\pi - 1)\theta$ when $\tilde{\theta}$ is θ and $\pi\theta$ when $\tilde{\theta}$ is 0. The contract has zero mean and provide full insurance to the buyers. In real operations, such a contract can be implemented by Credit Default Swap (CDS).



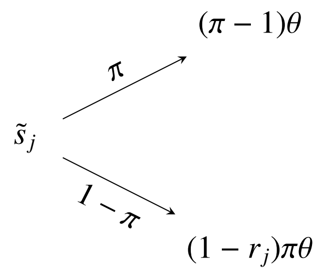
[6]All the results are preserved with concave utility functions. However, for tractability purpose, I use mean-variance utility in the model.

If protection seller j trades with a buyer, he can hedge the downside risk by investing in one unit hedging asset $\tilde{R}_j$. If seller j does not join the trading and clearing game, he will not invest in the hedging asset $\tilde{R}_j$. Hence, the outside option of seller j is 0 instead of $(1-\pi)r_j\pi\theta$. Different protection seller j has different payoff from the hedging asset $\tilde{R}_j$ at $t = 1$: $\tilde{R}_j$ is 0 when $\tilde{\theta}$ is θ and $r_j\pi\theta$ when $\tilde{\theta}$ is 0. r_j ranges from 0 to 1, which can be viewed as seller j's hedging capability. The assumption of heterogeneous hedging capability of protection sellers is not far from the reality. Dealers in OTC derivatives markets normally have their own specialty in managing their position risk (Perez Saiz, Fontaine, and Slive, 2013).



Each protection buyer is matched with one protection seller randomly.[7] So the trading volume between a buyer and a seller is either zero or one. Taking into account the payoff from the hedging asset $\tilde{R}_j$, the overall payment from seller j to his counterparty is denoted as $\tilde{s}_j$. $\tilde{s}_j$ is $(\pi-1)\theta$ when $\tilde{\theta}$ is θ and $(1-r_j)\pi\theta$ when $\tilde{\theta}$ is 0.

[7]The trading capacity of a seller is one. So, it is not the case that all the buyers will go for the seller with the best hedging capability ($r_j = 1$). Also, matching does not guarantee trading. If both trading parties do not benefit from trading, they have the outside option of no trade. I adopt the most simplified searching model here: random matching with Nash bargaining. Introducing more advanced search models will definitely have a better approximation of OTC derivatives markets. But that complicates the model unnecessarily, since the focus of my paper is the incentives of the CCP instead of the incentives of the trading parties. Interesting readers could refer to Koeppl, Monnet, and Temzelides (2012) and Biais, Heider, and Hoerova (2016) for example. In addition, since each trader only trades with one counterparty, I don't capture the netting efficiency provided by central clearing. Although that is a very important feature of central clearing, it does not matter that much when it comes to the SITG problem of CCPs. Interesting readers could refer to Duffie and Zhu (2011) and Anderson et al. (2013).



Ex ante, the protection buyer knows the hedging capability r_j of the protection seller j. Protection buyers and sellers have equal bargaining power in setting the price of the contract. To disincentivize protection sellers' defaults, the CCP impose a collateral requirement c and a default fund contribution αc for each protection seller. The unit cost of collateral and default fund contribution for each seller is δ.[8] In case of the default of seller j, the hedging asset $\tilde{R}_j$ will be seized by the buyer.

I assume that the collateral cost is not negligible, i.e., the collateral cost is large enough so that some seller j cannot provide full collateral to cover his potential loss at $t = 1$.

$$\delta > \frac{\theta\gamma(1-\pi)}{2} \equiv \underline{\delta}$$

CCP. There is one CCP that clears all the trades in the market. The CCP interposes itself between protection buyers and protection sellers. Through the novation process, the trading contract between buyers and sellers splits into two contracts: one is between protection buyers and the CCP; and the other is between protection sellers and the CCP. If protection sellers default, they default on the CCP. The CCP is effectively providing insurance against counterparty risk.

The CCP is a risk-neutral and profit-driven financial intermediary.[9] The CCP has capital K and the unit cost of capital is φ.[10] According to the size of capital, the CCP chooses the optimal collateral c to maximize her expected value. The collateral requirement is only for the sellers, since

[8] I assume linear cost of collateral and default fund contribution. The idea is that, in reality, the total financial resources contributed to CCP by large dealers (usually large banks) are normally a small fraction (1% or even less) of the dealers' available liquidity. Hence, it is realistic to assume linear collateral cost (Murphy, 2016).

[9] In section 3, I analyze the case of benevolent CCP. In that case, the benevolent CCP maximizes the total social welfare.

[10] Again, I assume linear capital cost here.

they are the trading party who has incentives to default.[11] Moreover, the collateral requirement c is position-specific instead of trader-specific. In other words, the CCP charges the same collateral for every seller despite their heterogeneous hedging capability.[12] As to the default fund contributed by the sellers, I assume it is proportional to the collateral, i.e., αc, where α is an exogenous variable.[13]

The CCP charges volume-based commission fee $\frac{f}{2}$ for both buyers and sellers when they use the clearing service.[14] The fee is exogenous and not controlled by the CCP.[15] Instead of increasing the fee, the CCP can increase the trading volume to maximize the profit. I assume the fee is small.

$$f < \pi\theta(1 - \pi) \equiv \bar{f}$$

For the default waterfall of the CCP, I follow the order outlined in Duffie (2015). In case of seller j's default, the loss will be covered as follows.

1. the collateral c contributed by seller j
2. the default fund contribution αc by seller j
3. CCP's capital K
4. the default fund contributed by the non-default sellers

If the CCP has not enough financial resources to cover the default losses, the CCP becomes insolvent. In that case, I assume that the protection buyers will bear the remaining losses. Hence, the protection buyers are not fully insured in that situation.

[11] In reality, I normally observe both counterparties deposit collateral because they both might default when risky payoff is realized. But in my model, I have an option type contract. Only the sellers will have incentive to default. The benefit of doing so is to separate the losses born by the non-default sellers (via default fund contribution) and losses born by the counterparties of default sellers (via partial insurance losses).

[12] In real operations, CCPs do charge credit add-on for credit risk. But in my model, I don't take that into account.

[13] In reality, the size of default fund is usually determined by stress tests and should satisfy the "Cover 2" principle (CPMI-IOSCO, 2012, 2014, 2016).

[14] For notation purpose, the CCP charges $\frac{f}{2}$ for both buyers and sellers, so that the total fee paid by a pair of buyer and seller for one unit of trading volume is f

[15] The assumption of exogenous fee is to mimic the OTC derivatives clearing reality that the commission fee is determined rather by the industry consensus.

2.2 First best allocation

I first study the first best allocation. Although sellers are heterogeneous, there is no asymmetric information between buyers and sellers. A buyer knows exactly the hedging capability of the seller he trades with. In other words, r_j is common knowledge for both buyers and sellers.[16] In the first best allocation, sellers will not default. Thus, the buyers are fully insured and receive utility gain from smoothing payoffs across states. For the sellers, they benefit from the expected payoff of the hedging asset $\tilde{R}_j$. The utility and outside options for the buyers and the sellers are as follows.

$$\begin{aligned} U^b &= m + \pi\theta - p_j, & D^b &= m + \pi\theta - \frac{\gamma}{2}(1-\pi)\pi\theta^2, \\ U^{s_j} &= p_j + (1-\pi) r_j \pi\theta, & D^{s_j} &= 0. \end{aligned}$$

The utility improvement for a pair of seller j and his buyer is[17]

$$\begin{aligned} \Delta U &= U^b + U^{s_j} - D^b - D^{s_j} \\ &= \underbrace{\frac{\gamma}{2}(1-\pi)\pi\theta^2}_{\text{utility gain}} + \underbrace{(1-\pi) r_j \pi\theta}_{\text{Expected return from } \tilde{R}_j} . \end{aligned} \tag{1}$$

Thus, the total welfare surplus in trading consist of two parts: (i) the utility gain due to buyers' risk aversion, and (ii) the expected return from the hedging asset $\tilde{R}_j$. Equation 2 shows the total surplus in the first best equilibrium.

$$\begin{aligned} W^{FB} &= \int_0^1 \Delta U \, \mathrm{d}r_j \\ &= \frac{1}{2}(1-\pi)\pi\theta(\gamma\theta + 1) \end{aligned} \tag{2}$$

[16] It is a rather tricky question whether r_j is known by the CCP or not. On the one hand, CCPs often have strict membership requirement that identify the credit-worthiness of clearing members. On the other hand, however, the collateral requirement is not member-specific. In the current setup, I assume that the collateral requirement set by the CCP is not contingent on the hedging capability r_j.

[17] In the following analysis, I always consider the utility improvement for a pair of buyer and seller, unless I study the Nash bargaining price and need to separate the utility improvement of the buyer from that of the seller.

3 Benevolent CCP

In this section, I analyze the case of a benevolent CCP. In other words, the benevolent CCP will maximize the total welfare surplus, including the utility improvement of buyers and sellers and the CCP's value, by setting the optimal collateral and capital. In reality, there are CCPs that are owned by clearing members. For example, Japanese Security Clearing Corporations (JSCC) and Swiss SIX X-clear Ltd are user-owned CCPs. These user-owned CCPs normally don't chase profits. Instead, their main purpose is to facilitate clearing and settlement among members.

The CCP maximizes the total welfare surplus.

$$\underset{K,c}{\text{Max}} \quad W^b + W^s + V_{CCP}$$

3.1 Collateral

Since protection seller j will lose both his collateral c and default fund contribution αc when he has (large) default loss, I take both collateral and default fund contributed by seller j together as seller j's collateralized financial resources.[18] In other words, the collateralized financial resources of seller j is $(1+\alpha)c$. Seller j defaults when the payment he need to make exceeds the collateralized financial resources. Protection seller j with hedging capability r_j will not default at $t = 1$ if and only if

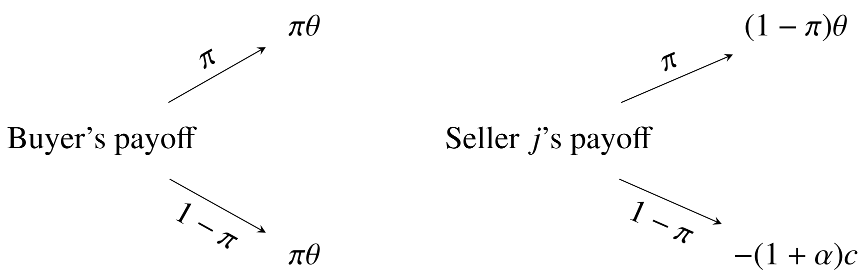
$$(1 + \alpha)c \geq (1 - r_j)\pi\theta.$$

In other words, with given c, seller j with hedging capability higher than $\frac{\pi\theta-(1+\alpha)c}{\pi\theta} \equiv \bar{r}$ will not default at $t = 1$. For the case that seller j does not default, i.e., $r_j \geq \bar{r}$, the overall utility improvement for this pair of traders is as follows.

[18] One could argue that when the default loss of seller j is between c and $(1 + \alpha)c$, he only lose part of the default fund contribution. In my model, I simplify that situation as seller j does not default.

$$\Delta U_{ND} = \underbrace{\frac{\gamma}{2}\pi(1-\pi)\theta^2}_{\text{utility gain}} + \underbrace{(1-\pi)r_j\pi\theta}_{\text{expected return from } \tilde{R}_j} - \underbrace{(1+\alpha)\delta c}_{\text{collateral cost}} - \underbrace{f}_{fee} \tag{3}$$

When seller j has a hedging capability lower than the threshold, i.e., $r_j < \bar{r}$, seller$j$ defaults with probability $(1-\pi)$ at $t = 1$. If seller j defaults, both the payoff of the hedging asset $r_j\pi\theta$ and the collateralized financial resources $(1+\alpha)c$ are seized by the buyer. Furthermore, since the CCP is benevolent and will not default, the rest of the default loss $((1-r_j)\pi\theta - (1+\alpha)c)$ will be covered by CCP's capital. Hence, the buyer is still fully insured. In fact, the benevolent CCP is subsidizing seller j, as seller j's collateralized financial resources establishes a "floor" for his downside risk.



In the case of seller j defaults, equation 4 shows the utility improvement for this pair of traders. Note that the utility improvement for a pair of buyer and default seller is invariant in the seller's hedging capability r_j.[19]

$$\Delta U_D = \underbrace{\frac{\gamma}{2}\pi(1-\pi)\theta^2}_{\text{utility gain}} + \underbrace{(1-\pi)(\pi\theta - (1+\alpha)c)}_{\text{expected gain from default}} - \underbrace{(1+\alpha)\delta c}_{\text{collateral cost}} - \underbrace{f}_{fee} \tag{4}$$

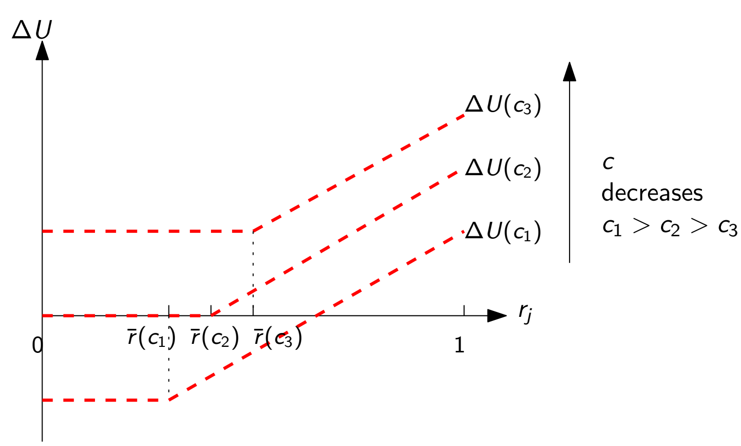
As shown in equation 3 and 4, the utility improvement is a monotonically decreasing function in collateral c. Because collateralized financial resources are costly. If the CCP sets a high collateral requirement, protection sellers who join the trading game need to bear a high collateral cost.[20] Moreover, for the sellers who have low hedging capability, the high collateral cost will drive the trading benefit to zero (or negative). Hence, with a high enough collateral requirement, the CCP

[19] I use default sellers to stand for sellers who will default with probability $(1-\pi)$ at $t = 1$.
[20] For simplicity, I use collateral cost to stand for cost of collateralized financial resources.

could separate the sellers with high hedging capability from those with low hedging capability. In that case, the trading volume is a decreasing function of collateral.

However, the trading volume is not always decreasing in collateral due to the fact that default sellers have a "floor" for their downside risk. Figure 2 shows the relationship between utility improvement and hedging capability. As mentioned before, there is a kink at $r_j = \bar{r}$. Let's call seller j with hedging capability $\bar{r}$ the "marginal seller", since sellers with hedging capability smaller than $\bar{r}$ may default at $t = 1$. Let $\bar{c}$ denotes the threshold of collateral that the utility improvement of the marginal seller is 0. Because all default sellers have the same utility improvement from trading; trading volume will jump to 1 when collateral is slightly below $\bar{c}$. The kink also means that default sellers all join the trading game when collateral is slightly lower than $\bar{c}$, but they all do not join when $c \geq \bar{c}$. Proposition 1 formalizes the idea.

Figure 2: Utility improvement with different collateral



Proposition 1. ***Trading volume (benevolent CCP)***

The trading volume in the case of benevolent CCP is as follows

$$v(c) = \begin{cases} 1 - \bar{r}, & c \geq \bar{c} \\ 1, & 0 \leq c < \bar{c} \end{cases} \tag{5}$$

where $\bar{r}$ is the hedging capability of the marginal seller and $\bar{c}$ is the threshold of collateral *that the utility improvement of the marginal seller is 0.*

$$\begin{aligned} \bar{r} &= \frac{\gamma\pi(1-\pi)\theta^2 - 2(1+\alpha)\delta c - 2f}{2(1-\pi)\pi\theta} \\ \bar{c} &= \frac{\pi\theta(1-\pi)(\gamma\theta+2) - 2f}{2(1+\alpha)(1-\pi+\delta)} \end{aligned} \tag{6}$$

Proof. See appendix.

3.2 Optimal capital and collateral for a benevolent CCP

Given the two fold effects of collateral, I analyze two different cases: in the first case, the benevolent CCP charges high collateral whereas no seller defaults at $t = 1$; in the second case, the benevolent CCP charges low collateral and some sellers may default at $t = 1$.

Case 1: no seller defaults at $t = 1$. When sellers who join the trading game don't have incentive to default at $t = 1$, there is no potential systemic risk. But in this case, the trading volume is not one. Hence, there is a trade-off between the decrease of systemic risk and the decrease of realized gain from trade (RGFT). The expected value of the benevolent CCP is

$$V_{CCP}^{ND} = fv(c) - \varphi K.$$

The total welfare surplus is

$$W^{ND} = \int_{\bar{r}}^{1} \Delta U_{ND} \, \mathrm{d}r_j + V_{CCP}^{ND} \tag{7}$$

Note that since no sellers will default at $t = 1$, there is no need for the benevolent CCP to hold capital in order to absorb losses. Also, in the case of benevolent CCP, the CCP maximizes total welfare improvement. There is no need for the benevolent CCP to hold capital in order to align incentives. Thus, the optimal capital in this case is 0. As to collateral c, from Figure 2, it is obvious that increasing collateral to more than $\bar{c}$ will only decrease trading volume, without the benefit of "screening out" default sellers. It means that the optimal collateral in this case should be as low as possible, while keeping default sellers out of the trading game. In other words, the optimal collateral should be $\bar{c}$. Lemma 1 summarizes the optimal capital and collateral in this case. The total welfare surplus with optimal capital and collateral in this case is less than the first best welfare surplus owing to two things: (i) the collateral is costly and (ii) the RGFT is not maximal.

Lemma 1. ***No default case (benevolent CCP)***

(i) For the benevolent CCP, the optimal capital and collateral in the no default case are

$$K_{ND}^* = 0, \qquad c_{ND}^* = \bar{c}. \tag{8}$$

(ii) The total welfare surplus $W^{ND}(c_{ND}^*, K_{ND}^*)$ is

$$W^{ND}(c_{ND}^*, K_{ND}^*) \quad = W^{FB} - \underbrace{\frac{(1+\alpha)\bar{c}}{\pi\theta}(1+\alpha)\delta\bar{c}}_{\text{collateral cost}} - \underbrace{\frac{\pi\theta - (1+\alpha)\bar{c}}{\pi\theta} W^{FB}}_{\text{not all traders trade}} \tag{9}$$

Proof. See appendix.

Case 2: some sellers default at $t = 1$. When the benevolent CCP impose a collateral requirement lower than $\bar{c}$, all sellers join the trading game because they all have positive utility improvement from trading. But the maximal RGFT goes hand in hand with increasing systemic risk. The benevolent CCP needs to hold capital to cope with potential default losses at $t = 1$. In this case, the expected value of the benevolent CCP is

$$V_{CCP}^D = f - (1-\pi)K - \varphi K.$$

The total welfare surplus is

$$W^D = \int_0^{\bar{r}} \Delta U_D \, \mathrm{d}r_j + \int_{\bar{r}}^1 \Delta U_{ND} \, \mathrm{d}r_j + V_{CCP}^D$$

As mentioned before, the benevolent CCP is actually subsidizing default sellers because the amount of collateralized financial resources limits the downside risk of default sellers. It means that all the default losses will be covered by the benevolent CCP. Let L denote the total default losses.

$$\begin{aligned} L &= \int_0^{\bar{r}} \left[(1-r_j)\pi\theta - (1+\alpha)c\right] \mathrm{d}r_j \\ &= \frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta} \end{aligned} \tag{10}$$

Lemma 2 summarizes the optimal capital and collateral in this case. It turns out that the optimal collateral is zero when capital cost is smaller than collateral cost. In order to cover the default losses, the benevolent CCP needs to hold a capital of $\frac{\pi\theta}{2}$. When capital cost is larger than collateral cost, it is better to charge some collateral so that the capital needed to cover the default losses is smaller.

In fact, in both cases, the value of the benevolent CCP with such capital and collateral is negative, i.e., the benevolent CCP is making loss instead of making profit with this arrangement. The total welfare surplus with optimal capital and collateral in this case is less than the first best welfare surplus because of the capital cost and the potential collateral cost.

Lemma 2. ***Default case (benevolent CCP)***

(i) For the benevolent CCP, the optimal capital and collateral in the default case are

$$K_D^* = \begin{cases} \frac{\pi\theta}{2}\frac{\delta^2}{\varphi^2}, & \varphi > \delta \\ \frac{\pi\theta}{2}, & \varphi \leq \delta \end{cases} \qquad c_D^* = \begin{cases} \frac{\pi\theta}{1+\alpha}\frac{\varphi-\delta}{\varphi}, & \varphi > \delta \\ 0, & \varphi \leq \delta \end{cases} \tag{11}$$

(ii) The total welfare surplus $W^D(c_D^*, K_D^*)$ is

$$W^D(c_D^*, K_D^*) = \begin{cases} W^{FB} - \underbrace{\frac{\pi\theta}{2}\frac{\delta^2}{\varphi}}_{\text{cost of capital}} - \underbrace{\frac{\delta\pi\theta(\varphi-\delta)}{\varphi}}_{\text{cost of collateral}}, & \varphi > \delta \\ W^{FB} - \underbrace{\frac{\pi\theta}{2}\varphi}_{\text{cost of capital}}, & \varphi \leq \delta \end{cases} \tag{12}$$

Proof. See appendix.

No default case v.s. default case. Which case will lead to a higher total welfare surplus? The answer depends on how large the capital cost is. To the extreme, if capital cost is zero, the welfare surplus in the default case would be the first best welfare surplus. Hence, the CCP should charge low collateral, in fact zero collateral, and let sellers default at $t = 1$ if the low state is realized. However, if the capital of the benevolent CCP is very expensive, the benefit of subsidizing the sellers will be outweighed by the cost of capital. In that scenario, the benevolent CCP should be conservative and charge high collateral. Proposition 2 formalizes this idea.

Proposition 2. ***Optimal capital and collateral for a benevolent CCP***

Let $\bar{\varphi}$ denote the threshold of capital cost.

$$\bar{\varphi} = \frac{2}{\pi\theta}\left(W^{FB} - W^{ND}(\bar{c}, 0)\right) \tag{13}$$

(i) When capital cost is higher than $\bar{\varphi}$, the default case will have a higher welfare surplus. The optimal capital and collateral of a benevolent CCP are

$$K^* = 0, \qquad c^* = \bar{c}. \tag{14}$$

(ii) When capital cost is lower than $\bar{\varphi}$, the no default case will have a higher welfare surplus. The optimal capital and collateral of a benevolent CCP are

$$K^* = \frac{\pi\theta}{2}, \qquad c^* = 0. \tag{15}$$

Proof. See appendix.

4 Profit-driven CCP

In this section, I study the case of profit-driven CCP. Different from the benevolent CCP who maximizes the total welfare surplus, the profit-driven CCP only cares about maximizing her own value. Moreover, the CCP has limited liability. In other words, the capital of the CCP is the maximum that she can lose. On the one hand, to chase profit, the CCP has incentive to lower risk management standard to maximize trading volume. On the other hand, because of the limited liability, the CCP does not (fully) internalize her externality on systemic risk. Hence, the profit-driven CCP will contribute low capital and set a low collateral, leading to insolvency problem of the CCP.

At the end of the default waterfall, when the CCP becomes insolvent, I assume that the protection buyers who trade with those default sellers will bear the rest of the losses. It is not far from the reality. In the recovery plan outlined by CPMI-IOSCO (2014), one way to recover an insolvent CCP is partially tear-up, which essentially ask the winning sides (protection buyers) to bear the losses caused by the losing side's (protection sellers') defaults.

It is worthwhile to point out that, for this section, I don't have a predetermined capital requirement for the CCP set by a regulator. Hence, the CCP raise capital and set collateral requirement spontaneously.[21] In section 5, I analyze the optimal capital requirement for the profit-driven CCP.

Let V^P_{CCP} denote the expected value of the profit-driven CCP. P in the superscript stands for profit-driven. The CCP's objective function is to maximize her own expected value instead of to maximize the total welfare surplus.

$$\underset{K,c}{\text{Max}} \quad f v(c) + (1-\pi)\max(-L,-K) - \varphi K \tag{16}$$

where L is the total default losses as a function of collateral c (see in equation 10).

[21] Currently, there are very few regulations on CCP's capital. EMIR sets a capital requirement of 25% of the CCP's operational risk, which is negligible compared to the size of default fund.

4.1 Collateralized and mutualized financial resources

When the CCP is profit-driven and has limited liability, the CCP has no incentive to hold enough capital for potential default losses at $t = 1$, which means that the default fund contributed by non-default sellers might be used to cover the losses caused by default sellers. In this case, it is important to distinguish mutualized financial resources from collateralized financial resources. Collateralized financial resources include collateral and default fund contributed by default sellers. Section 3 has discussed collateralized financial resources. Mutualized financial resources refer to the default fund contributed by non-default sellers. As I will show later, the possibility that the default fund contributed by non-default sellers could be used to cover the losses caused by the default sellers will disincentivize some non-default sellers to join the trading game, hence reducing the trading volume. In this subsection, I study two scenarios when the profit-driven CCP is solvent: (i) total financial resources are only covered by the CCP's capital, and (ii) mutualized financial resources are also used to cover default losses.

Mutualized financial resources remain untouched. According to the default waterfall, when the collateralized financial resources and the CCP's capital is large enough, the default fund contributed by non-default sellers will not be used. In other words, the CCP's capital K is larger than the total default losses L. From equation 10, mutualized financial resources are untouched when the following relationship holds.

$$K \geq \frac{[\pi\theta - (1+\alpha)c]^2}{2\pi\theta} \equiv \bar{K} \tag{17}$$

In this scenario, the utility improvement for traders is the same as equation 3 and 4. Hence, Proposition 1 holds when equation 17 holds.

Mutualized financial resources are used. When the CCP's capital K is not large enough to cover the total default losses, mutualized financial resources are used to cover the rest of the losses. Note that in this scenario, the CCP is still solvent. In other words, the buyers are fully insured. In terms of the relationship between c and K, it means that

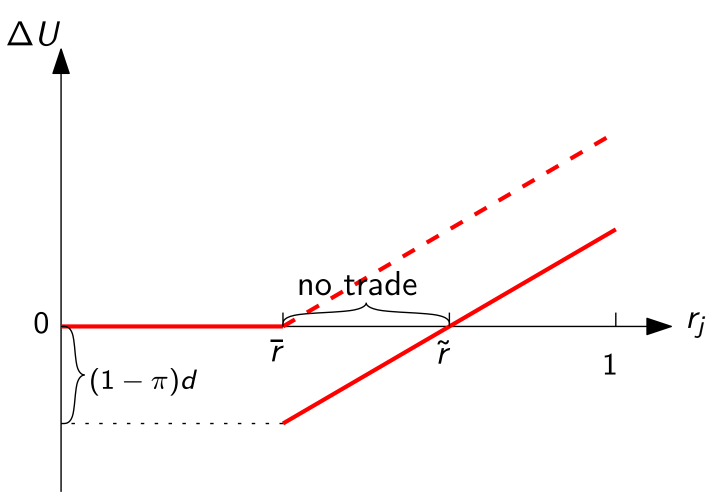
$$\tilde{K} \leq K < \bar{K} \tag{18}$$

where $\tilde{K}$ satisfies the following relationship.[22]

$$\tilde{K} = \frac{[\pi\theta - (1+\alpha)c]^2}{2\pi\theta} - \alpha c(1 - \tilde{r}) \tag{19}$$

In this scenario, the utility improvement for a pair of default seller and his counterparty is the same as equation 4. But the utility improvement of a pair of non-default seller and his buyer will decrease because of the expected loss from default fund contribution. Figure 3 visualizes this idea. Same as in Figure 2, seller j with hedging capability lower than $\bar{r}$ are the default sellers. But different from Figure 2, not every seller with hedging capability higher than $\bar{r}$ will join the trading game because of the expected loss from default fund contribution. Sellers with hedging capability between $\bar{r}$ and $\tilde{r}$ will not trade, where $\tilde{r}$ represents a second type of "marginal seller". Thus, $1 - \tilde{r}$ is the volume of non-default sellers.

Figure 3: Utility improvement when mutualized financial resources are used



I assume that non-default sellers share the losses evenly. Let d denote the default fund loss for

[22] In this equation, $\tilde{r}$ is also a function of K as shown in equation 20. I will solve this equation in Section 4.3.

each non-default seller.

$$d = \frac{L - K}{1 - \tilde{r}} \tag{20}$$

The utility improvement of a pair of non-default seller and his buyer is

$$\Delta U_{ND,M} = \frac{\gamma}{2}\pi(1-\pi)\theta^2 + (1-\pi)r_j\pi\theta - (1+\alpha)\delta c - f - \underbrace{\boldsymbol{(1-\pi)d}}_{\text{Expected loss from default fund}} . \tag{21}$$

Similar with the scenario when mutualized financial resources are not used, when the CCP decreases collateral requirement, utility improvement of traders will increase. Hence, trading volume will increase as well. Lemma 3 summarizes the relationship between trading volume and collateral when mutualized financial resources are used to cover default losses.

Lemma 3. ***Trading volume (mutualized financial resources used)***

When K and c satisfy the following relationship, the CCP remains solvent but has not enough capital to cover the total default losses. Mutualized financial resources are used to cover part of the losses.

$$0 < \frac{[\pi\theta - (1+\alpha)c]^2}{2\pi\theta} - K \leq \alpha c(1 - \tilde{r}) \tag{22}$$

The trading volume is as follows

$$v(c) = \begin{cases} 1 - \tilde{r} + \bar{r}, & \tilde{c} < c < \bar{c} \\ 1, & 0 \leq c \leq \tilde{c} \end{cases} \tag{23}$$

where $\tilde{r}$ and $\tilde{c}$ are as follows.

$$\tilde{r} = \frac{[\pi\theta(1-\pi)(\gamma\theta-2)+2f] + \sqrt{[\pi\theta(1-\pi)(\gamma\theta-2)+2f]^2 - 8(1-\pi)\left(2(1+\alpha)c\pi\theta(\delta-(1-\pi)^2-2K\right))}}{4(1-\pi)(\pi\theta)^2}$$
$$\tilde{c} = \frac{[\pi\theta(1-\pi)(\gamma\theta+4)-2f] + \sqrt{[\pi\theta(1-\pi)(4+\gamma\theta)-2f]^2 + 4\pi\theta(1-\pi)(3(1-\pi)+2\delta)(2K-\pi\theta)}}{2(1+\alpha)(3(1-\pi)+2\delta)} \tag{24}$$

Proof. See appendix.

4.2 End of default waterfall

When all the available financial resources of the CCP drain out, the CCP becomes insolvent. It is not an impossible situation. There are several clearinghouse failures in recent decades: the French Caisse de Liquidation in 1973, the Kuala Lumpur Commodities Clearing House in 1983, the Hong Kong Futures Exchange in 1987, the New Zealand Futures and Options Exchange in 1989, and the Korean exchange clearinghouse in 2014.

$$K + \alpha c(1-\tilde{r}) - \frac{[\pi\theta-(1+\alpha)c]^2}{2\pi\theta} < 0 \tag{25}$$

At the end of default waterfall, i.e., when the above inequality holds, I assume that the protection buyers who trade with those default sellers will bear the rest of the losses.[23] These protection buyers are not fully insured now. The utility improvement of a pair of default seller and his buyer is as follows.

$$\begin{aligned}\Delta U_{D,E} &= \frac{\gamma}{2}\pi(1-\pi)(\theta^2-\boldsymbol{w^2}) + (1-\pi)(\pi\theta-(1+\alpha)c-\boldsymbol{w}) - (1+\alpha)\delta c - f \\ &= \Delta U_D - E(w)\end{aligned} \tag{26}$$

where w denotes the wedge between the required payment and the (insufficient) financial resources and $E(w)$ is the utility loss from partial insurance.

[23] For simplification, it is implicitly assumed that the protection buyers share the losses evenly.

$$
\begin{aligned}
w &= \frac{L - K - \alpha c(1 - \tilde{r})}{\bar{r}} \\
E(w) &= \frac{\gamma}{2}\pi(1 - \pi)w^2 + (1 - \pi)w
\end{aligned}
\tag{27}
$$

Equation 26 and 27 show that when both collateral and capital are very small, $\Delta U_{D,E}$ could be negative, which means the default sellers and their counterparties will not join the trading and clearing game. In this case, there is no default from sellers at $t = 1$. But different from the previous situation when $c \geq \bar{c}$, now default sellers do not have incentive to trade because of the utility loss from partial insurance. Hence, $\Delta U_{D,E} \geq 0$ will pin down a threshold $\underline{\underline{K}}$, which is a function of c. I will elaborate $\underline{\underline{K}}$ in section 4.3.

As to the non-default sellers, they lose all the default fund that they contribute. Hence, the utility improvement of a pair of non-default seller and his counterparty is as follows.

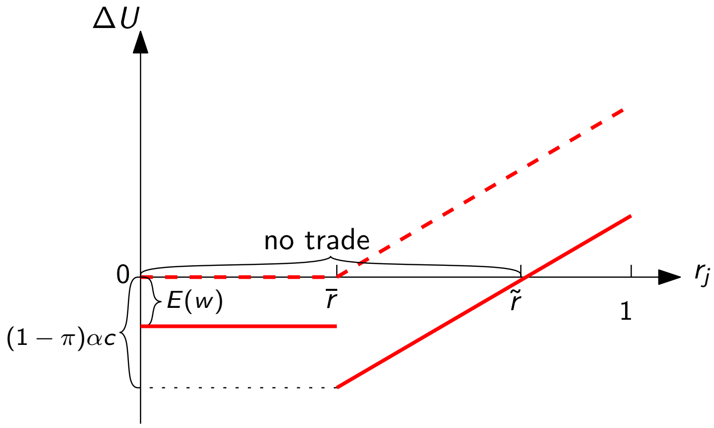
$$
\begin{aligned}
\Delta U_{ND,E} &= \frac{\gamma}{2}\pi(1 - \pi)\theta^2 + (1 - \pi)r_j\pi\theta - (1 + \alpha)\delta c - f - \boldsymbol{(1 - \pi)\alpha c} \\
&= \Delta U_{ND} - (1 - \pi)\alpha c
\end{aligned}
\tag{28}
$$

Comparing equation 26 and 28, I distinguish two cases: one in which the partial insurance loss $E(w)$ is smaller than the default fund loss $(1 - \pi)\alpha c$, and the other in which $E(w) > (1 - \pi)\alpha c$. The intuition is that when K and c satisfy the relationship in equation 25, if c remains unchanged and K decreases, the partial insurance loss $E(w)$ increase from 0 to some amount that is larger than the default fund loss $(1 - \pi)\alpha c$.

Small utility loss from partial insurance. Figure 4 shows the utility improvement when $E(w) \leq (1 - \pi)\alpha c$. When collateral is relatively high (but smaller than $\bar{c}$), the trading volume is $1 - \tilde{r} + \bar{r}$.[24] When the collateral decreases, the trading volume will increase continuously till 1 when collateral reaches $\underline{c}$.

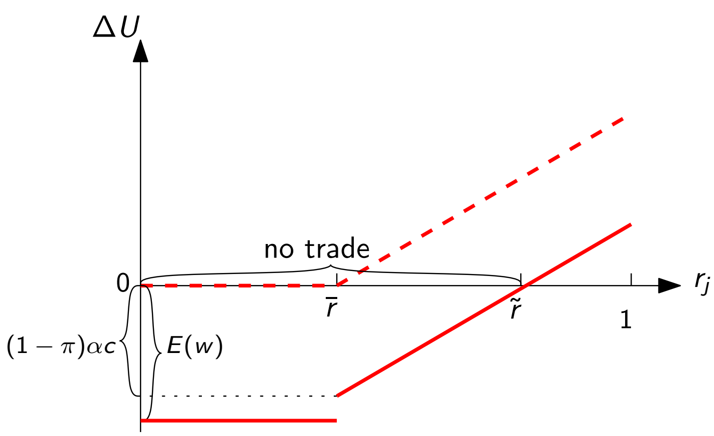
[24] I will show later that this is not an equilibrium since the CCP in this situation will lose their capital and has no incentive to charge a high collateral.

Figure 4: Utility improvement when utility loss from partial insurance is small



Large utility loss from partial insurance When the utility loss from partial insurance is larger than that from default fund loss, Figure 5 shows the utility improvement of traders. Under this situation, when collateral is high, trading volume is $1 - \tilde{r} + \bar{r}$. As collateral decreases, trading volume increases continuously until collateral reaches $\underline{c}$. When $0 \le c \le \underline{c}$, the trading volume is 1.[25] Lemma 4 summarizes the results.

Figure 5: Utility improvement when utility loss from partial insurance is large



[25]Note that default sellers and their counterparties will only trade when they could have positive utility improvement, i.e., when $K > \underline{\underline{K}}$. Since $\underline{\underline{K}}$ is a function of c, it is an implicit constraint for c when K is given.

Lemma 4. ***Trading volume (insolvent CCP)***

When K and c satisfy the following relationship, the CCP becomes insolvent and some buyers may be partial insured.

$$K + \alpha c(1 - \tilde{r}) - \frac{[\pi\theta - (1+\alpha)c]^2}{2\pi\theta} < 0 \tag{29}$$

The trading volume for small and large utility losses from partial insurance are as follows. The trading volume is as follows

$$v(c) = \begin{cases} 1 - \tilde{r} + \bar{r}, & \underline{c} < c < \bar{c} \\ 1, & 0 \leq c \leq \underline{c} \end{cases} \tag{30}$$

where $\underline{c}$ is as follows.

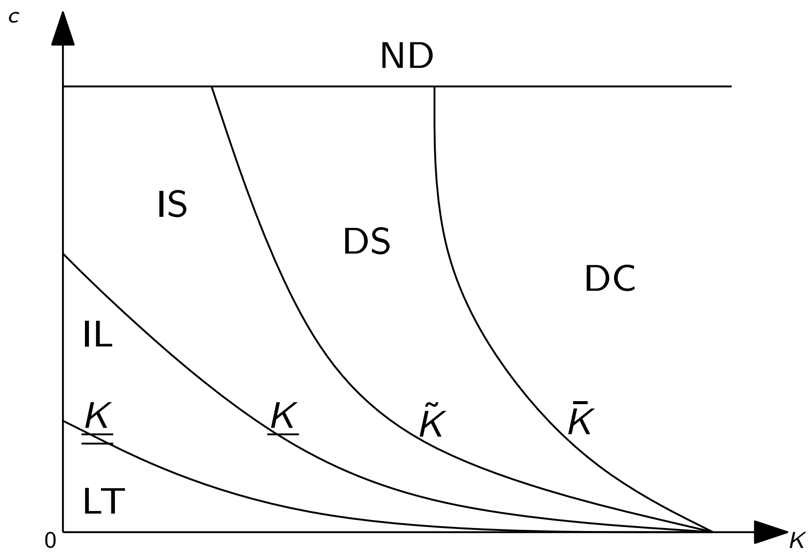
$$\underline{c} = \frac{\pi\theta(1-\pi)(\gamma\theta + 2) - 2f}{2(1+\alpha)(1-\pi+\delta) + 2\alpha(1-\pi)} \tag{31}$$

Proof. See appendix.

4.3 Traders' decision

I solve the equilibrium problem by backward deduction. I first analyze whether traders would join the trading and clearing game. Depending on the size of potential trading losses of sellers and the size of available financial resources of the CCP, there are six different cases where relationship between collateral c and capital K is different. Figure 6 shows the six different combinations of c and K. In other words, when traders observe a pair of (c, K), they "foresee" what would happen at $t = 1$ if the bad state is realized.

Figure 6: Six different cases



Case 1: No seller defaults at $t = 1$. When the trading losses of sellers are covered by the collateralized financial resources, no seller has incentive to default at $t = 1$. As discussed in section 3, collateral is costly and can disincentivize sellers from default. When $c \geq \bar{c}$, only the non-default sellers have positive utility surplus and will join the trading and clearing game. Hence, under such condition, there is no default losses for the CCP. I call such a case "**N**o **D**efault" case.

However, since not every trader joins the game and the trading volume is less than one, the RGFT is not fully realized. As collateral increases, the trading volume decreases further.

Case 2: Some sellers default at $t = 1$ and the CCP covers the losses. When the potential trading loss of a seller exceeds his collateralized financial resources, this seller may default at $t = 1$. If the potential trading losses of default sellers can be covered by the collateralized financial resources and the CCP's capital, the mutualized financial resources contributed by the non-default sellers remain untouched. In other words, $K \geq \bar{K}$ as shown in equation 17. There is no default fund losses for the non-default sellers. I call this case "**D**efault **C**overed" case.

In this case, trading volume is always one. The RGFT is fully realized. But the potential trading losses from default sellers raise systemic risk. As collateral c decreases, the total default losses will increase. The threshold $\bar{K}$ also need to increase to cover the losses. Hence, $\bar{K}$ is a decreasing function in c.

Case 3: Default fund is consumed and the CCP is solvent. When the CCP has not enough capital, the mutualized financial resources are also used to cover the potential trading losses of default sellers. As long as the default fund is large enough to cover the losses, the CCP remains solvent. I call this case "**D**efault **S**olvent" case.

When default fund contributed by the non-default sellers is used to cover the losses of default sellers, the utility improvement of a pair of non-default seller and his buyer $\Delta U_{ND,M}$ also depends on the CCP's capital K, as shown in equation 21. Solve $\Delta U_{ND,M}(\tilde{r}) = 0$, I have $\tilde{r}$ as a function of both c and K. Plug $\tilde{r}$ back to equation 34. I have the explicit form of $\tilde{K}$ as follows.

$$\tilde{K} = \frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta} - \frac{\alpha c\left[(1+\alpha)\delta c + f + \pi(1-\pi)(\gamma - \theta)\right]}{2(1-\pi)\theta} \tag{32}$$

Since $c < \bar{c}$, the default sellers will join the game. But their default losses will consume the default fund contributed by the non-default sellers. As lemma 3 shows, the trading volume in this case could be either one or smaller than one. When $\tilde{c} < c < \bar{c}$, trading volume will be smaller than one. Some of the non-default sellers will not like to join the game because the expected default fund losses make it unprofitable for them to trade.[26] When $0 \leq c \leq \tilde{c}$, although the non-default sellers (and their counterparties) still "subsidize" the default sellers (and their counterparties), the amount of subsidy is small enough that all non-default sellers could have non-negative utility improvement from trading. Hence, the trading volume is one when $0 \leq c \leq \tilde{c}$.

Case 4: CCP is insolvent and the utility loss from partial insurance is small. When all the available financial resources of the CCP drains out, the CCP becomes insolvent. At the end of

[26] Murphy (2016) studies the incentives created by CCP's financial resources. One important element is default fund. The risk of losing default fund will increase the cost for the traders, hence decreasing the trading volume. In my model, the potential default fund breach also increase the cost for the traders. But since the CCP can choose the collateral requirement, the CCP will compensate the traders by lowering collateral requirement. On one hand, lowering collateral requirement will increase the possibility of default fund breach. But on the other hand, lowering collateral requirement can directly reduce the collateral cost. Hence, the CCP still can maximize trading volume by lowering collateral requirement, at the cost of larger potential default losses.

default waterfall, the remaining losses will be born by the buyers who trade with those default sellers. In other words, these buyers are partially insured. When the utility loss from partial insurance $E(w)$ is smaller than the expected default fund losses $(1-\pi)\alpha c$, I call it "**I**nsolvent **S**mall" case.

As discussed in section 4.2, there is a threshold $\underline{K}$ that satisfies $E(w) \leq (1-\pi)\alpha c$. I first solve the $\tilde{r}$ from $\Delta U_{ND,E}(\tilde{r}) = 0$, and then solve the inequality $E(w) \leq (1-\pi)\alpha c$. Hence, the condition for this case is $\underline{K} \leq K < \tilde{K}$ where

$$\underline{K} = \frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta} - \alpha c - \frac{(1+\alpha)c}{\theta\gamma(1-\pi)\pi^2}\left[\sqrt{(1-\pi)\theta[(1-\pi)\theta + 2\theta\gamma(1+\alpha)]} - (1-\pi)(1+\pi\gamma\alpha c)\right] \tag{33}$$

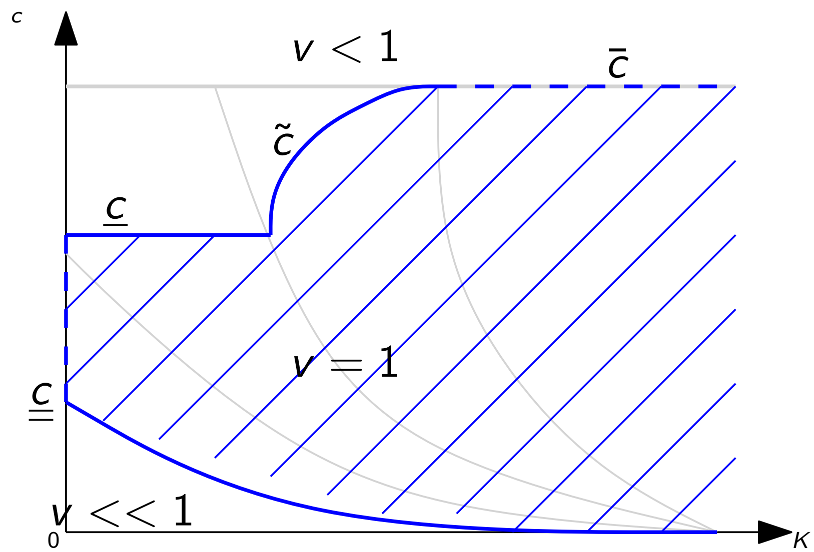
Case 5: CCP is insolvent and the utility loss from partial insurance is large. When the CCP is insolvent and the utility loss from partial insurance is larger than the default fund loss, I call it "**I**nsolvent **L**arge" case.

$$\begin{aligned}\underline{\underline{K}} = &\frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta} - \alpha c \\ &- \frac{(1+\alpha)c}{\theta\gamma\pi^2}\left[\sqrt{1-\pi+2\pi\gamma[(1+\alpha)(1-\pi+\delta)c+f] - (1-\pi)(\gamma\theta-2)\theta\gamma\pi^2} - (1+\pi\gamma\alpha c)\right]\end{aligned} \tag{34}$$

Case 6: Extremely small capital and collateral. When capital K is smaller than $\underline{\underline{K}}$, i.e., when both capital and collateral are extremely small, the losses from partial insurance are so large that the default sellers and their counterparties will not trade with each other. In this case, there is no default at $t = 1$ when the bad state is realized. However, given the collateral is so small, the trading volume of non-default sellers is also very small. Hence, compared to the **ND** case, the trading volume is also negligible.

Figure 7 combines the results from lemma 3, 4 and figure 6. The blue shaded area is the parameter space that trading volume will be one. Note that the dashed blue line means that the boundary is not included. For instance, when $c = \bar{c}$, the trading volume is smaller than one.

Figure 7: Trading volume



4.4 Optimal collateral and capital for a profit-driven CCP

As stated in equation 16, the optimization problem of a profit-driven CCP is

$$\underset{K,c}{\text{Max}} \quad f v(c) + (1-\pi)\max(-L, -K) - \varphi K.$$

As shown in figure 6, when $c \geq \bar{c}$, only non-default sellers and their counterparties join the trading game. There is no default loss for the CCP at $t = 1$. Hence, the expected value of the CCP only consists of the volume-based fee income and the cost of capital.

When $0 \leq c < \bar{c}$, V_{CCP} takes three different formula, depending on how large the CCP capital is. When $K \geq \bar{K}$, default sellers and their counterparties join the trading game. The CCP will cover all default losses, i.e., $\frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$, at $t = 1$ when the bad state is realized. When $\underline{\underline{K}} < K < \tilde{K}$, the CCP only contributes his capital but does not cover all default losses when the bad state is realized. When $0 \leq K < \underline{\underline{K}}$, the default sellers and their counterparties will not join the trading game because of large losses from partial insurance. In this case, there is no default losses for the CCP. Equation 35 summarizes the idea.

$$V_{CCP} = \begin{cases} fv(c) - \varphi K, & c \geq \bar{c} \\ fv(c) - (1-\pi)\frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta} - \varphi K, & 0 \leq c < \bar{c}, K \geq \bar{K} \\ fv(c) - (1-\pi)K - \varphi K, & 0 \leq c < \bar{c}, \underline{\underline{K}} \leq K < \bar{K} \\ fv(c) - \varphi K, & 0 \leq c < \underline{c}, 0 < K < \underline{\underline{K}} \end{cases} \tag{35}$$

Optimal collateral as a function of capital. Although the CCP choose optimal collateral and capital simultaneously, I separate the decision procedure into two steps in order to facilitate the comparison between the CCP's choice and the optimal level of collateral and capital in terms of maximizing social welfare, which will be discussed in section 5.

There are several important observations from equation 35. First, it shows that when $0 < K < \underline{\underline{K}}$, setting collateral higher than $\bar{c}$ gets a higher CCP value than setting collateral lower than $\bar{c}$. Because trading volume in this case is increasing in c. Second, when $K \geq \bar{K}$, the CCP trades off between large trading volume and large default losses. On the one hand, the CCP could set collateral higher than $\bar{c}$ to minimize default losses. But the trading volume will be low. On the other hand, the CCP could set collateral lower than $\bar{c}$ to maximize trading volume, hence maximizing fee income. But the default losses will be high. The optimal collateral depends on which leads to a higher expected value of CCP. Fee would be a crucial element in determining optimal collateral. Intuitively, when fee is low, the "temptation" for the CCP to increase trading volume is small. So the CCP cares more about the expected default losses and will set a high collateral. However, when the fee is high, increasing one unit of trading volume will bring large profit. The CCP has strong incentive to maximize trading volume and will go for a low collateral. Third, when $\underline{\underline{K}} \leq K < \bar{K}$, the limited CCP capital break the trade-off between high trading volume and large default losses, as the CCP does not cover all the default losses. As K reduces, the CCP tends to chase high trading volume since she has very little to lose. Thus, when K is smaller than some threshold $\hat{K}$, the CCP will set collateral c within the blue shared area in figure 7 so that the trading volume is one. All RGFT will be realized. But the CCP does not cover all the default losses. If $\hat{K} \geq \underline{K}$, the default losses will be covered by the default fund contributed by other non-default sellers. If $\hat{K} < \underline{K}$, the default losses will also be covered by the buyers that traded with the default sellers. Proposition 3 summarizes the optimal collateral requirement with given K.[27]

Proposition 3. ***(Optimal collateral given specific capital)***

The optimal collateral when fee is lower than $\underline{f}$ and higher than $\underline{f}$ are

$$c^*(K) = \begin{cases} \bar{c}, & K \geq \hat{K}(\bar{c}) \\ \tilde{c}, & \tilde{K}(\underline{c}) \leq K < \hat{K}(\bar{c}) \\ \underline{c}, & 0 \leq K < \tilde{K}(\underline{c}) \end{cases} \qquad c^*(K) = \begin{cases} [\bar{c}]^-, & K \geq \bar{K}(\bar{c}) \\ \tilde{c}, & \tilde{K}(\underline{c}) \leq K < \bar{K}(\bar{c}) \\ \underline{c}, & 0 \leq K < \tilde{K}(\underline{c}) \end{cases} \tag{36}$$

where the threshold $\underline{f}$ and $\hat{K}$ are

$$\begin{aligned} \underline{f} &= \frac{(1-\pi)\pi\theta[2(1-\pi+\delta)+\gamma\theta+2]}{6-4\pi+4\delta} \\ \hat{K} &= f(1-\frac{(1+\alpha)c}{\pi\theta}) \end{aligned} \tag{37}$$

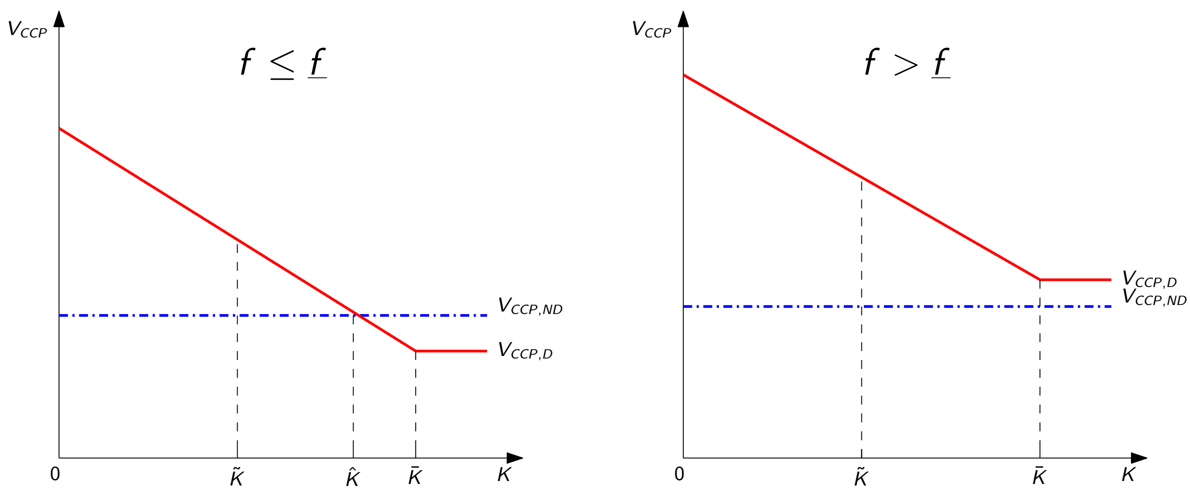
Proof. See appendix.

Figure 8 visualizes the expected value of the CCP as a function of K. Without loss of generality, I set $\varphi = 0$ since the capital cost applies to all different cases. The red solid line represents CCP value when default sellers join the game, while the blue dashed line is CCP value when only non-default sellers join the game. When only non-default sellers join the game, the expected value of the CCP is invariant in CCP's capital, as there is no default losses need to be covered. But when default sellers also join the game, the expected value decreased in K until $\bar{K}$. The reason is, when $K < \bar{K}$, the CCP contributes all her capital to cover the default losses. But when $K \geq \bar{K}$, the CCP's capital is larger than the total default losses. Thus, the expected value is also invariant in K. The two subplots show the two cases when fee is higher or lower than $\underline{f}$. As discussed before, fee is an important factor in altering CCP's incentives. When $f > \underline{f}$, the CCP will always maximize

[27] I use the notation $[X]^-$ to denote the amount that is slightly smaller than X and $[X]^+$ to denote the amount that is slightly larger than X.

trading volume no matter how large her capital is. Because the fee income from the default sellers (and their counterparties) is larger than the expected default losses. In this case, the lower is the capital, the higher is the CCP value. When $f \geq \underline{f}$, the fee income from the default sellers (and their counterparties) is smaller than the expected default losses. Depending on how low the fee level is, the CCP will trade-off the fee income and the expected loss of her capital. In this case, the CCP with large capital, i.e., $K \geq \hat{K}$, will be conservative and set high collateral to disincentivize sellers' default at $t = 1$. As fee decreases, the threshold $\hat{K}$ decreases. But as long as fee is positive, $\hat{K} > 0$, which means when capital is very small, the CCP still will go for large trading volume since she has very little to lose.

Figure 8: CCP's value



Optimal capital for a profit-driven CCP. As outlined in figure 8, the largest expected value of the CCP is achieved at $K = 0$. It means that the CCP will choose zero capital to minimize her exposure to potential defaults at $t = 1$ when the bad state is realized. Since the CCP capital is zero, she will not have incentives to do set a high collateral to avoid default sellers. Instead, the CCP will set a low collateral to attract default sellers, hence to maximize trading volume. The collateral cannot be too low neither, since low collateral leads to too many defaults when the bad state is realized, which in return will jeopardize the counterparties of default sellers. Proposition 4 presents the optimal capital and collateral for a profit-driven CCP.

Proposition 4. ***(Profit-driven CCP's optimal capital and collateral)*** *The optimal capital and collateral for a profit-driven CCP are*

$$K^* = 0, \qquad c^* = \underline{c} \tag{38}$$

Proof. See appendix.

Note that in section 3, the benevolent CCP also set $K = 0$ when the capital cost is high. But the benevolent CCP sets a high collateral to avoid defaults at $t = 1$ when the bad state is realized. Because a benevolent CCP cares about the total welfare surplus. On contrary, the profit-driven CCP in this section has no incentive to set a high collateral. The low collateral $\underline{c}$ set by the profit-driven CCP maximizes the CCP's value; but it does not maximize the total social welfare surplus.

5 Optimal capital requirement for a profit-driven CCP

In the previous section, there is no capital requirement for a profit-driven CCP. The CCP choose the capital and collateral simultaneously to maximize her profit. Now I introduce a regulator that maximizes the total welfare surplus W.

$$\underset{K}{\text{Max}} \quad W^b + W^s + V_{CCP}$$

From proposition 3, I have the optimal collateral that a profit-driven CCP will choose when the capital is given. Equation 39 shows the total welfare surplus.

$$W(c^*) = \int_0^{\frac{(1+\alpha)c^*}{\pi\theta}} \Delta U^{ND}(r_j)\, \mathrm{d}r_j + \int_{\frac{(1+\alpha)c^*}{\pi\theta}}^{v(c^*)} \Delta U^D\, \mathrm{d}r_j + V_{CCP}(c^*) \tag{39}$$

With the optimal collateral, I calculate the total welfare surplus in different cases. First of all, from proposition 3, the profit-driven CCP set high collateral and hence there is no default when $K \geq \hat{K}(\bar{c})$ and $f \leq \underline{f}$. In this case, the welfare surplus is lower than the first best welfare surplus because of collateral cost, not-fully-realized gains from trade, and capital cost. Second, if $0 \leq K < \tilde{K}(\underline{c})$, the profit-driven CCP will become insolvent. Hence, the welfare surplus is lower than the first best one because of collateral cost, partial insurance loss and capital cost. Third, if K is in between $\tilde{K}(\underline{c})$ and $\hat{K}(\bar{c})$ (or $\bar{K}(\bar{c})$) when fee is lower (or higher) than $\underline{f}$, all gains from trade are realized and the CCP remains solvent. Hence, the total welfare surplus is lower than the first best one only because of collateral cost and capital cost. Lemma 5 presents the welfare surplus when K and f are different.

Lemma 5. ***(Total welfare surplus given specific capital)***

When $f \leq \underline{f}$, the total welfare surplus is as follows.

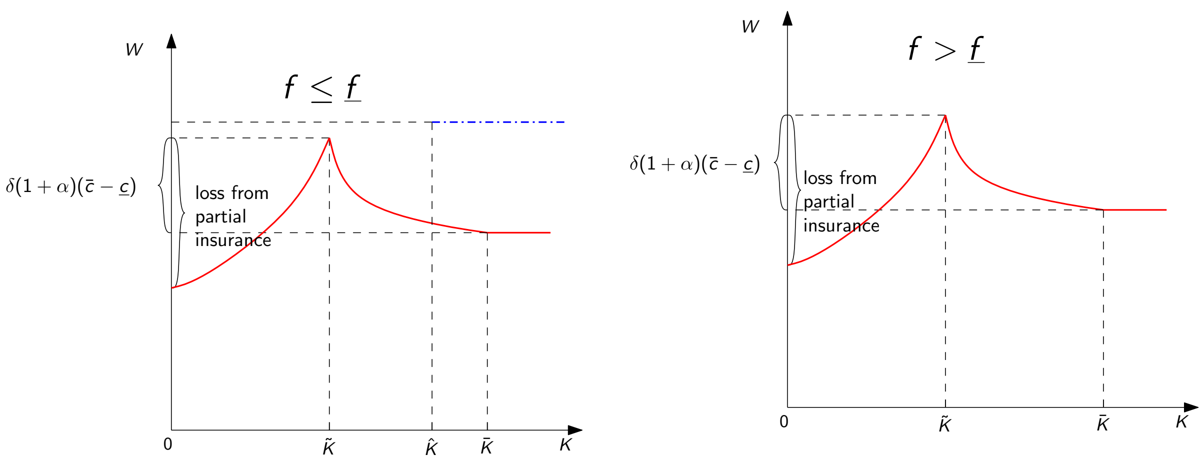
$$W = \begin{cases} W^{FB} - \underbrace{(1+\alpha)\delta\bar{c}\frac{(1+\alpha)\bar{c}}{\pi\theta}}_{\text{collateral cost}} - \underbrace{(1-\pi)\frac{(\pi\theta-(1+\alpha)\bar{c})^2}{2\pi\theta}}_{\text{not all traders trade}} - \underbrace{\varphi K}_{\text{capital cost}}, & K \geq \hat{K}(\bar{c}) \\ W^{FB} - \underbrace{(1+\alpha)\delta\tilde{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & \tilde{K}(\underline{c}) \leq K < \hat{K}(\bar{c}) \\ W^{FB} - \underbrace{(1+\alpha)\delta\underline{c}}_{\text{collateral cost}} - \underbrace{\left(1-\frac{(1+\alpha)\underline{c}}{\pi\theta}\right)\frac{\gamma}{2}\pi(1-\pi)w^2}_{\text{loss from partial insurance}} - \underbrace{\varphi K}_{\text{capital cost}}, & 0 \leq K < \tilde{K}(\underline{c}) \end{cases} \tag{40}$$

When $f > \underline{f}$, the total welfare surplus is as follows.

$$W = \begin{cases} W^{FB} - \underbrace{(1+\alpha)\delta\bar{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & K \geq \bar{K}(\bar{c}) \\ W^{FB} - \underbrace{(1+\alpha)\delta\tilde{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}, & \tilde{K}(\underline{c}) \leq K < \bar{K}(\bar{c}) \\ W^{FB} - \underbrace{(1+\alpha)\delta\underline{c}}_{\text{collateral cost}} - \underbrace{\left(1-\frac{(1+\alpha)\underline{c}}{\pi\theta}\right)\frac{\gamma}{2}\pi(1-\pi)w^2}_{\text{loss from partial insurance}} - \underbrace{\varphi K}_{\text{capital cost}}, & 0 \leq K < \tilde{K}(\underline{c}) \end{cases} \tag{41}$$

Figure 9 shows the total welfare surplus when $\varphi = 0$. The red solid line is the total welfare surplus when there is no default and the blue dashed line is that when there are defaults at $t = 1$ when the bad state is realized. Since the total welfare surplus is decreasing in collateral and the optimal collateral set by the CCP is increasing in K, the welfare surplus with default (the red solid line) is decreasing in K when $K \geq \tilde{K}(\underline{c})$. But when $0 \leq K < \tilde{K}(\underline{c})$, the welfare surplus with default is increasing in K due to the partial insurance loss. For the welfare surplus without default (the blue dashed line), it is constant in K because $\varphi = 0$.

Figure 9: Total welfare surplus



I compare the total welfare surplus with different capital in lemma 5. The optimal capital requirement depends on the fee level f and the capital cost φ. When capital cost is very high, i.e., $\varphi > \bar{\bar{\varphi}}$, the high cost of imposing capital outweighs the benefit of a safe CCP. Hence, the optimal capital requirement in this case will be zero. When $\varphi \leq \bar{\bar{\varphi}}$, it is socially optimal to have a safe CCP. A safe CCP in this context means a solvent CCP. In other words, the financial resources of the CCP can cover the potential default losses. But it does not mean that the CCP's own capital will cover all default losses. Whether that should happen or not depends on the fee level. When $f > \underline{\underline{f}}$, the clearing business is so profitable that the profit-driven CCP will always chase high trading volume to maximize her profits. In this case, high capital does not help to increase total welfare surplus. Instead, high capital requirement leads to high collateral and makes transaction expensive for traders. The optimal capital in this case is to maintain a safe CCP with lowest collateral possible. Hence, $K^* = \tilde{K}(\underline{c})$. When $f \leq \underline{\underline{f}}$, high capital makes the profit-driven CCP conservative. In this case, $K^* = \hat{K}(\bar{c})$ leads to a high collateral $\bar{c}$ and disincentivize sellers' defaults. Proposition 5 summarizes the optimal capital requirement for a profit-driven CCP.

Proposition 5. ***(Optimal capital requirement for a profit-driven CCP)***

The optimal capital requirement for a profit-driven CCP depends on the fee level f and the capital cost φ.

(i) When $\varphi > \bar{\bar{\varphi}}$, the optimal capital requirement is $K^* = 0$.

(ii) When $\varphi \leq \bar{\bar{\varphi}}$ and $f > \underline{\underline{f}}$, the optimal capital requirement is $K^* = \tilde{K}(\underline{c})$.

(iii) When $\varphi \leq \bar{\bar{\varphi}}$ and $f \leq \underline{\underline{f}}$, the optimal capital requirement is $K^* = \hat{K}(\bar{c})$.

The thresholds are as follows.

$$\begin{aligned} \bar{\bar{\varphi}} &= \gamma\pi(1-\pi)w \\ \underline{\underline{f}} &= \frac{\pi\gamma\theta^2(1-\pi)(\gamma\theta+2)(\gamma\theta+2+\alpha(\gamma\theta+6))}{4(\gamma\theta+1)(\gamma\theta+2+\alpha(\gamma\theta+4))} \\ &\quad - \frac{\sqrt{(\pi\gamma\theta^2(1-\pi)(\gamma\theta+2)(\gamma\theta+2+\alpha(\gamma\theta+6)))^2 - 8\alpha\gamma\pi^2\theta^3(\gamma\theta+1)(\gamma\theta+2)^2(\gamma\theta+2+\alpha(\gamma\theta+4))}}{4(\gamma\theta+1)(\gamma\theta+2+\alpha(\gamma\theta+4))} \end{aligned} \tag{42}$$

Proof. See appendix.

6 Conclusion

To the best of my knowledge, this paper is the first in the literature that models CCP's insolvency. CCPs are not benevolent organizations. Instead, many CCPs operate as profit-driven public companies. The profit-driven character, coupled with limited liability constraint, gives rise to potentially misaligned incentives for a CCP to lower collateral requirement in exchange for higher trading volume. I show that a profit-driven CCP will choose zero capital and set a low collateral requirement to maximize her expected value, when there is no capital requirement. A benevolent CCP will choose a minimum capital when capital cost is high; but the benevolent CCP will set a high collateral requirement, since she cares about not only her own expected value but also the utility improvement of traders.

As pointed out before, CCPs are different from banks because of the mutualized financial resources. Hence, the optimal capital requirement should be designed to tailored to such features. My model suggests that the optimal capital requirement for profit-driven CCPs should depend on both capital cost and volume-based fee. For the capital cost, it is a similar story as the capitalization problem for banking. When capital cost is very high, the optimal capital requirement should be zero because the high capital cost outweighs the benefit of a safe CCP. For the low capital cost, the optimal capitalization depends on how profitable is clearing business. Since the main profits of CCPs come from commission fee which is greatly dependent on trading volume, high volume-based fee represents a great "temptation" for the CCP. Hence, when volume-based fee is high enough, the CCP will always go for large trading volume. The optimal capital requirement in this case is to have a safe CCP with lowest collateral possible, as collateral is also costly in my model. When volume-based fee is low, imposing a high capital requirement makes a profit-driven CCP more conservative and will charge a high collateral to disincentivize traders' default.

There are several possible extensions for the current model. First, it is possible to introduce fire sale cost and bail out cost in the case of CCP insolvency. I currently assume that the counterparties that trades with default sellers bear the remaining losses, similar to a partial tear-up. There is no fire sale cost for the collateral when there is a large amount of sellers' defaults in the market. Such assumption could be relaxed by incorporating a "cash-in-the-market" mechanism (Acharya and Yorulmazer, 2008). To bail out or to resolve a CCP depends on how many sellers default and how large the fire sale cost will be. Second, it is possible to endogenize volume-base fee and introduce competition among CCPs, following the circular road model (Salop, 1979). Since fee and collateral both decrease traders' utility, a profit-driven CCP will try to lower these two to maximize their own trading volume. I expect that the competition between CCPs will first drive down the fee level. But as competition gets fierce, profit-driven CCPs will also try to lower collateral requirement to increase trading volume.

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Appendix

A Variable summary

Variable	Definition
α	the ratio between default fund contribution and initial margin
δ	convex collateral cost coefficient for protection sellers
π	probability of the high state
$\tilde{\tau}$	protection seller j's payment to protection buyer
θ	the high realized value of the risky asset
γ	per unit capital cost of the CCP
c	collateral from the sellers to disincentivize the sellers' defaults
d	the share of default fund contribution that survival sellers need to pay
f	volume-based fee charged by the CCP
K	capital of the CCP
L	total default losses
m	cash endowment of protection buyers
r_j	protection seller j's capability to lower the downside risk
$\tilde{s}_j$	protection seller j's real payment to protection buyer
v	trading volume
w	wedge between the required payment and the available financial resources
W	total welfare

B Proof

Proposition 1

Proof. As figure 2 shows, as collateral decreases, traders' utility improvement increases. I first get the hedging capability $\tilde{r}$ of the marginal non-default sellers by setting $\Delta U_{ND}(\tilde{r}) = 0$. From equation 3, I have

$$\tilde{r} = \frac{\gamma\pi(1-\pi)\theta^2 - 2(1+\alpha)\delta c - 2f}{2(1-\pi)\pi\theta} \tag{A1}$$

Also, I need to take into account the fact that $\tilde{r}$ cannot be smaller than $\bar{r}$. Because when r_j is smaller than $\bar{r}$, the seller j will defaults. Thus, equation A1 and $\bar{r} = \frac{\pi\theta-(1+\alpha)c}{\pi\theta}$ pin down a threshold $\bar{c}$ that when $c \geq \bar{c}$, $\tilde{r} \leq \bar{r}$.

$$\bar{c} = \frac{\pi\theta(1-\pi)(\gamma\theta+2) - 2f}{2(1+\alpha)(1-\pi+\delta)} \tag{A2}$$

Hence, when $c \geq \bar{c}$, only non-default sellers have positive utility improvement from trading. Hence, the trading volume is $1 - \tilde{r}$. When $0 \leq c < \bar{c}$, both default and non-default sellers will join the game since they both have positive utility improvement. Trading volume is 1.

■

Lemma 1

Proof. In the case of no default at $t = 1$, holding capital is only adding cost for the benevolent CCP. Hence, the optimal capital in this case is 0. As to collateral, to have no default at $t = 1$, collateral needs to satisfy $c \geq \bar{c}$. Take the first order derivative of W^{ND} with respect to c leads to

$$\frac{\partial W^{ND}}{\partial c} < 0, \quad \text{if } c \geq \bar{c}. \tag{A3}$$

Thus, the optimal c is $\bar{c}$. Plug in c^*_{ND} and K^*_{ND} into equation 7, I directly have

$$W^{ND}(c^*_{ND}, K^*_{ND}) = W^{FB} - \frac{(1+\alpha)\bar{c}}{\pi\theta}(1+\alpha)\delta\bar{c} - \frac{\pi\theta - (1+\alpha)\bar{c}}{\pi\theta}W^{FB} \tag{A4}$$

■

Lemma 2

Proof. Given equation 3 and 4, the objective function of the benevolent CCP is rewritten as

$$\begin{aligned} \underset{K,c}{\text{Max}} \quad & \frac{1}{2}\pi(1-\pi)\theta^2 + \frac{1}{2}(1-\pi)\pi\theta - (1+\alpha)\delta c - \varphi K \\ s.t. \quad & K \geq \frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta} \\ & 0 \leq c < \bar{c} \end{aligned} \tag{A5}$$

Since the objective function is decreasing in K, the optimal K is achieved when $K \geq \frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$ is binding. Hence I could plug in $K = \frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$ into the objective function. Take the first order derivative of the objective function with respect to c and I could see that the optimal c depends on how large is φ. When $\varphi \leq \delta$, the objective function is decreasing in c. Thus, the optimal collateral is zero. When $\varphi > \delta$, the optimal collateral is achieved at $\frac{\pi\theta}{1+\alpha}\frac{\varphi-\delta}{\varphi}$.

With the optimal c^*_D, I could have the optimal K^*_D by plugging c^*_D in $K = \frac{(\pi\theta-(1+\alpha)c)^2}{2\pi\theta}$. Hence, K^*_D is $\frac{\pi\theta}{2}$ when $\varphi \leq \delta$ and is $\frac{\pi\theta}{2}\frac{\delta^2}{\varphi^2}$ when $\varphi > \delta$.

With the c^*_D and K^*_D, I have the total welfare surplus as

$$W^D(c^*_D, K^*_D) = \begin{cases} W^{FB} - \frac{\pi\theta}{2}\frac{\delta^2}{\varphi} - \frac{\delta\pi\theta(\varphi-\delta)}{\varphi}, & \varphi > \delta \\ W^{FB} - \frac{\pi\theta}{2}\varphi, & \varphi \leq \delta \end{cases}$$

■

Proposition 2

Proof. From lemma 1 and 2, I have the optimal capital and collateral for a benevolent CCP in no default case and default case, respectively. Which case leads to a higher total welfare surplus depends on how large is the capital cost φ. Because the total welfare surplus in default case $W^D(c^*_D, K^*_D)$ is decreasing in φ, while that in no default case $W^{ND}(c^*_{ND}, K^*_{ND})$ is invariant in φ. I first discuss the situation that $\varphi \leq \delta$.

When $\varphi \leq \delta$, the total welfare surplus in default case $W^D(c^*_D, K^*_D)$ is

$$\begin{aligned} W^D(c^*_D, K^*_D) &= W^{FB} - \frac{\pi\theta}{2}\varphi \\ &\geq W^{FB} - \frac{\pi\theta}{2}\delta \end{aligned} \tag{A6}$$

Let $f(\delta)$ denote the function of the difference between $W^{ND}(c^*_{ND}, K^*_{ND})$ and $W^{FB} - \frac{\pi\theta}{2}\delta$.

$$\begin{aligned} f(\delta) &= W^{ND}(c^*_{ND}, K^*_{ND}) - (W^{FB} - \frac{\pi\theta}{2}\delta) \\ &= \frac{\pi\theta}{2}\delta - \frac{(1+\alpha)\bar{c}}{\pi\theta}(1+\alpha)\delta\bar{c} - \frac{\pi\theta - (1+\alpha)\bar{c}}{\pi\theta}W^{FB} \end{aligned} \tag{A7}$$

Since $\bar{c} = \frac{\pi\theta(1-\pi)(\gamma\theta+2)-2f}{2(1+\alpha)(1-\pi+\delta)}$, I have the first order derivative of $f(\delta)$ w.r.t. δ as

$$\frac{\partial f(\delta)}{\partial \delta} < 0. \tag{A8}$$

As I assume the collateral cost is large enough that $\delta > \underline{\delta}$; and $f(\underline{\delta}) < 0$, I have $f(\delta) < 0$ for

$\delta > \underline{\delta}$. In other words, $W^{ND}(c^*_{ND}, K^*_{ND}) < W^{FB} - \frac{\pi\theta}{2}\delta \leq W^D(c^*_D, K^*_D)$, when $\varphi \leq \delta$. The default case leads to a higher total welfare surplus for the benevolent CCP.

When $\varphi > \delta$, the total welfare surplus in default case $W^D(c^*_D, K^*_D)$ is

$$W^D(c^*_D, K^*_D) = W^{FB} - \frac{\pi\theta}{2}\frac{\delta^2}{\varphi} - \frac{\delta\pi\theta(\varphi - \delta)}{\varphi}. \tag{A9}$$

Let $g(\varphi)$ denote the function of the difference between $W^{ND}(c^*_{ND}, K^*_{ND})$ and $W^D(c^*_D, K^*_D)$.

$$\begin{aligned} g(\varphi) \quad &= W^{ND}(c^*_{ND}, K^*_{ND}) - W^D(c^*_D, K^*_D) \\ &= \frac{\pi\theta}{2}\frac{\delta^2}{\varphi} + \frac{\delta\pi\theta(\varphi - \delta)}{\varphi} - \frac{(1+\alpha)\bar{c}}{\pi\theta}(1+\alpha)\delta\bar{c} - \frac{\pi\theta - (1+\alpha)\bar{c}}{\pi\theta}W^{FB} \end{aligned} \tag{A10}$$

Take the first order derivative of $g(\varphi)$ w.r.t. φ, I have

$$\frac{\partial g(\varphi)}{\partial \varphi} > 0. \tag{A11}$$

Let $\bar{\varphi}$ satisfy $g(\bar{\varphi}) = 0$, I have $g(\varphi) > 0$ when $\varphi > \bar{\varphi}$, i.e., $W^{ND}(c^*_{ND}, K^*_{ND}) > W^D(c^*_D, K^*_D)$. Solve $g(\bar{\varphi}) = 0$, I have

$$\bar{\varphi} = \frac{2}{\pi\theta}\left(W^{FB} - W^{ND}(\bar{c}, 0)\right). \tag{A12}$$

■

Lemma 3

Proof. When $0 < \frac{[\pi\theta-(1+\alpha)c]^2}{2\pi\theta} - K \leq \alpha c(1 - \tilde{r})$, the CCP remains solvent but has not enough capital to cover the total default losses. Mutualized financial resources are used to cover part of the losses. The utility improvement of a pair of default seller and his buyer is as the same as equation 4. As long as $c < \bar{c}$, the default sellers and their counterparties will join the trading game.

But since the mutualized financial resources contributed by non-default sellers are used to cover the losses of default sellers, the utility improvement of a pair of non-default seller and his buyer is lower, as written in equation 21.

$$\Delta U_{ND,M} = \frac{\gamma}{2}\pi(1-\pi)\theta^2 + (1-\pi)r_j\pi\theta - (1+\alpha)\delta c - f - \underbrace{\boldsymbol{(1-\pi)d}}_{\text{Expected loss from default fund}} \tag{A13}$$

The marginal non-default seller $\tilde{r}$ should satisfy $\Delta U_{ND,M}(\tilde{r}) = 0$. Solve the equation, I have

$$\tilde{r} = \frac{[\pi\theta(1-\pi)(\gamma\theta-2)+2f] + \sqrt{[\pi\theta(1-\pi)(\gamma\theta-2)+2f]^2 - 8(1-\pi)\left(2(1+\alpha)c\pi\theta(\delta-(1-\pi)^2-2K)\right)}}{4(1-\pi)(\pi\theta)^2} \tag{A14}$$

When $\tilde{r} > \bar{r}$, the trading volume is $1 - \tilde{r} + \bar{r}$. Solve $\tilde{r} > \bar{r}$, I have $c > \tilde{c}$, where $\tilde{c}$ is as follows. Note that $\tilde{c}$ is a function of K where $\tilde{c}(\bar{K}) = \bar{c}$ and $\tilde{c}(\underline{K}) = \underline{c}$.

$$\tilde{c} = \frac{[\pi\theta(1-\pi)(\gamma\theta+4)-2f] + \sqrt{[\pi\theta(1-\pi)(4+\gamma\theta)-2f]^2 + 4\pi\theta(1-\pi)(3(1-\pi)+2\delta)(2K-\pi\theta)}}{2(1+\alpha)(3(1-\pi)+2\delta)} \tag{A15}$$

Hence, when $0 \le c \le \tilde{c}$, the trading volume is one.

■

Lemma 4

Proof. When $K + \alpha c(1-\tilde{r}) - \frac{[\pi\theta-(1+\alpha)c]^2}{2\pi\theta} < 0$, the CCP becomes insolvent.[28] All the default fund contribution is used to cover the losses. Equation 28 gives the utility improvement of a pair of default seller and his buyer.

$$\begin{aligned}\Delta U_{ND,E} &= \frac{\gamma}{2}\pi(1-\pi)\theta^2 + (1-\pi)r_j\pi\theta - (1+\alpha)\delta c - f - \boldsymbol{(1-\pi)\alpha c} \\ &= \Delta U_{ND} - (1-\pi)\alpha c\end{aligned} \tag{A16}$$

When seller j with $\bar{r}$ and his counterparty have positive utility improvement, the trading volume

[28] Note that $K > \underline{\underline{K}}$ also holds here. I leave the discussion of K in section 4.3.

is one. Solve $\Delta U_{ND,E}(\bar{r}) = 0$, I have

$$c = \frac{\pi\theta(1-\pi)(\gamma\theta+2) - 2f}{2(1+\alpha)(1-\pi+\delta) + 2\alpha(1-\pi)} \equiv \underline{c}. \tag{A17}$$

Hence, when $\underline{c} < c < \bar{c}$, the trading volume is $1 - \tilde{r} + \bar{r}$, while the trading volume is one when $0 \leq \underline{c}$

■

Proposition 3

Proof. From equation A18, one can see that the CCP will set a collateral to maximize $v(c)$ when $c \geq \bar{c}$. From proposition 2, I know that $\bar{c}$ is the optimal collateral in this case. As to $c < \bar{c}$, figure 7 shows the collateral level that achieves maximum trading volume. But which collateral level maximizes the CCP value still depends on the fee level f. I first plug in the collateral that maximizes trading volume into equation 35.

$$V_{CCP} = \begin{cases} f\frac{\pi\theta(1-\pi)(\gamma\theta+2)-2f}{2\pi\theta(1-\pi+\delta)} - \varphi K, & c \geq \bar{c} \\ f - \frac{(1-\pi)(\pi\theta((1-\pi)\gamma\theta+2\delta)+2f)^2}{8\pi\theta(1-\pi+\delta)^2} - \varphi K, & 0 \leq c < \bar{c}, K \geq \bar{K} \\ f - (1-\pi)K - \varphi K, & 0 \leq c < \bar{c}, \underline{\underline{K}} \leq K < \bar{K} \end{cases} \tag{A18}$$

Since when $\underline{\underline{K}} \leq K < \bar{K}$, $f - (1-\pi)K - \varphi K > f - \frac{(1-\pi)(\pi\theta((1-\pi)\gamma\theta+2\delta)+2f)^2}{8\pi\theta(1-\pi+\delta)^2} - \varphi K$. The comparison is mainly between $f\frac{\pi\theta(1-\pi)(\gamma\theta+2)-2f}{2\pi\theta(1-\pi+\delta)} - \varphi K$ and $f - (1-\pi)K - \varphi K$. Let $h(f, K)$ denote the difference between $f\frac{\pi\theta(1-\pi)(\gamma\theta+2)-2f}{2\pi\theta(1-\pi+\delta)} - \varphi K$ and $f - (1-\pi)K - \varphi K$.

$$\begin{aligned} h(f, K) &= f\frac{\pi\theta(1-\pi)(\gamma\theta+2)-2f}{2\pi\theta(1-\pi+\delta)} - \varphi K - (f - (1-\pi)K - \varphi K) \\ &= f\frac{\pi\theta((1-\pi)\gamma\theta-2\delta)-2f}{2\pi\theta(1-\pi+\delta)} + (1-\pi)K \end{aligned} \tag{A19}$$

Since $\underline{\underline{K}} \leq K < \bar{K}$, I could solve the inequality $h(f, K) \geq 0$ as $f \leq \underline{f}$ and $K \geq \hat{K}(\bar{c})$ where

$$
\begin{aligned}
\underline{f} &= \frac{(1-\pi)\pi\theta[2(1-\pi+\delta)+\gamma\theta+2]}{6-4\pi+4\delta} \\
\hat{K} &= f(1-\frac{(1+\alpha)c}{\pi\theta})
\end{aligned}
\tag{A20}
$$

■

Proposition 4

Proof. From proposition 3, I have optimal collateral with given capital K. Since $\hat{K} > 0$, the combination of $(\underline{c}, 0)$ will always give the highest CCP value. Hence, the optimal capital and collateral for a profit-driven CCP are

$$
K^* = 0, \qquad c^* = \underline{c} \tag{A21}
$$

■

Lemma 5

Proof. From proposition 3, I have optimal collateral with given capital K. I plug in the optimal collateral into equation 39.

When $f \leq \underline{f}$ and $K \geq \hat{K}(\bar{c})$, the optimal collateral is $\bar{c}$. No sellers default at $t = 1$ when the bad state is realized. The welfare surplus consists of two parts: the utility improvement from the non-default sellers and the CCP value. Since fee is a pure transfer from the traders to the CCP, it has no impact on the total welfare surplus directly. Hence, f disappears in the final expression. However, note that fee does matter for the thresholds on collateral and capital. Thus, the level of fee has indirect impact on the welfare surplus.

$$
\begin{aligned}
W &= \int_0^{\frac{(1+\alpha)\bar{c}}{\pi\theta}} \Delta U_{ND} dr_j + f v(\bar{c}) - \varphi K \\
&= W^{FB} - \underbrace{(1+\alpha)\delta\bar{c}\frac{(1+\alpha)\bar{c}}{\pi\theta}}_{\text{collateral cost}} - \underbrace{(1-\pi)\frac{(\pi\theta-(1+\alpha)\bar{c})^2}{2\pi\theta}}_{\text{not all traders trade}} - \underbrace{\varphi K}_{\text{capital cost}}
\end{aligned}
\tag{A22}
$$

When $f > \underline{f}$ and $K \geq \bar{K}(\bar{c})$, the optimal collateral is $[\bar{c}]^-$. The trading volume is one and some sellers default at $t = 1$. The welfare surplus is the sum of the utility improvement from the default and non-default sellers and the CCP value.

$$
\begin{aligned}
W &= \int_0^{\frac{(1+\alpha)\bar{c}}{\pi\theta}} \Delta U_{ND} dr_j + \int_0^{\frac{(1+\alpha)\bar{c}}{\pi\theta}} \Delta U_D dr_j + f - (1-\pi)\frac{(\pi\theta - (1+\alpha)c)^2}{2\pi\theta} - \varphi K \\
&= W^{FB} - \underbrace{(1+\alpha)\delta\bar{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}
\end{aligned} \tag{A23}
$$

When $f > \underline{f}$ and $\tilde{K}(\underline{c}) \leq K < \bar{K}(\bar{c})$, the optimal collateral is $\tilde{c}$. The same optimal collateral also applies to the case when $f \leq \underline{f}$ and $\tilde{K}(\underline{c}) \leq K < \hat{K}(\bar{c})$. In this case, the trading volume is one and some sellers default at $t = 1$. The utility improvement for the default sellers and their counterparties are the same as before; but that for the non-default sellers and their counterparties is different from the previous case because of the losses from default fund contribution.

$$
\begin{aligned}
W &= \int_0^{\frac{(1+\alpha)\tilde{c}}{\pi\theta}} \Delta U_{ND,M} dr_j + \int_0^{\frac{(1+\alpha)\tilde{c}}{\pi\theta}} \Delta U_D dr_j + f - (1-\pi)K - \varphi K \\
&= W^{FB} - \underbrace{(1+\alpha)\delta\tilde{c}}_{\text{collateral cost}} - \underbrace{\varphi K}_{\text{capital cost}}
\end{aligned} \tag{A24}
$$

When $\leq K < \tilde{K}(\underline{c})$, the optimal collateral is $\underline{c}$. In this case, the CCP becomes insolvent. The utility improvement for the default sellers and their counterparties is different because of the losses from partial insurance. For the non-default sellers and their counterparties, they loss all the default fund contribution and have a lower utility improvement as well. But the CCP in this case has a higher expected value. That is also why they will choose zero capital if there is no capital requirement.

$$
\begin{aligned}
W &= \int_0^{\frac{(1+\alpha)\underline{c}}{\pi\theta}} \Delta U_{ND,E} dr_j + \int_0^{\frac{(1+\alpha)\underline{c}}{\pi\theta}} \Delta U_{D,E} dr_j + f - (1-\pi)K - \varphi K \\
&= W^{FB} - \underbrace{(1+\alpha)\delta\underline{c}}_{\text{collateral cost}} - \underbrace{\left(1 - \frac{(1+\alpha)\underline{c}}{\pi\theta}\right)\frac{\gamma}{2}\pi(1-\pi)w^2}_{\text{loss from partial insurance}} - \underbrace{\varphi K}_{\text{capital cost}}
\end{aligned} \tag{A25}
$$

■

Proposition 5

Proof. When $0 \leq K < \tilde{K}(\underline{c})$, whether W is increasing or decreasing in K depends on how large the capital cost φ is. From equation 41, I have the total welfare surplus when $0 \leq K < \tilde{K}(\underline{c})$ as follows.

$$W = W^{FB} - (1+\alpha)\delta\underline{c} - \left(1 - \frac{(1+\alpha)\underline{c}}{\pi\theta}\right)\frac{\gamma}{2}\pi(1-\pi)w^2 - \varphi K \tag{A26}$$

Note that w is also a function of K as shown in equation 27. Plug in w and take the first order derivative of W w.r.t. K, I have the following.

$$\frac{\partial W}{\partial K} = \gamma\pi(1-\pi)w - \varphi \tag{A27}$$

Let $\bar{\bar{\varphi}} = \gamma\pi(1-\pi)w$. When $\varphi > \bar{\bar{\varphi}}$, W is decreasing in K. Hence, the optimal capital requirement is zero.

When $\varphi \leq \bar{\bar{\varphi}}$, W is increasing in K. Hence, for the total welfare surplus with default, the highest welfare surplus is achieved at $\tilde{K}(\underline{c})$. I know from proposition 3 that when $f > \underline{f}$, the CCP will have low collateral to maximize trading volume. Hence, for $\varphi \leq \bar{\bar{\varphi}}$ and $f > \underline{f}$, I have the optimal capital requirement $K^* = \tilde{K}(\underline{c})$. For the case when $f \leq \underline{f}$, I need to compare the welfare surplus when $K = \tilde{K}(\underline{c})$ and that when $K = \hat{K}(\bar{c})$. Let $l(f)$ denote the difference between these two welfare surplus. From equation 40, I have the following.

$$\begin{aligned} l(f) \; &= W^{FB} - (1+\alpha)\delta\bar{c}\frac{(1+\alpha)\bar{c}}{\pi\theta} - (1-\pi)\frac{(\pi\theta-(1+\alpha)\bar{c})^2}{2\pi\theta} - \varphi\bar{K}(\bar{c}) - (W^{FB} - (1+\alpha)\delta\underline{c} - \varphi\tilde{K}(\underline{c}) \\ &= (1+\alpha)\delta\underline{c} - (1+\alpha)\delta\bar{c}\frac{(1+\alpha)\bar{c}}{\pi\theta} - (1-\pi)\frac{(\pi\theta-(1+\alpha)\bar{c})^2}{2\pi\theta} + \varphi(\tilde{K}(\underline{c}) - \bar{K}(\bar{c})) \end{aligned} \tag{A28}$$

Solve $l(f) = 0$, I have

$$\begin{aligned} \underline{\underline{f}} \; &= \frac{\pi\gamma\theta^2(1-\pi)(\gamma\theta+2)(\gamma\theta+2+\alpha(\gamma\theta+6))}{4(\gamma\theta+1)(\gamma\theta+2+\alpha(\gamma\theta+4))} \\ &- \frac{\sqrt{(\pi\gamma\theta^2(1-\pi)(\gamma\theta+2)(\gamma\theta+2+\alpha(\gamma\theta+6)))^2 - 8\alpha\gamma\pi^2\theta^3(\gamma\theta+1)(\gamma\theta+2)^2(\gamma\theta+2+\alpha(\gamma\theta+4))}}{4(\gamma\theta+1)(\gamma\theta+2+\alpha(\gamma\theta+4))} \end{aligned} \tag{A29}$$

Hence, when $f \leq \underline{\underline{f}}$, $l(f) \geq 0$, which means the total welfare surplus without default is higher. The optimal capital requirement is $\hat{K}(\bar{c})$. When $f > \underline{\underline{f}}$, $l(f) < 0$. The optimal capital requirement is $\tilde{K}(\underline{c})$.

■