Government Debt, the Zero Lower Bound and Monetary Policy*

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Abstract

The financial crisis and the subsequent world-wide recession has led to a ballooning of government debt. This paper examines the implications of high government debt for optimal monetary policy in response to a large recessionary shock in a Blanchard-Yaari economy. In the model, the required risk premium on government debt is a function of the debt to GDP ratio and conventional monetary policy is constrained by the lower bound on the riskless short-term interest rate. We find that under the optimal policy the central bank reduces the risk premium on government debt to stabilise the economy. In the process, it expands its balance sheet and needs to rely less on forward guidance.

Keywords: Optimal monetary policy, zero lower bound, government debt, risk premia
JEL codes: E4, E5

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Contents

1. Introduction

The financial crisis that fully erupted following the failure of Lehman Brothers and the subsequent worldwide recession have triggered a rapid, large and at times coordinated response of monetary and fiscal authorities across the world. As a result, nominal short-term interest rates are close to their effective lower bound in the major industrial countries, government budget deficits and public debt have ballooned and central bank balance sheets have increased very significantly. On average public debt in the advanced economies is now reaching 100 percent of GDP, levels that are unprecedented in peace time.[1]

Rising government debt complicates monetary policy in a number of ways. First, to the extent that the necessary fiscal consolidation programmes have a negative short-term impact on economic activity and constrain an active use of fiscal policy including the automatic stabilizers, it puts a larger burden on monetary policy to stabilize the economy. This may not be straightforward, if standard monetary policy is constrained by the zero lower bound on nominal short-term interest rates. In that case, non-conventional measures including forward guidance and large-scale asset purchases may have to be used, but their effectiveness is uncertain.

Second, to the extent that long-term government debt is issued in nominal terms it increases the pressure to reduce the real burden of the debt and nominal entitlement programmes by unexpected inflation. It may also increase the pressure to rely on alternative sources of government finance such as seigniorage. These pressures risk undermining the credibility and the independence of the central bank to maintain price stability and may thereby give rise to higher inflation expectations.

Finally, the increasing riskiness of government debt may undermine the proper functioning of financial markets and the transmission of monetary policy. By reducing the value and quantity of safe collateral it may increase the price of risk and liquidity premia. Moreover, to the extent that government interest rates set a floor for the cost of financing of private firms and households, it increases the cost of private financing. Finally, a reduction in the value of government bonds will reduce the capital ratio of banks holding these government bonds and may thereby lead to a credit crunch as those banks try to adjust and deleverage.

In this paper, we focus on the first issue and examine the implications of high government debt for optimal monetary policy in response to a large recessionary shock in a Blanchard-Yaari economy in which the required risk premium on government debt is a function of the debt to GDP ratio and conventional monetary policy is constrained by the lower bound on the riskless short-term interest rate. Importantly, we assume that the central bank can credibly commit to minimizing a standard quadratic loss function in deviations of inflation

[1] See IMF (2011).

from an inflation objective and the output gap, and that the government is committed to ultimately stabilize the government debt to GDP ratio around a medium-term target. Our analysis is closely related to the analysis of Eggertsson and Woodford (2003) and Levin et al (2010) which investigate the implications of the zero lower bound for optimal monetary policy in the basic New Keynesian model. We extend this analysis in two directions.

First, the overlapping generations structure of the Blanchard-Yaari model implies that Ricardian equivalence no longer holds and that government debt and central bank money are net wealth for the private sector. Using a similar framework, Devereux (2011) emphasized that debt-financed fiscal spending may have significantly larger multiplier effects in particular when interest rates are constrained by the zero lower bound. In contrast to what happens in a Ricardian world, a debt or money-financed increase in lump-sum transfers will also be expansionary. The quantitative impact very much depends on the horizon of the overlapping generations and whether the short-term interest rate is constrained by the lower bound. However, without an endogenous risk premium, the effects of a classical open market operation whereby the central bank issues money to buy government bonds continue to be zero.[2]

Second, we allow for an endogenous risk premium on government debt. The risk premium is assumed to be a convex function of the debt to GDP ratio. A higher risk premium on government debt has a negative impact on the real economy and inflation because it increases the return on saving and reduces consumption. As a result, a rise in government debt may have a positive or negative impact on the real economy depending on whether the net wealth effect dominates the substitution effect. This will amongst others depend on the initial level of debt.

In this environment, we then investigate the implications of a large recessionary shock for the optimal monetary policy response. Importantly, we assume that fiscal policy is active in the sense that it does not attempt to stabilize government debt for the first two years of the recession. Under a classical Taylor rule, the central bank's interest rate hits the zero lower bound for about 8 quarters, government debt and risk premia rise substantially and output and inflation fall significantly. We then compare the Taylor rule with optimal monetary policy when the central bank can commit to minimizing a quadratic objective function in deviations of inflation from an inflation objective and variations of the output gap. As argued in Eggertsson and Woodford (2003), a policy geared at keeping interest rates low for longer can significantly stabilize the economy by increasing inflation expectations and lowering the real long-term interest rate. In our model with "risky" government debt, the central bank can alternatively reduce the risk premium on government debt by expanding

[2]See Eggertsson and Woodford (2003) and Curdia and Woodford (2011) for a discussion of the irrelevance result of traditional open market operations at the zero lower bound.

its balance sheet. We find that in our benchmark calibration optimal policy indeed uses the imperfect substitutability of the government debt to expand its balance sheet and stabilize the economy. It thereby needs to rely less on forward guidance. The extent to which the central bank relies on forward guidance depends on the risk premium and the initial level of debt. We also show that a Taylor rule which credibly and aggressively responds to the risk premium on government debt can closely replicate the optimal policy response. A policy that promises to keep interest rates low for longer (than the Taylor rule would suggest) can also stabilize the economy, but suffers from multiple equilibria.

There are various caveats to our analysis. First, as mentioned before the analysis is done under the assumption that, in the medium to long run, monetary policy pursues an inflation objective and the fiscal authorities credibly adjust primary balances to ultimately target a certain debt to GDP level. More importantly, however, in the short run the government does not stabilize the level of government debt while the nominal interest rate is at the zero lower bound. In other words, in Leeper's (1991) terminology the economy is operating in a passive monetary – active fiscal policy regime in the short run while it runs in an active monetary - passive fiscal policy regime in the medium to long run. We believe that in the short run the fall-out of the financial crisis has increased the probability of a switch to an active fiscal – passive monetary policy regime as interest rates are bound at zero and rising government debt has brought public finances closer to the fiscal limit, at least temporarily. However, in this paper, we retain the assumption that in the medium to longer run, the government stabilizes government debt and the central bank can achieve its inflation objective. It is beyond the scope of this paper to examine the implications of accounting for e.g. unfunded pension and other liabilities in the long term that may undermine the credibility of the central bank's price stability objective, see e.g. Leeper (2011). Second, we assume that the risk premium is an ad-hoc increasing function of the government debt to GDP ratio, but at the same time we do not allow for the possibility of an actual default on government debt. Finally, we assume that government's fiscal policy reaction function is independent of the central bank's policy response. Allowing for imperfect credibility of the long-run sustainability of debt, the possibility of default and strategic interaction between fiscal and monetary authorities may fundamentally change the optimal monetary policy prescriptions derived in this paper. We leave this for future research.

The paper is structured as follows. Section 3 presents the Blanchard-Yaari model. The calibration and parameterization of the model is discussed in section 4. The results are presented in section 5. Finally, we end the paper by summarizing our findings and the resulting policy implications in Section 6.

2. Government Debt and Monetary Policy in a Blanchard-Yaari model

In order to study the interaction between government debt and monetary policy, we develop a Blanchard-Yaari-type macro-economic model of overlapping generations along the lines of Devereux (2011).[3] As a special case, the model nests the standard model with infinitely lived households in which all generations are identical. We choose the Blanchard-Yaari model since it implies a departure from Ricardian equivalence. That is, changes in e.g. lump-sum transfers have real effects in contrast to the standard model. Oh and Reis (2011) have documented that across OECD countries, transfers to households have increased more than any other part of public spending in the great recession.[4] In the standard model, a debt-financed increase in transfers has no effects while it does in the Blanchard-Yaari framework. Below, we shall set up the model such that transfers to households rise substantially as part of a systematic fiscal policy response in the wake of a recession.

In the model, every period new households are born with a fraction $1-\delta$ of total population and die with a probability of $1-\delta$. Because households have no bequest motive, the overlapping generation nature of the population structure implies that government bonds and money are net wealth: The usual Ricardian equivalence in dynamic models with infinitely-lived households breaks down. A debt-financed increase in lump-sum transfers to households will have a positive effect on spending because a part of the government debt will be paid back by future generations. This makes the model particularly suitable for studying the impact of government debt on the economy.

Each household consumes a bundle of consumption goods, enjoys the benefits from holding money, supplies Labor and saves in the form of nominal government bonds or money holdings. There is no capital in the model. Money demand is assumed to be satiated at

[3] We extend the model presented in Devereux (2011) along several dimensions. First, we add money to the model by adopting a money-in-utility specification with a satiation point. Second, we introduce endogenous sovereign risk premia and distortionary taxation to the model. Third, we allow government bonds to be held by the public as well as by the central bank and thereby focus on the monetary-fiscal interactions more explicitely. Fourth, we study optimal monetary policy. Fifth, we examine Taylor rule based and unconventional monetary policies that results in allocations which are similar to those under optimal monetary policy. Sixth, we economize on the assumption of a monotonically decreasing labor productivity profile during each generation's lifetime. Although this is interesting per se, empirical evidence would suggest an inverted U-shape for the labor productivity profile during lifetime.

[4] An alternative framework that allows for deviations from Ricardian equivalence are models in which a share of households is liquidity constrained, see e.g. Coenen and Straub (2005) and Gali et al (2007). For our purposes, we believe that this framework is too restrictive since a common assumption in those models is that liquidity constrained households are infinitely lived and have no access to financial markets and thereby do not hold e.g. government debt. By contrast, in the Blanchard-Yaari environment, all households hold government debt. More importantly, the burden of repaying government debt is distributed unequally across generations. Younger generations typically bear most of that burden, i.e. repay debt issued in the past with higher taxes or reduced transfers.

a specific level of real money balances. Intermediate firms produce the differentiated consumption goods using Labor and set their prices in a monopolistic competitive market with price stickiness as in Calvo (1983). Price stickiness gives rise to a New Keynesian Phillips curve and implies that monetary policy has real effects in the short term.

We study the implications of two alternative specifications for the conduct of monetary policy in the model. First, we assume that the central bank pursues optimal policy by minimizing a loss function along the lines of Svensson (2011) and the references therein. Second, we assume that the central bank follows a Taylor rule when the short-term nominal interest rate is positive and reverts to a money supply rule at the zero lower bound. Both types of policies respect the zero lower bound explicitly. The monetary authority transfers part of its profits to the government and invests the other part in government bonds. The fiscal authority issues government bonds, raises distortionary Labor taxes and adjusts lump-sum transfers to households in order to target a 60 percent government debt to GDP ratio[5].

More importantly, we shall assume that the government pays a premium on the policy rate controlled by the central bank. The premium is a function of the deviation of total government debt to GDP from its steady state level of 60 percent. Our approach captures endogenous sovereign risk premia due to e.g. default risk in a mimimalistic way. We shall calibrate a sovereign risk premium along the lines of Laubach (2009) as well as Corsetti et al (2011).

In this framework, bond and money holdings enter the dynamic Euler equation of the households and will have real effects on the savings decisions of the households. For example, it turns out that the ratio of government debt and real money held by the households to GDP will have a positive impact on the steady state real interest rate. However, at the zero lower bound when money balances are satiated and in the absence of endogenous sovereign risk premia, a pure open market operation consisting of a swap of government bonds for money will have no impact on the economy.

In order to investigate the interaction of the zero lower bound on interest rates and the accumulation of government debt, we use a calibrated version of the Blanchard-Yaari model to simulate a great-recession type of shock. The calibration of the model aims at roughly mimicking the quantitative impact of the great recession in the euro area. We assume

[5]We do not address the optimal level of debt. Recently, Leith et al (2011) analyse the optimal level of public debt in a Blanchard-Yaari model. Another interesting recent paper is Adam (2011) who analyses the implications of nominal government debt for the optimal response to productivity shocks. In his framework, higher government debt requires lowering the average level of public spending and exposes fiscal budgets to increased risks following technology shocks or – more generally – fluctuations in the tax base. These budget risk considerations can provide quantitatively important incentives to reduce government debt over time. The results in this paper suggest that debt optimally converges to zero over time and that the optimal speed of debt reduction tends to increase if governments cannot adjust their spending plans following fluctuations in the tax base.

that the economy is hit by a large and persistent rise in the discount factor similar to e.g. Christiano, Eichenbaum and Rebelo (2011, CER henceforth). Such a rise in the discount factor can stand in for a tightening of credit constraints or increased precautionary saving due to a rise in uncertainty. In addition, in line with euro area data, we assume that in the model the debt to GDP ratio just before the onset of the great recession is at 70 percent compared to its steady state value of 60 percent.

2.1. Households

We adopt the specification of the Blanchard (1985) and Yaari (1965) model of perpetual youth in discrete time similar to Devereux (2011). Each period, households die with probability $1-\delta$. Futher, in each period a new generation j is born that represents a fraction $1-\delta$ of total population.[6] Thus, the size of generation j at time t is therefore: $(1-\delta)\delta^{t-j}$ while total population has measure 1. Households in each generation j maximize

$$\max_{c_t^j, M_t^j, B_t^{H,j}, n_t^j} E_0 \sum_{t=0}^{\infty} (\beta\delta)^t \sigma_{t-1} \left[\log c_t^j - \frac{\nu_t}{2} \left(\max\left\{ \frac{\overline{M_t^j}}{P_t} - \frac{M_t^j}{P_t}, 0 \right\} \right)^2 + A \log(1-n_t^j) \right]$$

subject to

$$P_t c_t^j + \frac{B_t^{H,j}}{R_t^{gov}} + M_t^j = (1-\tau_t) W_t n_t^j + \Theta_t^j + TR_t^j + \frac{1}{\delta} \left(B_{t-1}^{H,j} + M_{t-1}^j \right)$$

where c_t^j, M_t^j, n_t^j and $B_t^{H,j}$ are consumption, nominal money, hours worked and government bonds of households of generation j. $\overline{M_t}^j$ denotes the satiation level of money balances. P_t is the aggregate nominal price level. We assume a competitive labor market. The common nominal wage is denoted by W_t. Further, Θ_t^j are the share of profits of intermediate goods producers that go to generation j. Moreover, σ_{t-1} is a shock to utility, realized in the previous period.[7] TR_t^j are lump-sum transfers from the government to generation j households.

We assume that

$$R_t^{gov} = \gamma_t R_t$$

[6] Thus, average household lifetime is $\sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta}$. For conventional quarterly calibrations of δ, typically in the range of 0.95 to 0.99, the implied lifetime is small compared to the data. However, an alternative and empirically more plausible interpretation of $\frac{1}{1-\delta}$ is that it reflects the effective planning horizon of households. We shall adopt the planning horizon interpretation in this paper.

[7] In equilibrium, the ratio $\frac{\sigma_t}{\sigma_{t-1}}$ will be a shifter of the discount factor β in the Euler equation. That is, a positive realization of $\frac{\sigma_t}{\sigma_{t-1}}$ will induce a rise in the effective discount factor so that households want to save more. This will trigger a fall in consumption today and lead to a recession possibly implying a binding zero lower bound of nominal interest rates.

where γ_t drives a wedge between the nominal interest rate controlled by the central bank, R_t, and the nominal interest rate paid on government debt, R_t^{gov}. In other words, an increase of γ_t leads to a fall of the price of government debt which we will interpret as an increase in sovereign risk. We adopt the following functional form for γ:

$$\gamma_t = \max\left\{\exp\left(\varkappa\left[\frac{B_t^G}{4P_t y_t} - \frac{b^G}{4y}\right]\right), 1\right\}$$

where B_t^G denotes total government debt and y_t is aggregate output. Further, $\frac{b^G}{4y}$ denotes the annual debt to GDP ratio in steady state.

Similar to Blanchard (1995), we assume a full annuities market, i.e. a perfectly competitive life insurance industry. In that environment, borrowers pay a premium to cover their posthumous debt while savers get a premium on lending to cover their unintended bequests.[8]

The first order conditions at an interior solution can be written as:

$$\begin{aligned}
\frac{M_t^j}{P_t} &= \overline{\frac{M_t^j}{P_t}} - \left[\frac{R_t^{gov} - 1}{R_t^{gov}}\right]\frac{1}{\nu_t c_t^j} \\
\frac{A c_t^j}{1 - n_t^j} &= (1 - \tau_t)\frac{W_t}{P_t} \\
1 &= \beta\frac{\sigma_t}{\sigma_{t-1}} E_t\left[\frac{c_t^j}{c_{t+1}^j}\frac{R_t^{gov}}{\Pi_{t+1}}\right]
\end{aligned}$$

2.2. Aggregation

Aggregation implies the following relationship between a generation specific variable, say z_t^j, and its associated aggregate representation z_t:

$$z_t = \sum_{j=-\infty}^{t} (1-\delta)\delta^{t-j} z_t^j.$$

The appendix provides the details on the aggregation. Since we will study deterministic simulations below, we ignore Jensen's inequality as well as drop the expectation operator. The Euler equation in its aggregate representation reads as:

$$\beta\frac{\sigma_t}{\sigma_{t-1}} c_t \frac{R_t^{gov}}{\Pi_{t+1}} = \frac{1-\delta}{\delta\mu_{t+1}\Pi_{t+1}}\left[\frac{B_t^H}{P_t} + \frac{M_t}{P_t}\right] + c_{t+1}.$$

[8]Thus, full annuity markets imply that rates of return are grossed up to cover the probability of death. Put differently, households have no bequest motive. They sell contingent claims on their assets to perfectly competitive insurance companies. Assets from the $(1-\delta)$ exiting households are transferred to all non-exiting and newborn households. Hence, each surviving generation receives a premium payment, per unit of asset, of $(1-\delta)/\delta$. Therefore, the gross return on the insurance contract is $1 + (1-\delta)/\delta = 1/\delta > 1$ which is the factor multiplying asset income per household.

with $\mu_t = 1 + \frac{\sigma_t}{\sigma_{t-1}}\delta\beta\mu_{t+1}$. Note that for $\delta < 1$, government debt and money held by households represent net wealth and thereby affect consumption spending. Moreover, observe that in steady state, $R^{gov} = \frac{\Pi}{\beta} + \frac{(1-\delta)(1-\delta\beta)}{\delta\beta}\left[\frac{b^H}{y} + \frac{m}{y}\right]$. This implies that more debt held by the public results in a higher nominal interest rate in order to maintain a given inflation rate.

The labor-leisure trade-off can be written as:

$$\frac{Ac_t}{1-n_t} = (1-\tau_t)\frac{W_t}{P_t}.$$

The money demand equation can be written as:

$$\frac{M_t}{P_t} = \frac{\overline{M_t}}{P_t} - \left[\frac{R_t^{gov}-1}{R_t^{gov}}\right]\frac{1}{\nu_t c_t}.$$

Note that for $R_t^{gov} = 1$ it follows that $M_t = \overline{M_t}$, i.e. nominal money holdings attain the satiation level.[9] Finally, the aggregate budget constraint is given by:

$$P_t c_t + \frac{B_t^H}{R_t^{gov}} + M_t = (1-\tau_t)W_t n_t + \Theta_t + TR_t + B_{t-1}^H + M_{t-1}.$$

2.3. Final Goods Firms

Competitive final goods firms maximize profits

$$\Pi_{t,i}^f = \max_{y_{t,i}}\left[P_t\left(\int y_{t,i}^{\frac{1}{\omega}}\,di\right)^{\omega} - \int P_{t,i}y_{t,i}di\right]$$

subject to the Dixit-Stiglitz production function $y_t = \left(\int y_{t,i}^{\frac{1}{\omega}}\,di\right)^{\omega}$ with $\omega > 1$. Optimality implies the standard input demand function, $y_{t,i} = \left(\frac{P_t}{P_{t,i}}\right)^{\frac{\omega}{\omega-1}} y_t$ where the aggregate input price index is given by $P_t = \left(\int P_{t,i}^{\frac{1}{1-\omega}}di\right)^{1-\omega}$. Note too, that $\int \Pi_{t,i}^f\,di = \Pi_t^f = 0$ in equilibrium

2.4. Intermediate Good Firms

Intermediate goods firms are monopolistically competitive. It is useful to consider two cases: i) flexible prices and ii) Calvo sticky prices.

[9] Due to the quadratic form for real money balances in the utility function, the generation specific money demand function is non-linear in consumption. In the appendix we derive an approximate aggregate money demand equation based on a first order Taylor series expansion. Note that all other relationships in the model are aggregated exactly.

2.4.1. Flexible Prices

Under flexible prices, profit maximization solves

$$\Pi^I_{t,i} = \max_{P_{t,i}} \left[P_{t,i} y_{t,i} - (1-\chi_t) W_t n_{t,i}\right]$$

subject to

$$y_{t,i} = \left(\frac{P_t}{P_{t,i}}\right)^{\frac{\omega}{\omega-1}} y_t \text{ and } y_{t,i} = n_{t,i}$$

where χ_t denotes a subsidy. Note that marginal costs are given by $MC_t = (1-\chi_t)W_t$. Optimality implies:

$$P_{t,i} = \omega MC_t = \omega(1-\chi_t)W_t.$$

Hence, in equilibrium, all firms set the same price $P_{t,i} = P_t$. Accordingly, $y_{t,i} = y_t$, $n_{t,i} = n_t$ and $y_t = n_t$. Therefore, aggregate equilibrium profits are given by $\int \Pi^I_{t,i}\, di = \Pi^I_t = (\omega - 1)(1-\chi_t)W_t y_t$.

2.4.2. Sticky Prices

Following Calvo (1983), firms may set an optimal price with probability $1-\xi_p$. Conversely, with probability ξ_p, firms have to keep last periods updated by steady state inflation Π. In this case, profit maximization solves:

$$\max_{\tilde{P}_t}\; E_0 \sum_{j=0}^{\infty} \left(\xi_p \beta \delta\right)^j \Lambda_{t+j} \left[\Pi^j \tilde{P}_t y_{t+j,i} - MC_{t+j} y_{t+j,i}\right]$$

subject to

$$y_{t+j,i} = \left(\frac{P_{t+j}}{\Pi^j \tilde{P}_t}\right)^{\frac{\omega}{\omega-1}} y_{t+j}.$$

Similar to e.g. Christiano, Trabandt and Walentin (2011), optimal price setting can be expressed by the following three recursive non-linear equations, see the appendix for the details:

$$\begin{aligned}
K_t &= \lambda_t y_t \omega mc_t + \xi_p \beta \delta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{\omega}{\omega-1}} K_{t+1} \\
F_t &= \lambda_t y_t + \xi_p \beta \delta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{1}{\omega-1}} F_{t+1} \\
\frac{K_t}{F_t} &= \left[\frac{1-\xi_p \left(\frac{\Pi_t}{\Pi}\right)^{\frac{1}{\omega-1}}}{1-\xi_p}\right]^{1-\omega}
\end{aligned}$$

where $mc_t = \frac{MC_t}{P_t}$ denotes real marginal cost and λ_t is the Lagrange multiplier on the aggregate household budget constraint. In linearized form, these three equations can be combined to obtain the standard New Keynesian Phillips curve.

In the presence of shocks, price dispersion arises among intermediate goods producers. Formally, let $1/\mathring{p}_t$ denote a measure of price dispersion. The appendix derives the following recursive representation:

$$\mathring{p}_t^{\frac{\omega}{1-\omega}} = (1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\Pi}{\Pi_t}\right)^{\frac{1}{1-\omega}}}{1-\xi_p}\right)^{\omega} + \xi_p\left(\frac{\Pi}{\Pi_t}\mathring{p}_{t-1}\right)^{\frac{\omega}{1-\omega}}.$$

Further, there is a relationship between aggregate inputs and output, that takes losses in terms of aggregate output due to price dispersion among intermediate goods producers into account:

$$y_t = \mathring{p}_t^{\frac{\omega}{\omega-1}} n_t.$$

2.5. Government

The government is subject to the following budget constraint:

$$B^G_{t-1} + TR_t + \chi_t W_t \int y_{t,i} = \frac{B^G_t}{R^{gov}_t} + \tau_t W_t \int n_{t,i} + S_t$$

where B^G_t denotes total debt issued by the government. S_t is a transfer received from the central bank. We assume that the distortionary labor income tax rate is constant over time. More importantly, we shall assume that transfers to households, TR_t, adjust to balance the budget according to the following rule:

$$\frac{TR_t}{P_t} = tr - \theta_{TRB,t}\left(\frac{B^G_{t-1}}{P_{t-1}} - b^G\right) - \theta_{TRY}\left(\frac{y_t}{y} - 1\right)$$

where tr denotes steady state real transfers and b^G is real total government debt in steady state. The fiscal rule consists of a debt stabilizing part and a part that we assume to be a stand in for automatic stabilizers. Below we shall assume that in the wake of a large shock that drives the economy into a deep recession, the debt stabilizing part is switched off temporarily. That is, $\theta_{TRB,t} = 0$ for some $t = 0, .., T$ and $\theta_{TRB,t} > 0$ if $t > T$. This setup resembles a regime in which fiscal policy is active, i.e. does not stabilize government debt. Further, we assume that $\theta_{TRY} > 0$ throughout. As a result, automatic stabilizers lead to an increase in transfers and thereby fuel the buildup of government debt in addition to the shortfall in revenues in the wake of a recession.

Finally, note that lump-sum transfers in our model have real effects due to the overlapping generation structure of the model. In contrast to the standard infinitely lived representative agent framework, a debt financed increase of transfers during the recession increases consumption in our model. Households take into account that they may have exited the economy already at the time when the government reduces future transfers to repay the debt.

2.6. Central Bank

The central bank faces the following budget constraint:

$$\frac{B_t^M}{R_t^{gov}} + S_t = B_{t-1}^M + M_t - M_{t-1}$$

where B_t^M denotes sovereign debt held by the central bank. S_t denotes a transfer from the central bank to the government which is set according to the following rule:

$$\frac{S_t}{P_t} = s + \theta_C \left(\frac{B_t^M}{P_t} - b^M \right)$$

where s are steady state transfers from the central bank to the government and b^M is government debt held by the central bank in steady state.

We study the implications of two alternative specifications for the conduct of monetary policy in the model. First, we assume that the central bank pursues optimal policy by minimizing the following loss function along the lines of Eggertsson and Woodford (2003) or Svensson (2011) and the references therein:[10]

$$L = \min \ \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[(\Pi_t - \Pi)^2 + \lambda \left(\frac{y_t}{y} - 1 \right)^2 \right]$$

subject to the private and public sector equilibrium equations as well as subject to the zero lower bound constraint, $R_t \geq 1$.

Second, we assume that the central bank follows a Taylor rule, subject to the zero lower bound contraint:

$$R_t = \max \left\{ R + \phi_\pi (\Pi_t - \Pi) + \phi_y \left(\frac{y_t}{y} - 1 \right), 1 \right\}.$$

We assume that the zero lower bound is binding for the policy rate even though we have not modelled an explicit asset market that trades at R_t when risk premia arise. One interpretation of this assumption is that the economy simply looses its safe asset with positive

[10] An alternative criterion function would be a social welfare function that summarizes the utility functions of all generations in the economy. In order to facilitate comparision with e.g. Eggertsson and Woodford, we leave this alternative approach for future research.

government bond risk premia. This case resembles closely a situation when the monetary policy transmission channel is severely disturbed so that monetary policy becomes less effective.

As an alternative to assuming a binding zero lower bound for R_t in the presence of risk premia, we could have adapted our model suitably to allow for segmented asset markets. One asset market would trade government bonds as before. In an additional market, arbitrage monetary financial institutions (MFI's) would be assumed to either hold money or reserves at the central bank with the latter paying gross interest R_t. In this case, R_t would have a zero lower bound since MFI's would be able to have unbounded profits if $R_t < 1$. Since none of our results would be affected by this modelling complication, we shall proceed with our analysis.

2.7. Equilibrium

In equilibrium, all markets clear. It is straightforward to show that, by consolidating the households, fiscal and central bank budget constraints, the aggregate resource constraint becomes $c_t = y_t$. See the appendix for the details.

2.7.1. Monetary Policy follows Taylor Rule

The equilibrium when the central bank follows a Taylor rule can be summarized as follows:

$$
\begin{aligned}
\text{Bond Market Clearing (e1)} &: \quad b_t^G = b_t^M + b_t^H \\
\text{Central Bank Budget (e2)} &: \quad \frac{b_t^M}{R_t^{gov}} + s_t = \frac{b_{t-1}^M}{\Pi_t} + m_t - \frac{m_{t-1}}{\Pi_t} \\
\text{Transfer from CB to Gov. (e3)} &: \quad s_t = s + \theta_C \left(b_t^M - b^M \right) \\
\text{Government Budget (e4)} &: \quad \frac{b_{t-1}^G}{\Pi_t} + tr_t = \frac{b_t^G}{R_t^{gov}} + \tau w_t n_t + s_t \\
\text{Fiscal Rule for Transfers (e5)} &: \quad tr_t = tr - \theta_{TRB,t} \left(b_{t-1}^G - b^G \right) - \theta_{TRY} \left(\frac{y_t}{y} - 1 \right) + \varepsilon_t \\
\text{Leisure/Labor Trade-off (e6)} &: \quad \frac{A y_t}{1 - n_t} = (1 - \tau) w_t \\
\text{Gov. Bond Interest Rate (e7)} &: \quad R_t^{gov} = \gamma_t R_t \\
\text{Sovereign Risk Premium (e8)} &: \quad \gamma_t = \max \left\{ \exp \left(\varkappa \left[\frac{b_t^G}{4 y_t} - \frac{b^G}{4y} \right] \right), 1 \right\} \\
\text{Euler Equation Bonds (e9)} &: \quad \beta \frac{\sigma_t}{\sigma_{t-1}} y_t \frac{R_t^{gov}}{\Pi_{t+1}} = \frac{1 - \delta}{\delta \mu_{t+1} \Pi_{t+1}} \left[b_t^H + m_t \right] + y_{t+1} \\
\text{Recursive Discounting (e10)} &: \quad \mu_t = 1 + \frac{\sigma_t}{\sigma_{t-1}} \delta \beta \mu_{t+1} \\
\text{Real Money Demand (e11)} &: \quad m_t = \overline{m} - \left[\frac{R_t^{gov} - 1}{R_t^{gov}} \right] \frac{1}{\nu_t y_t} \\
\text{Optimal Price Setting 1 (e12)} &: \quad F_t = \sigma_{t-1} + \delta \xi_p \beta \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\frac{1}{\omega - 1}} F_{t+1} \\
\text{Optimal Price Setting 2 (e13)} &: \quad K_t = \sigma_{t-1} \omega w_t + \delta \xi_p \beta \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\frac{\omega}{\omega - 1}} K_{t+1} \\
\text{Optimal Price Setting 3 (e14)} &: \quad \frac{K_t}{F_t} = \left[\frac{1 - \xi_p \left(\frac{\Pi_t}{\Pi} \right)^{\frac{1}{\omega - 1}}}{1 - \xi_p} \right]^{1 - \omega} \\
\text{Inv. Price Dispersion (e15)} &: \quad \mathring{p}_t^{\frac{\omega}{1-\omega}} = \left(1 - \xi_p \right) \left(\frac{1 - \xi_p \left(\frac{\Pi_t}{\Pi} \right)^{\frac{1}{\omega - 1}}}{1 - \xi_p} \right)^{\omega} + \xi_p \left(\frac{\Pi}{\Pi_t} \mathring{p}_{t-1} \right)^{\frac{\omega}{1-\omega}} \\
\text{Production (e16)} &: \quad y_t = n_t \mathring{p}_t^{\frac{\omega}{\omega - 1}} \\
\text{Taylor Rule (e17)} &: \quad R_t = \max \left\{ R + \phi_\pi \left(\Pi_t - \Pi \right) + \phi_y \left(\frac{y_t}{y} - 1 \right), 1 \right\}
\end{aligned}
$$

Note that the ratio $\frac{\sigma_t}{\sigma_{t-1}}$ is exogenous and subject to shocks.

2.7.2. Optimal Monetary Policy

As an alternative, we assume that the central bank minimizes the following loss function:

$$\min \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[(\Pi_t - \Pi)^2 + \lambda \left(\frac{y_t}{y} - 1 \right)^2 \right]$$

subject to equations (e1) to (e16) and the zero lower bound constraint $R_t \geq 1$. Following Woodford (2003), we assume that the central bank acts under commitment and takes a so-called "timeless" perspective. We solve for the optimal first order conditions as well as for the fully non-linear equilibrium paths in response to a large discount factor shock that triggers the zero lower bound to bind. That is, the Lagrange multiplier of the zero lower bound constraint will be binding for several periods before the exit occurs, say in period $t = T$. We employ standard numerical methods to determine T and to solve for the non-linear system of equations for the periods $t \leq T$ and $t > T$.[11]

3. Calibration and Steady State

Tables 1 and 2 provide an overview of the calibration and parameterization of the model. Time is discrete and taken to be quarters. Our calibration is geared toward the Euro Area. We set the discount factor $\beta = 0.999$ and steady state inflation to 1.9 percent. Together with the parameters discussed below, this implies a real interest rate of about 1.5 percent and a nominal interest rate of roughly 3.5 percent. The survival probability of households, δ, is set to 0.97. Our choice represents an intermediate value between Devereux (2011) who uses a value of $\delta = 0.945$ and the case of $\delta = 1$ which implies representative and infinitely lived households. The Calvo price stickiness parameter, ξ_p, is set to 0.95 which implies a rather flat Phillips curve. Our choice is motivated by at least two reasons. First, the value is in the ballpark of the reported estimates of the ECB's New Area-Wide Model, see Coenen et al (2008) and the Smets and Wouters (2003) model for the euro area. Second, inflation did not fall much during the great recession. A substantially lower value of ξ_p would imply a fall of inflation in the model which would be counterfactually strong compared to the data. Further, we set the gross steady state markup, ω, equal to 1.35 also in line with Coenen et al. (2008).

In terms of monetary policy, we set the reaction coefficients for inflation and output equal to standard values of ϕ_π =1.5 and ϕ_y =0.5. In case of optimal monetary policy, we assume a weight of output deviations in the loss function of $\lambda = 0.001$. In steady state, annual transfers of the central bank to the government are about 0.1 percent of GDP. Moreover, we assume a relatively small reaction coefficient $\theta_C = 0.01$ for the rule that the central bank

[11] The appendix provides an example for a much simpler and linearized version of our model.

uses to determine transfers to the government outside the steady state. Finally, we assume that $\chi_t = 0 \ \forall t$.

In terms of fiscal policy, we assume a steady state labor income tax, $\tau = 0.5$ which is in line with the total tax wedge on labor due to labor and consumption taxes reported in Uhlig and Trabandt (2011, 2012). Moreover, we set the annual total government debt to GDP ratio, $b^G/y = 0.6$ and assume that debt held by the public as a share of GDP equals $b^H/y = 0.5$. Accordingly, the central bank holds 10 percent of total government debt in terms of GDP in our model in steady state. As a result, annual government transfers in steady state amount to roughly 9 percent of GDP. In the simulations, we shall assume that the debt stabilizing part of the fiscal rule, i.e. the coefficient $\theta_{TRB,t} = 0$ for $t = 0, .., 7$, i.e. for the first two years after the onset of the recession. In other words, fiscal policy is active during this period. Thereafter, we shall assume $\theta_{TRB} = 0.1$, similar to Coenen et al (2008). We set the feedback rule coefficient on output equal to $\theta_{TRY} = 0.45$. As discussed in the previous section, we interpret this feedback coefficient as a stand in for automatic stabilizers. We have chosen this particular value in order to match the peak response of government debt to GDP of about 90 percent in the baseline simulations, see the next section.

We set the level parameter of utility from real money balances, $\nu = 0.1$ and the annual steady state real money to GDP ratio, m/y equal to 0.25. Together, these values imply a satiation level of real money balances to GDP of about 45 percent.

In addition, we assume that households work one third of their total time endowment in steady state, i.e. $n = 1/3$. Furthermore, we set subsidies to firm's marginal cost equal to zero in steady state as well as dynamically, i.e. $\chi = 0 \ \forall \ t$.

Similar to CER (2011), we assume a two percent increase of the household discount factor. In our model, this corresponds to a two percent increase of the ratio $\frac{\sigma_t}{\sigma_{t-1}}$ initially. Reflecting, euro area savings rate dynamics during the crisis, we assume a that the ratio $\frac{\sigma_t}{\sigma_{t-1}}$ reverts back to the steady state with a first order autoregressive coefficient of 0.8. In addition, in line with euro area data, we assume that in the model the debt to GDP ratio just before the onset of the great recession is at 70 percent compared to its steady state value of 60 percent.

Finally, we assume a slope coefficient for the sovereign risk premium of $\varkappa = 0.025$. Our choice implies that in response to a one percentage point increase of government debt to GDP from 60 percent, the interest rate on government bonds increases by about 10 annual basis points. This sensitivity of the risk premium is within the range of estimates provided by e.g. Laubach (2009) of 4 annual basis points based on US data and Corsetti et al (2011) of 15 annual basis points based on a cross section of OECD countries.

4. Results

4.1. Baseline Results

Figure 1 plots the economy's response to the discount factor shock in the baseline version of the model. Consider the equilibrium when monetary policy follows a Taylor rule. The shock leads to a fall in real GDP of about 8 percent and a more muted drop in inflation towards zero. As a result, the central bank lowers the short-term nominal interest rate to the effective lower bound, where it stays for about 8 quarters. The drop in interest rates leads to a rise in money demand which is accommodated by the central bank. The fall in output reduces Labor tax revenues and increases government transfers. As a result, the government deficit rises and government debt increases by about 20 percentage points of GDP compared to its initial value.[12] Note that due to the increase of government debt and the presence of the endogenous sovereign risk premium, the interest rate on government debt rises relative to the policy rate controlled by the central bank. Higher risk premia in turn reduce output and increase government debt even more. Note that as a result, the central bank keeps its interest rate at the lower bound for longer than without sovereign risk premia. Figure A9 in the appendix contains the details.[13]

Figure 1 also contains the allocations when the central bank pursues optimal monetary policy under the zero lower bound constraint.[14] Output and inflation do not fall as much as under the Taylor rule. It turns out to be optimal to reduce the implied risk premium on government bonds during the recession. This in turn triggers a substantial increase of real money balances. It is optimal to accommodate this demand by expanding the central bank balance sheet by acquiring government debt in exchange for real money balances. Interestingly, the exit date from the zero lower bound is similar than the one under the Taylor rule equilibrium. Eggertsson and Woodford (2003) have emphasized the importance of "forward guidance", i.e. optimal policy keeps nominal interest rates longer at the zero lower bound in order to stimulate the economy to lessen the fall of output in the recession. The presence of the sovereign risk channel in our model appears to reduce the scope or necessity of forward guidance substantially. More precisely, in our model with "risky" government debt, the central bank can alternatively reduce the risk premium on government debt by expanding

[12] Section A in the appendix discusses the evolution of the crisis based on key macroeconomic data for the euro area as well as for the UK, Japan and the US. Figures A1 to A8 contain a graphical representation. It turns out that our stylized model accounts well for most of the features of the data, at least qualitatively if not quantitatively.

[13] We have also examined the implications of Blanchard-Yaari vs. infinitely lived households. Figure A10 in the appendix shows the dynamics in response to the discount factor shock. The figure reveals interesting differences between the two specifications. In particular, the positive impact of the rise in government debt on consumption and thereby output under the Blanchard-Yaari OLG structure appears to tbe quite powerful.

[14] Note that if the zero lower bound is ignored, full stabilization of inflation and output is attainable by setting the nominal interest rate to a sufficiently negative value.

its balance sheet. We find that in our benchmark calibration optimal policy indeed uses the imperfect substitutability of the government debt to expand its balance sheet and stabilize the economy. It thereby needs to rely less on forward guidance. If anything, even though the exit date of optimal and Taylor rule based policies are identical, optimal policy appears to return the nominal interest rate faster to the steady state.

Finally, note that we have verified that if we shut off the sovereign risk channel in our model, the standard Eggertsson and Woodford result re-emerges, see Figure A9 in the appendix for the details.

4.2. Reaction to Long-Term Sovereign Spread

A natural question that arises after examining Figure 1 is: what does it take to implement - or at least come close to - the allocations that result under the prescriptions of optimal monetary policy?

From Figure 1 it is clear that the response of the government bond interest rate is quite different under Taylor and optimal policy. In the first case, the implied spread between the nominal 10 year government bond rate and the 10 year implied policy rate increases on impact by about 1.4 percentage points and stays persistently high for many quarters. In contrast, under optimal policy the spread increase is much more modest. It is therefore worth investigating whether adding a systematic response to the long-term interest rate spread in the Taylor rule allows to come closer to the equilibrium allocations under optimal policy.

Formally, we augment the Taylor rule as follows:

$$R_t = \max\left\{R + \phi_\pi\left(\Pi_t - \Pi\right) + \phi_y\left(\frac{y_t}{y} - 1\right) + \phi_s\Theta_t, 1\right\}$$

where

$$\Theta_t = \left[\prod_{i=0}^{39} R_{t+i}^{gov}\right]^{\frac{1}{40}} - \left[\prod_{i=0}^{39} R_{t+i}\right]^{\frac{1}{40}}.$$

Figure 2 provides the results for the drastic case when $\phi_s = -10000$. It turns out that optimal and Taylor rule based allocations are indeed very similar in this case. Although interest rates are constrained by the zero lower bound during the first eight quarters, the credible threat to continue to keep interest rates low in response to high sovereign spreads has a powerful impact on the current spread and sets in motion a positive spiral whereby lower spreads stimulate the economy and reduce the accumulation of government debt, which in turn allows for lower spreads. In this process, both real money demand by the household sector and government debt held by the central bank significantly expand compared to the Taylor rule without a response to the sovereign spread. In equilibrium, there is no need to keep interest rates low for longer than under the simple Taylor rule because spreads have fallen to very low levels after eight quarters.

Figure 3 provides a sensitivity analysis with respect to alternative values of ϕ_s. Note that for any value $\phi_s < 0$, the recession is less severe and the inflation response is more muted. By contrast to the output and inflation responses, the duration of the zero lower bound is not monotonic in ϕ_s. For low values of ϕ_s, the exit occurs earlier, but the interest rate increase after exit from the zero lower bound is very gradual. In contrast, for higher values of ϕ_s the exit occurs later, but the exit is much steeper. Overall, a credible and aggressive reaction to the long-term spread appears to work quite effectively in reducing the impact of the initial shock.

4.3. Commitment to Longer ZLB

Non-conventional monetary policies may be an alternative policy instrument to reduce the output costs of a rise in government debt in the Taylor rule equilibrium. Figure 4 takes the scenario with optimal monetary policy as a baseline and shows the effects of a credible central bank commitment to keep policy rates at zero for various additional quarters longer than implied by the standard Taylor rule (i.e. $\phi_s = 0$). More precisely, we assume that the central bank commits to keep the nominal interest rate at zero for 8, 9, 10, etc quarters after the wake of the recessionary shock. As a result of this policy compared with the baseline case, long-term interest rates fall by considerably more and output and inflation by significantly less. These beneficial effects on the economy imply a much less pronounced increase in government debt and a much smaller rise in the risk premium in turn contributing to a smaller recession. Of course, these quite powerful results depend on the credibility of the commitment to keep interest rates low for longer.

Interestingly, the same promise to keep interest rates low for longer (than the Taylor rule would suggest) can also destabilize the economy. Figure 5 shows that the equilibrium equations also satisfy allocations in which output falls much more compared to our baseline optimal policy results. In other words, the commitment under a Taylor rule appears to suffer from multiple equilibria, at least numerically so far. This equilibrium appears to result in dynamics of the sort: "if the central bank keeps interest rates at zero that long - the recession must be *really* bad". In other words, the beneficial effects of the announcement may be easily overturned based on non-fundamental features such as self-fulfilling beliefs.

4.4. A Money Boost at the ZLB

Another difference between the equilibrium outcome under the Taylor and optimal monetary policy in Figure 1 is the much larger purchases of government debt by the central bank and the increase in real money under optimal policy. It is therefore worth exploring an alternative monetary policy which boosts the money supply by buying government debt when interest rates hit the lower bound.

In the spirit of Eggertsson and Woodford (2003), we assume that at the zero lower bound, the central bank reverts to a money supply rule.[15] That is, it supplies any quantity of money demanded. In addition, the central bank may find it useful to inject further money at the zero lower bound, i.e. "a money boost". Clearly, at the zero lower bound and in the absence of sovereign risk premia, $R_t^{gov} = R_t = 1$, money demand attains its satiation level $\overline{\frac{M_t}{P_t}}$. Hence, a money boost by the central bank is ineffective in this case in our specification. However, in the presence of sovereign risk premia and rising government debt, it is possible that $R_t^{gov} > R_t = 1$ and thereby money demand falls with rising government debt. In this case, the central bank might want to inject money. Since Blanchard-Yaari households consider the increase in money balances as partly net wealth, they start spending more and thereby reduce the severity of the recession with further beneficial effects for e.g. sovereign risk premia. In the next version of this paper, we shall simulate the model under this policy and provide the quantitative results.

5. Conclusions

The financial crisis and the subsequent world-wide recession has led to a ballooning of government debt. This paper examines the implications of high government debt for optimal monetary policy in response to a large recessionary shock in a Blanchard-Yaari economy in which the required risk premium on government debt is a function of the debt to GDP ratio and conventional monetary policy is constrained by the lower bound on the riskless short-term interest rate. We find that under the optimal policy the central bank reduces the risk premium on government debt to stabilize the economy. In the process, it expands its balance sheet and needs to rely less on forward guidance.

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Tables and Figures

Table 1: Parameters and Imposed Steady States

Parameter	Value	Description
β	0.999	Discount factor
δ	0.97	Survival probability of households
ξ_p	0.95	Calvo price stickiness
ω	1.35	Gross price markup
ϕ_π	1.5	Taylor rule coefficient on inflation
ϕ_y	0.5	Taylor rule coefficient on output
λ	0.001	Weight on output in loss function
θ_C	0.01	CB to gov. transfer rule coefficient
θ_{TRB}	0 or 0.1	Gov. to households transfer rule coef.
θ_{TRY}	0.45	Gov. to households transfer rule coef.
ν	0.1	Level parameter utility of real money
$\varkappa$	0.025	Slope coefficient sovereign risk premium
ρ_σ	0.8	AR(1) of discount factor shock
ε_σ	2	Initial shock to discount factor, in percent
		Imposed steady states
τ	0.5	Distortionary tax rate (tax wedge on labor)
π	1.9	Annual inflation rate
m/y	0.25	Annual money to GDP ratio
b^G/y	0.6	Annual total gov. debt to GDP ratio
b^H/y	0.5	Annual gov. debt held by public to GDP ratio
n	1/3	Hours worked
χ	0	Subsidy to firms
σ/σ_{-1}	1	Discount factor shock

Table 2: Steady States and Implied Parameters for Different Households (HH)

Variable	Infinitely lived HH ($\delta = 1$)	Blanchard/Yaari HH ($\delta = 0.97$)	Description
r	2.3	3.45	Nominal interest rate
$\bar{m}/y$	0.3814	0.4465	Satiation: annual money to GDP ratio
tr/y	0.0933	0.0919	Annual transfer to GDP ratio
w	0.74		Real wage
A	0.75		Level parameter disutiliy of labor
y	0.33		Real GDP
b^M/y	0.1		Annual gov. debt to GDP ratio held by CB
s/y	0.001		Annual CB transfers to gov. as ratio to GDP

Figure 1: Baseline Results – OLG, Endog. Sovereign Risk Premium, Responses to Discount Factor Shock

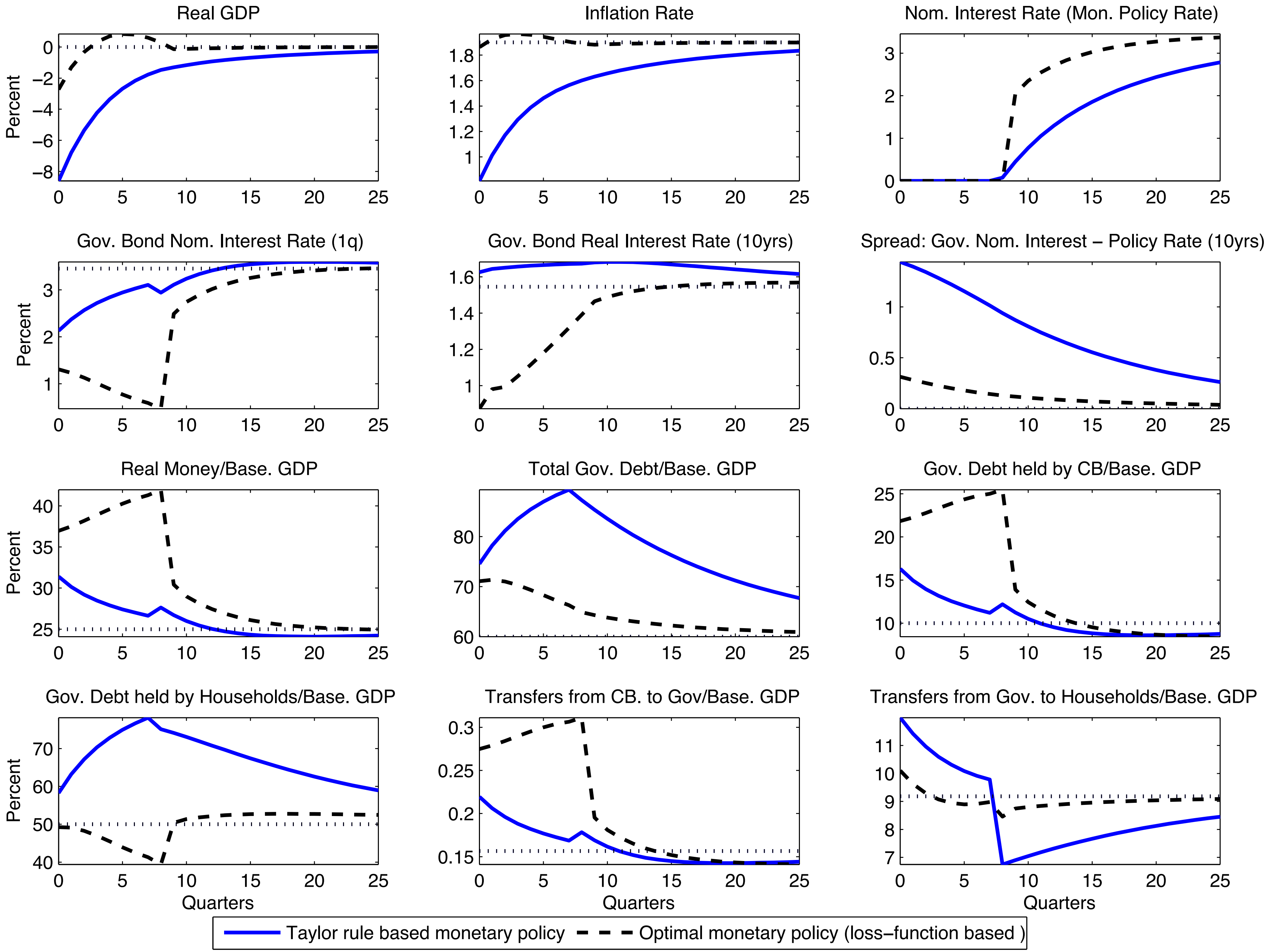


Figure 2: Sovereign Spread (10yrs) in Taylor rule – OLG, Endog. Sovereign Risk Premium, Responses to Discount Factor Shock

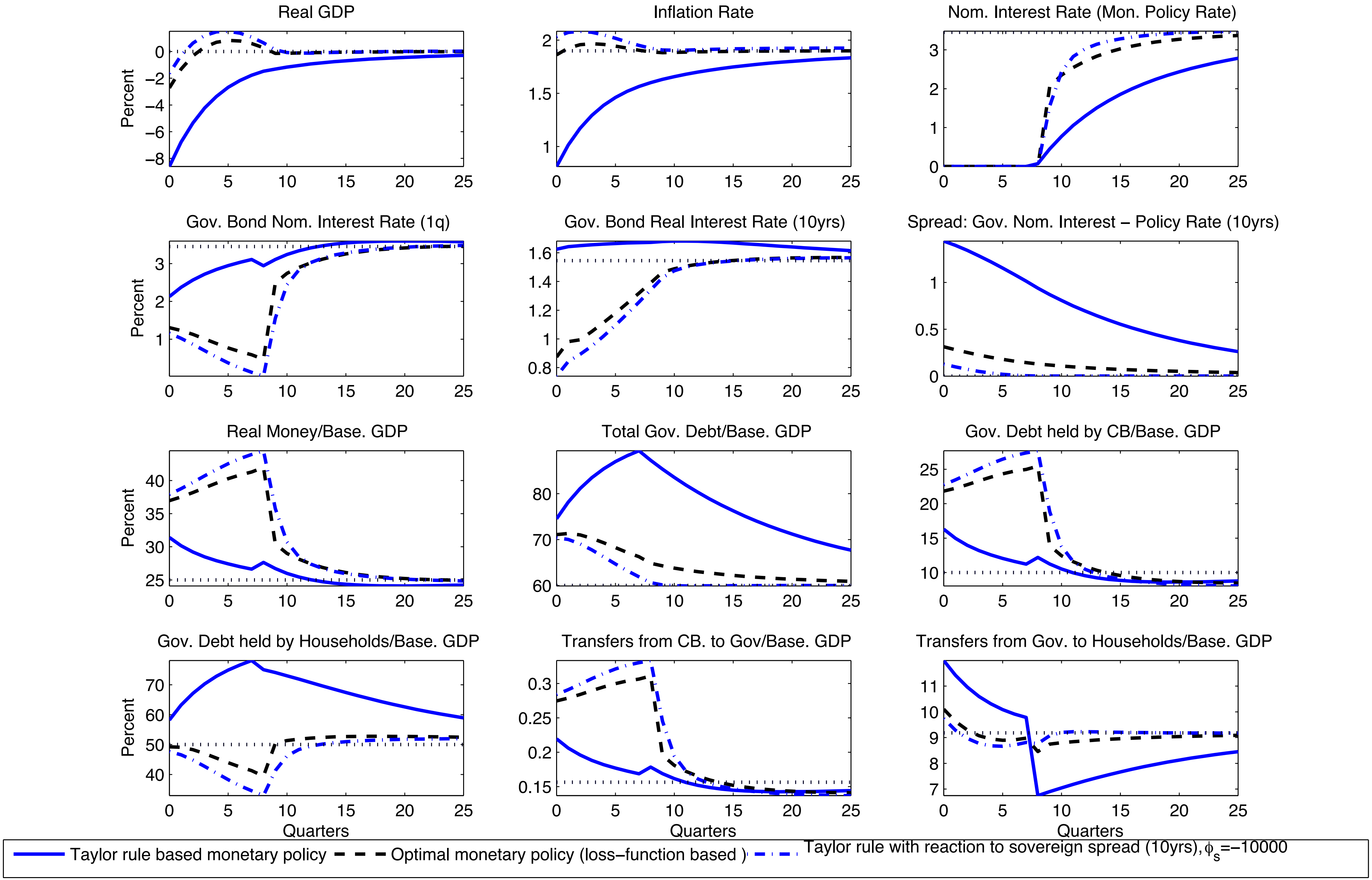
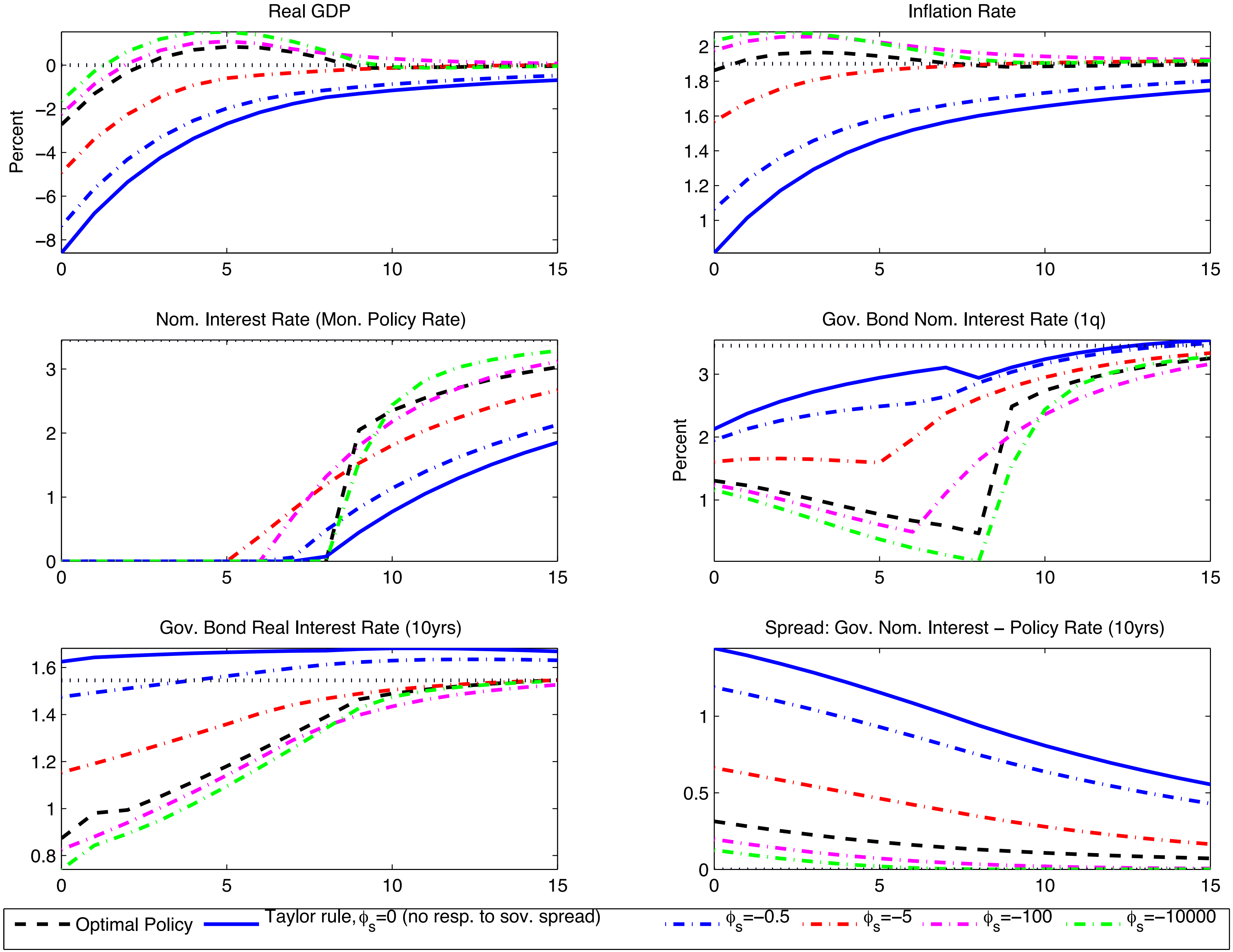


Figure 3: Sensitivity of Sovereign Spread (10yrs) in Taylor rule – OLG, Endog. Sov. Risk Premium, Discount Factor Shock



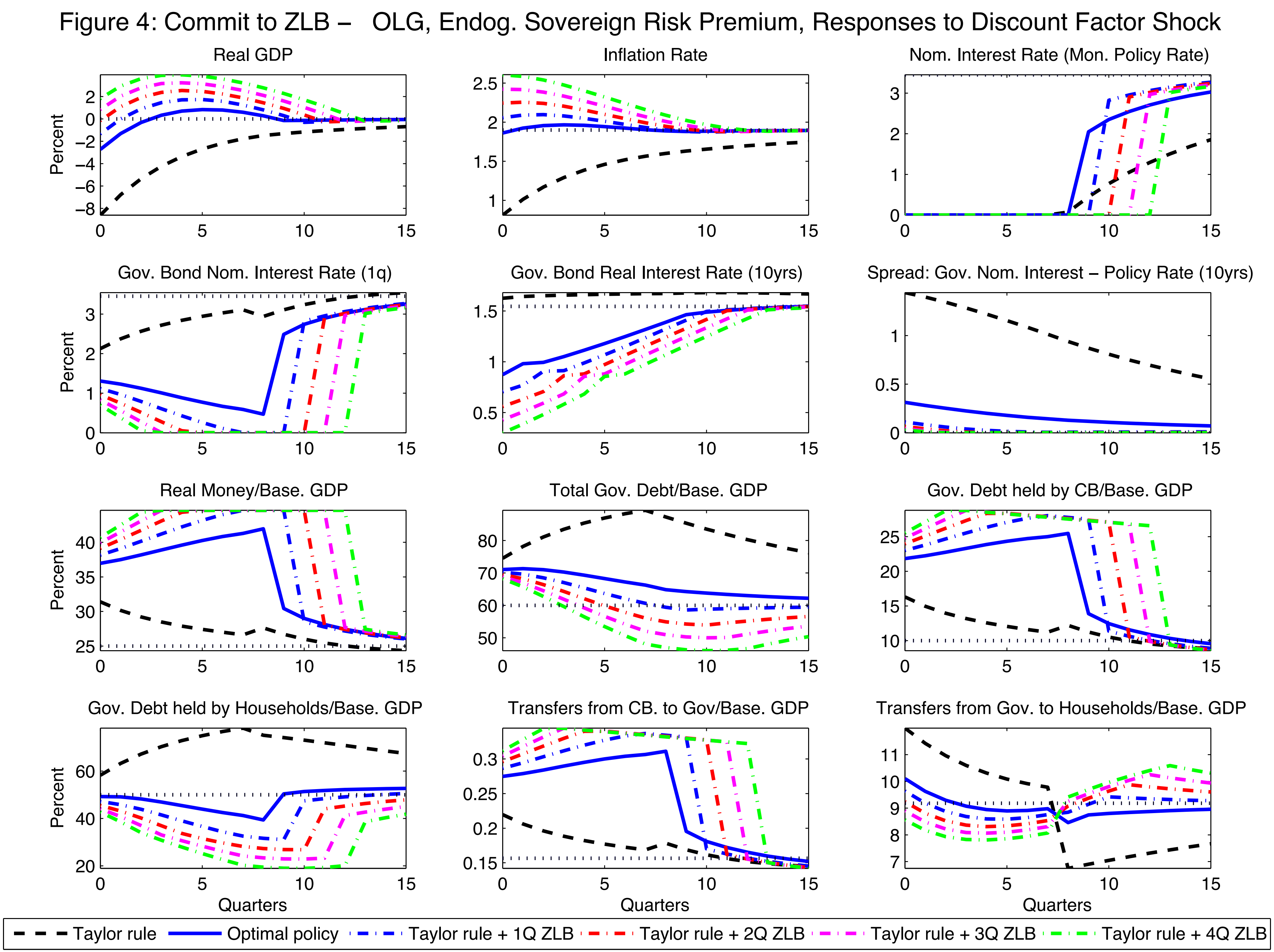
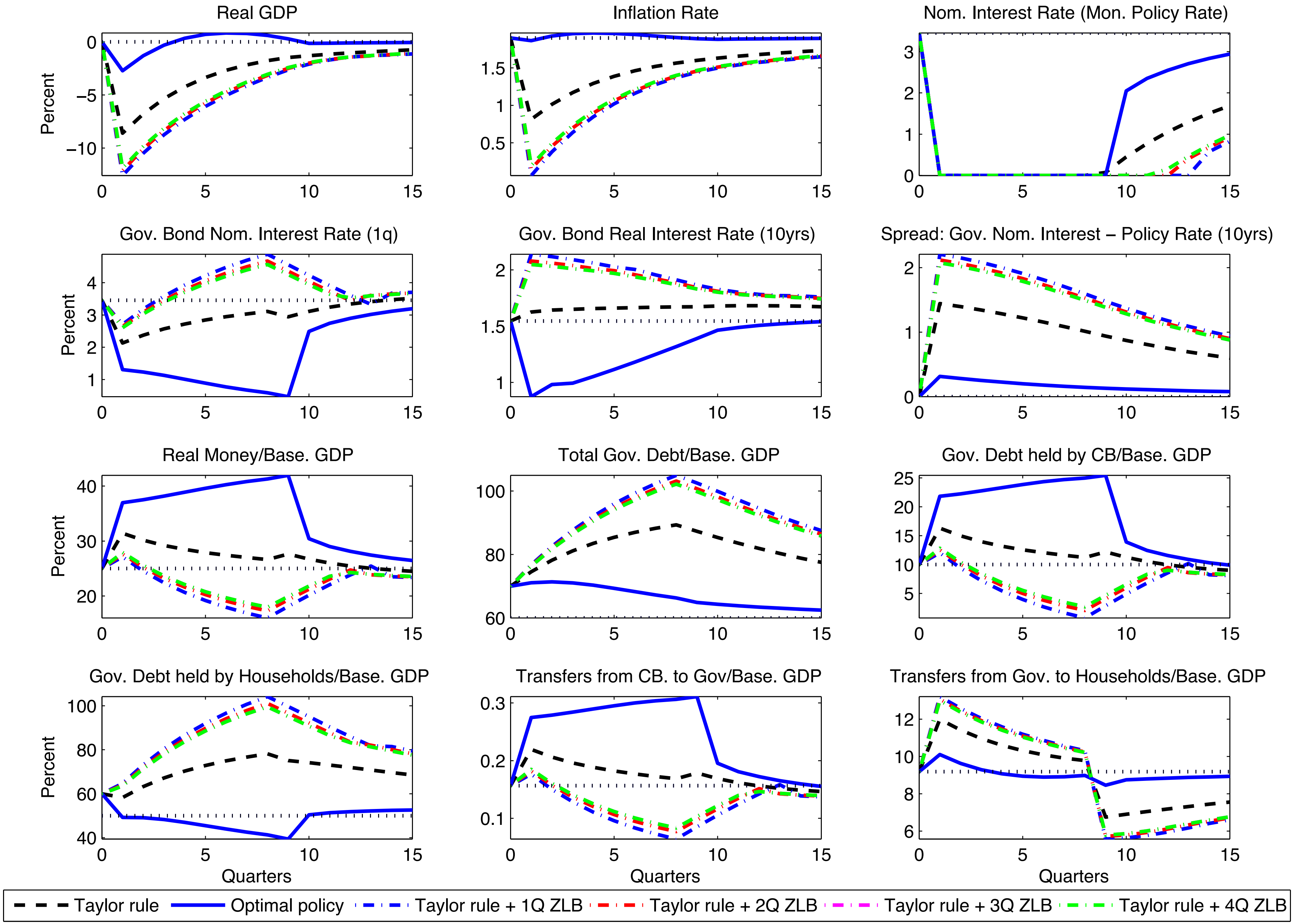


Figure 4: Commit to ZLB – OLG, Endog. Sovereign Risk Premium, Responses to Discount Factor Shock
Real GDP
Inflation Rate
Nom. Interest Rate (Mon. Policy Rate)
Gov. Bond Nom. Interest Rate (1q)
Gov. Bond Real Interest Rate (10yrs)
Spread: Gov. Nom. Interest – Policy Rate (10yrs)
Real Money/Base. GDP
Total Gov. Debt/Base. GDP
Gov. Debt held by CB/Base. GDP
Gov. Debt held by Households/Base. GDP
Transfers from CB. to Gov/Base. GDP
Transfers from Gov. to Households/Base. GDP
Percent
Quarters
Taylor rule
Optimal policy
Taylor rule + 1Q ZLB
Taylor rule + 2Q ZLB
Taylor rule + 3Q ZLB
Taylor rule + 4Q ZLB

Figure 5: Commit to ZLB – OLG, Endog. Sovereign Risk Premium, Responses to Discount Factor Shock



A. Appendix

A.1. Background

This section we briefly describes and compares the behavior of growth, inflation, short-term interest rates, the size of the balance sheet of the central bank, the general government deficit and debt and the long-term government bond rate in the euro area, the United States, Japan and the United Kingdom during the financial crisis and its aftermath.

Figure A1 shows how, following the failure of Lehman Brothers in September 2008 and the resulting collapse of the interbank market and rise in interest rate spreads, annual GDP growth collapsed with a trough of about minus 5 percent in both the euro area and the United States, and a significantly larger drop in the United Kingdom and Japan. As a result of the world-wide fall in demand, oil and commodity prices fell from their peaks in 2008 and contributed to a quite rapid fall in consumer prices which reached negative annual rates in 2009 before bouncing back in 2010, as shown in Figure A2. One exception is the United Kingdom where annual inflation remained above 1 percent partly due to a sharp depreciation of the pound sterling.

In response to the rapid fall in demand in the last quarter of 2008 and the beginning of 2009 and the risks of deflation, monetary and fiscal authorities in the major advanced economies eased policy rapidly and very significantly. On the monetary policy side, Figure A3 plots the short-term nominal interest rates in the euro area and the United States. Policy-controlled short-term interest rates were rapidly reduced to levels close to the zero lower bound. Moreover, various non-conventional monetary policy measures, which aimed at avoiding that liquidity shortages in various financial markets (in particular in the money market) translated into an outright systemic collapse, resulted in a sharp increase in the size of the balance sheet of the central bank (Figure A4) and a gradual reduction of money market spreads.

In the euro area, the enhanced credit support implemented by the ECB in the course of 2009 consisted of (i) changing the provision of liquidity from variable-rate financing to full allotment at a fixed interest rate, (ii) broadening the collateral base which financial institutions could use to obtain central bank refinancing, (iii) lengthening the maturity of the refinancing operations, (iv) providing dollar refinancing through foreign exchange swaps; and (v) supporting the covered bond market which is an important source of long-term financing for financial institutions in the euro area through the Covered Bond Purchases Programme (CBPP). In addition, as the sovereign debt crisis broke out in 2010, the Securities Market Programme (SMP) consisted of the purchase of selected government bond securities to alleviate malfunctioning in the government bond market and support the transmission of monetary policy throughout the euro area. Nevertheless, the share of purchases of government securities in the increase of the central bank's balance sheet is significantly larger in the United States and the United Kingdom due to the various LSAP (Large-Scale Asset Purchases) and QE (Quantitative Easing) programmes in those countries. As the sovereign debt crisis in the euro area intensified in 2011, the expanded liquidity provision by the ECB including 3-year Long-Term Refinancing Operations and a re-activation of the SMP led to an additional expansion of the ECB's balance sheet.

On the fiscal policy side, the deterioration of the economic outlook, discretionary fiscal stimulus programmes and to a lesser extent support to the financial sector resulted in a sharp increase in the general government deficit and a rapid rise in public debt in all four countries. Figure A5 shows that both the total and the structural government deficit increased by less in the euro area than in the United States, Japan and the United Kingdom. As a result, government debt rose more rapidly in the latter countries and surpassed the net debt to GDP ratio in the euro area in 2011, see Figure A6. Nevertheless, long-term interest rates on government bonds fell to historic lows, partly driven by the historically low short-term interest rates and the large provision of central bank liquidity, see Figure A7. The outbreak of the sovereign debt crisis in the euro area in 2010 contributed to a rising gap between average bond yields in the euro area and those in the United States, Japan and the United Kingdom.

A.2. Deriving Aggregate Household Budget Constraint

Note that the following are the budget constraints of different generations in the economy multiplied by the number (share) of households in that generation at time t :

$$
\begin{aligned}
(1-\delta)\left(P_t c_t^j+\frac{B_t^{H,j}}{R_t^{gov}}+M_t^j\right) &= (1-\delta)\left[(1-\tau_t)\,W_t n_t^j+\Theta_t^j+TR_t^j\right] \\
(1-\delta)\delta\left(P_t c_t^j+\frac{B_t^{H,j}}{R_t^{gov}}+M_t^j\right) &= (1-\delta)\delta\left[(1-\tau_t)\,W_t n_t^j+\Theta_t^j+TR_t^j\right]+(1-\delta)\delta\frac{1}{\delta}\left(B_{t-1}^{H,j}+M_{t-1}^j\right) \\
(1-\delta)\delta^2\left(P_t c_t^j+\frac{B_t^{H,j}}{R_t^{gov}}+M_t^j\right) &= (1-\delta)\delta^2\left[(1-\tau_t)\,W_t n_t^j+\Theta_t^j+TR_t^j\right]+(1-\delta)\delta^2\frac{1}{\delta}\left(B_{t-1}^{H,j}+M_{t-1}^j\right) \\
(1-\delta)\delta^3\left(P_t c_t^j+\frac{B_t^{H,j}}{R_t^{gov}}+M_t^j\right) &= (1-\delta)\delta^3\left[(1-\tau_t)\,W_t n_t^j+\Theta_t^j+TR_t^j\right]+(1-\delta)\delta^3\frac{1}{\delta}\left(B_{t-1}^{H,j}+M_{t-1}^j\right) \\
&\ldots
\end{aligned}
$$

Denote the relation of aggregate and generation specific variables as follows

$$
\begin{aligned}
c_t &= \sum_{j=-\infty}^{t} (1-\delta)\delta^{t-j} c_t^j \\
c_{t-1} &= \sum_{j=-\infty}^{t-1} (1-\delta)\delta^{t-1-j} c_{t-1}^j
\end{aligned}
$$

So that the aggregate budget constraint is

$$
P_t c_t + \frac{B_t^H}{R_t^{gov}} + M_t = (1-\tau_t) W_t n_t + \Theta_t + TR_t + B_{t-1}^H + M_{t-1}
$$

A.3. Deriving Aggregate First Order Conditions

A.3.1. Labor/Leisure Trade-off

It is straightforward to show that the first order condition for the labor/leisure trade-off can be aggregated to

$$
\frac{Ac_t}{1-n_t} = (1-\tau_t) \frac{W_t}{P_t}
$$

A.3.2. Money Demand

It is convenient to repeat the money demand equation

$$
\frac{M_t^j}{P_t} = \overline{\frac{M_t^j}{P_t}} - \left[\frac{R_t^{gov}-1}{R_t^{gov}}\right] \frac{1}{\nu_t c_t^j}
$$

The individual money demand equations for each j can not be aggregated easily due to the presence of the term $\frac{1}{c_t^j}$. We will approximate the latter term by a Taylor series expansion and examine the approximation error later on.

First, rewrite the equation as

$$
\frac{M_t^j}{P_t} = \overline{\frac{M_t^j}{P_t}} - \left[\frac{R_t^{gov}-1}{\nu_t R_t^{gov}}\right] g(c_t^j)
$$

where $g(c_t^j) = \frac{1}{c_t^j}$. A first order Taylor series expansion of $g(c_t^j)$ around aggregate consumption c_t yields

$$
\begin{aligned}
g(c_t^j) &\approx g(c_t) - \frac{1}{c_t^2}(c_t^j - c_t) \\
&\approx \frac{2}{c_t} - \frac{1}{c_t^2} c_t^j
\end{aligned}
$$

Substituting back into the money demand equation gives:

$$\frac{M_t^j}{P_t} \approx \overline{\frac{M_t^j}{P_t}} - \left[\frac{R_t^{gov}-1}{\nu_t R_t^{gov}}\right]\left[\frac{2}{c_t} - \frac{1}{c_t^2}c_t^j\right]$$

Aggregating the approximated individual money demand equations gives:

$$\frac{M_t}{P_t} = \overline{\frac{M_t}{P_t}} - \left[\frac{R_t^{gov}-1}{R_t^{gov}}\right]\frac{1}{\nu_t c_t}$$

A.3.3. Euler Equation

Obtaining an aggregate expression for the Euler equation for bonds is more involved, although does not rely on any approximations. As a strategy, we will derive an expression for the net present value representation of the household budget constraint per generation, then aggregate this and combine it with the aggregate household budget also using the Euler equation for bonds.

Rewrite household budget:

$$P_t c_t^j - \underbrace{(1-\tau_t)\, W_t n_t^j - \Theta_t^j - TR_t^j}_{\Delta_t^j} + \frac{B_t^{H,j}}{R_t^{gov}} + M_t^j = \frac{1}{\delta}\left(B_{t-1}^{H,j} + M_{t-1}^j\right)$$

Or

$$P_t c_t^j - \Delta_t^j + \frac{B_t^{H,j}}{R_t^{gov}} + M_t^j = \frac{B_{t-1}^{H,j}}{\delta} + \frac{M_{t-1}^j}{\delta}$$

Write this out for periods $t+1, t+2$,etc...

$$\left[P_{t+1}c_{t+1}^j - \Delta_{t+1}^j + \frac{B_{t+1}^{H,j}}{R_{t+1}^{gov}} + M_{t+1}^j\right]\delta - M_t^j = B_t^{H,j}$$

$$\left[P_{t+2}c_{t+2}^j - \Delta_{t+2}^j + \frac{B_{t+2}^{H,j}}{R_{t+2}^{gov}} + M_{t+2}^j\right]\delta - M_{t+1}^j = B_{t+1}^{H,j}$$

Substituting for $B_t^{H,j}, B_{t+1}^{H,j}$

$$\begin{aligned} P_t c_t^j + P_{t+1}c_{t+1}^j\frac{\delta}{R_t^{gov}} + P_{t+2}c_{t+2}^j\frac{\delta}{R_{t+1}^{gov}}\frac{\delta}{R_t^{gov}} - \Delta_t^j - \Delta_{t+1}^j\frac{\delta}{R_t^{gov}} - \Delta_{t+2}^j\frac{\delta}{R_{t+1}^{gov}}\frac{\delta}{R_t^{gov}} \\ + \left(\frac{R_t^{gov}-1}{R_t^{gov}}\right)M_t^j + \left(\frac{R_{t+1}^{gov}-1}{R_{t+1}^{gov}}\right)\frac{\delta}{R_t^{gov}}M_{t+1}^j \\ + \frac{B_{t+2}^{H,j}}{R_{t+2}^{gov}}\frac{\delta}{R_{t+1}^{gov}}\frac{\delta}{R_t^{gov}} + M_{t+2}^j\frac{\delta}{R_{t+1}^{gov}}\frac{\delta}{R_t^{gov}} = \frac{B_{t-1}^{H,j}}{\delta} + \frac{M_{t-1}^j}{\delta} \end{aligned}$$

Iterating forward

$$\sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} P_{t+i} c_{t+i}^j + \sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} \left[\left(\frac{R_{t+i}^{gov} - 1}{R_{t+i}^{gov}} \right) M_{t+i}^j - \Delta_{t+i}^j \right] +$$

$$+ \lim_{i \to \infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} \left[\frac{B_{t+i}^{H,j}}{R_{t+i}^{gov}} + M_{t+i}^j \right] = \frac{B_{t-1}^{H,j}}{\delta} + \frac{M_{t-1}^j}{\delta}$$

Impose transversality:

$$\lim_{i \to \infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} \left[\frac{B_{t+i}^{H,j}}{R_{t+i}^{gov}} + M_{t+i}^j \right] = 0$$

So that

$$\sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} P_{t+i} c_{t+i}^j + \sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} \left[\left(\frac{R_{t+i}^{gov} - 1}{R_{t+i}^{gov}} \right) M_{t+i}^j - \Delta_{t+i}^j \right] = \frac{B_{t-1}^{H,j}}{\delta} + \frac{M_{t-1}^j}{\delta}$$

We need to work on the expression:

$$\sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} P_{t+i} c_{t+i}^j$$

Consider the first order condition for bonds:

$$1 = \beta E_t \frac{\sigma_t}{\sigma_{t-1}} \frac{c_t^j}{c_{t+1}^j} R_t^{gov} \frac{P_t}{P_{t+1}}$$

Ignoring Jensen's inequality from now on:

$$\frac{1}{\beta R_t^{gov}} \frac{\sigma_{t-1}}{\sigma_t} P_{t+1} c_{t+1}^j = P_t c_t^j$$

$$\frac{1}{\beta R_{t+1}^{gov}} \frac{\sigma_t}{\sigma_{t+1}} P_{t+2} c_{t+2}^j = P_{t+1} c_{t+1}^j$$

Substitute out $P_{t+1} c_{t+1}^j$

$$\frac{1}{\beta R_t^{gov}} \frac{\sigma_{t-1}}{\sigma_{t+1}} \frac{1}{\beta R_{t+1}^{gov}} P_{t+2} c_{t+2}^j = P_t c_t^j$$

Iterate forward:

$$\frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} P_{t+i} c_{t+i}^j \frac{1}{(\delta\beta)^i} \frac{\sigma_{t-1}}{\sigma_{t+i-1}} = P_t c_t^j$$

Or

$$\frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} P_{t+i} c_{t+i}^j = P_t c_t^j \left(\frac{1}{(\delta\beta)^i} \frac{\sigma_{t-1}}{\sigma_{t+i-1}} \right)^{-1}$$

Note that the left hand side is identical to the expression in the net present value representation of the budget constraint. After substitution

$$P_t c_t^j \sum_{i=0}^{\infty} (\delta\beta)^i \frac{\sigma_{t+i-1}}{\sigma_{t-1}} + \sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} \left[\left(\frac{R_{t+i}^{gov} - 1}{R_{t+i}^{gov}} \right) M_{t+i}^j - \Delta_{t+i}^j \right] = \frac{B_{t-1}^{H,j}}{\delta} + \frac{M_{t-1}^j}{\delta}$$

Rearrange

$$P_t c_t^j = \frac{1}{\sum_{i=0}^{\infty} (\delta\beta)^i \frac{\sigma_{t+i-1}}{\sigma_{t-1}}} \left(\frac{B_{t-1}^{H,j}}{\delta} + \frac{M_{t-1}^j}{\delta} + \sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} \left[\Delta_{t+i}^j - \left(\frac{R_{t+i}^{gov} - 1}{R_{t+i}^{gov}} \right) M_{t+i}^j \right] \right)$$

Aggregate

$$P_t c_t = \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty} (\delta\beta)^i \sigma_{t+i-1}} \left(B_{t-1}^H + M_{t-1} + \sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} \left[\Delta_{t+i} - \left(\frac{R_{t+i}^{gov} - 1}{R_{t+i}^{gov}} \right) M_{t+i} \right] \right)$$

$$P_t c_t = \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty} (\delta\beta)^i \sigma_{t+i-1}} \left(\begin{array}{c} B_{t-1}^H + M_{t-1} + \Delta_t - \left(\frac{R_t^{gov} - 1}{R_t^{gov}} \right) M_t \\ + \sum_{i=1}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i}^{gov}} \left[\Delta_{t+i} - \left(\frac{R_{t+i}^{gov} - 1}{R_{t+i}^{gov}} \right) M_{t+i} \right] \end{array} \right)$$

Rearrange and add and substract $\frac{\delta}{R_t^{gov}} \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty} (\delta\beta)^i \sigma_{t+i-1}} \left(B_t^H + M_t \right)$:

$$\begin{aligned} P_t c_t \quad = \quad & \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty} (\delta\beta)^i \sigma_{t+i-1}} \left(B_{t-1}^H + M_{t-1} + \Delta_t - \left(\frac{R_t^{gov} - 1}{R_t^{gov}} \right) M_t \right) \\ & - \frac{\delta}{R_t^{gov}} \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty} (\delta\beta)^i \sigma_{t+i-1}} \left(B_t^H + M_t \right) \\ & + \frac{\delta}{R_t^{gov}} \frac{1}{\sum_{i=0}^{\infty} (\delta\beta)^i \frac{\sigma_{t+i-1}}{\sigma_{t-1}}} \left(B_t^H + M_t + \sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i+1}^{gov}} \left[\Delta_{t+i+1} - \left(\frac{R_{t+i+1}^{gov} - 1}{R_{t+i+1}^{gov}} \right) M_{t+i+1} \right] \right) \end{aligned}$$

Note that

$$P_{t+1} c_{t+1} = \frac{\sigma_t}{\sum_{i=0}^{\infty} (\delta\beta)^i \sigma_{t+i}} \left(B_t^H + M_t + \sum_{i=0}^{\infty} \frac{\delta^i}{\Pi_{i=0}^{i-1} R_{t+i+1}^{gov}} \left[\Delta_{t+i+1} - \left(\frac{R_{t+i+1}^{gov} - 1}{R_{t+i+1}^{gov}} \right) M_{t+i+1} \right] \right)$$

So that

$$P_t c_t = \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty} (\delta\beta)^i \sigma_{t+i-1}} \left[\begin{array}{c} B_{t-1}^H + M_{t-1} + \Delta_t - \left(\frac{R_t^{gov} - 1}{R_t^{gov}} \right) M_t - \frac{\delta}{R_t^{gov}} \left(B_t^H + M_t \right) \\ + \frac{\delta}{R_t^{gov}} P_{t+1} c_{t+1} \frac{\sum_{i=0}^{\infty} (\delta\beta)^i \sigma_{t+i}}{\sigma_t} \end{array} \right]$$

Use the aggregate budget constraint

$$P_t c_t + \frac{B_t^H}{R_t^{gov}} + M_t = \Delta_t + B_{t-1}^H + M_{t-1}$$

to get

$$P_t c_t = \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty}\left(\delta\beta\right)^i \sigma_{t+i-1}} \left[\begin{array}{c} P_t c_t + \frac{B_t^H}{R_t^{gov}} + M_t - \left(\frac{R_t^{gov}-1}{R_t^{gov}}\right) M_t - \frac{\delta}{R_t^{gov}}\left(B_t^H + M_t\right) \\ +\frac{\delta}{R_t^{gov}} P_{t+1}c_{t+1}\frac{\sum_{i=0}^{\infty}(\delta\beta)^i \sigma_{t+i}}{\sigma_t} \end{array} \right]$$

or

$$\left[1 - \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty}\left(\delta\beta\right)^i \sigma_{t+i-1}}\right] P_t c_t = \frac{\sigma_{t-1}}{\sum_{i=0}^{\infty}\left(\delta\beta\right)^i \sigma_{t+i-1}} \left[\begin{array}{c} (1-\delta)\frac{B_t^H}{R_t^{gov}} + \frac{1}{R_t^{gov}}(1-\delta) M_t \\ +\frac{\delta}{R_t^{gov}} P_{t+1}c_{t+1}\frac{\sum_{i=0}^{\infty}(\delta\beta)^i \sigma_{t+i}}{\sigma_t} \end{array} \right]$$

Further,

$$\frac{(\mu_t - 1)}{\delta} P_t c_t R_t^{gov} = \frac{(1-\delta)}{\delta} B_t^H + \frac{(1-\delta)}{\delta} M_t + P_{t+1}c_{t+1}\mu_{t+1}$$

where

$$\begin{aligned} \mu_t &= \sum_{i=0}^{\infty} (\delta\beta)^i \frac{\sigma_{t+i-1}}{\sigma_{t-1}} \\ \mu_t &= 1 + \sum_{i=1}^{\infty} (\delta\beta)^i \frac{\sigma_{t+i-1}}{\sigma_{t-1}} \\ \mu_t &= 1 + \frac{\sigma_t \delta\beta}{\sigma_{t-1}} \sum_{i=0}^{\infty} (\delta\beta)^i \frac{\sigma_{t+i}}{\sigma_t} \\ \mu_t &= 1 + \frac{\sigma_t}{\sigma_{t-1}} \delta\beta\mu_{t+1} \end{aligned}$$

So that

$$\beta\frac{\sigma_t}{\sigma_{t-1}} P_t c_t R_t^{gov} = \frac{(1-\delta)}{\mu_{t+1}\delta}\left[B_t^H + M_t\right] + P_{t+1}c_{t+1}$$

Or in real terms

$$\beta\frac{\sigma_t}{\sigma_{t-1}} c_t \frac{R_t^{gov}}{\Pi_{t+1}} = \frac{1-\delta}{\delta\mu_{t+1}\Pi_{t+1}}\left[\frac{B_t^H}{P_t} + \frac{M_t}{P_t}\right] + c_{t+1}$$

A.4. Derivations for Intermediate Good Firms With Sticky Prices

Profit maximization:

$$\max_{\tilde{P}_t} \; E_0 \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \Lambda_{t+j} \left[\Pi^j \tilde{P}_t y_{t+j,i} - MC_{t+j} y_{t+j,i}\right]$$

subject to

$$y_{t+j,i} = \left(\frac{P_{t+j}}{\Pi^j \tilde{P}_t}\right)^{\frac{\omega}{\omega-1}} y_{t+j}$$

Substitute out demand

$$\max_{\tilde{P}_t} \; E_0 \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \Lambda_{t+j} y_{t+j} P_{t+j} \left[\left(\frac{P_{t+j}}{\Pi^j \tilde{P}_t}\right)^{\frac{\omega}{\omega-1}-1} - \frac{MC_{t+j}}{P_{t+j}} \left(\frac{P_{t+j}}{\Pi^j \tilde{P}_t}\right)^{\frac{\omega}{\omega-1}} \right]$$

Differentiate

$$E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \Lambda_{t+j} y_{t+j} P_{t+j} \left[\left(\frac{\Pi^j \tilde{P}_t}{P_{t+j}}\right)^{1-\frac{\omega}{\omega-1}} - \omega \frac{MC_{t+j}}{P_{t+j}} \left(\frac{\Pi^j \tilde{P}_t}{P_{t+j}}\right)^{-\frac{\omega}{\omega-1}} \right] = 0$$

Note that

$$\begin{aligned} \frac{\Pi^j \tilde{P}_t}{P_{t+j}} &= \frac{\Pi^j}{P_{t+j}} \frac{P_{t+j-1}}{P_{t+j-1}} \cdots \frac{P_{t+1}}{P_{t+1}} P_t \frac{\tilde{P}_t}{P_t} \\ &= \frac{\Pi^j}{\Pi_{t+j}\Pi_{t+j-1}\ldots\Pi_{t+1}} \frac{\tilde{P}_t}{P_t} \\ &= X_{t,j}\tilde{p}_t \end{aligned}$$

where

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t}, X_{t,j} = \frac{\Pi^j}{\Pi_{t+j}\Pi_{t+j-1}\ldots\Pi_{t+1}}$$

So that

$$E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \Lambda_{t+j} y_{t+j} P_{t+j} \left[\left(X_{t,j}\tilde{p}_t\right)^{1-\frac{\omega}{\omega-1}} - \omega mc_{t+j} \left(X_{t,j}\tilde{p}_t\right)^{-\frac{\omega}{\omega-1}} \right] = 0$$

with

$$mc_{t+j} = \frac{MC_{t+j}}{P_{t+j}}$$

Or

$$E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \Lambda_{t+j} P_{t+j} y_{t+j} \left(X_{t,j}\right)^{-\frac{\omega}{\omega-1}} \left[X_{t,j}\tilde{p}_t - \omega mc_{t+j}\right] = 0$$

Or

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} X_{t,j}^{-\frac{\omega}{\omega-1}} \omega mc_{t+j}}{E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} X_{t,j}^{-\frac{1}{\omega-1}}} = \frac{K_t}{F_t}$$

where

$$\lambda_{t+j} = \Lambda_{t+j} P_{t+j}$$

Note that

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} X_{t,j}^{-\frac{\omega}{\omega-1}} \omega mc_{t+j} \\
&= \lambda_t y_t \omega mc_t + E_t \sum_{j=1}^{\infty} \left(\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} X_{t,j}^{-\frac{\omega}{\omega-1}} \omega mc_{t+j}
\end{aligned}
$$

Note that

$$
\begin{aligned}
X_{t+1,j-1} &= \frac{\Pi^{j-1}}{\Pi_{t+1+j-1}\Pi_{t+1+j-2}...\Pi_{t+1+1}} \\
X_{t+1,j-1}\frac{\Pi}{\Pi_{t+1}} &= \frac{\Pi}{\Pi_{t+1}}\frac{\Pi^{j-1}}{\Pi_{t+j}\Pi_{t+j-1}...\Pi_{t+2}} = X_{t,j}
\end{aligned}
$$

So that

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} X_{t,j}^{-\frac{\omega}{\omega-1}} \omega mc_{t+j} \\
&= \lambda_t y_t \omega mc_t + E_t \sum_{j=1}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} \left(X_{t,j}\right)^{-\frac{\omega}{\omega-1}} \omega mc_{t+j} \\
&= \lambda_t y_t \omega mc_t + E_t \sum_{j=1}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} \left(X_{t+1,j-1}\frac{\Pi}{\Pi_{t+1}}\right)^{-\frac{\omega}{\omega-1}} \omega mc_{t+j} \\
&= \lambda_t y_t \omega mc_t + \left(\frac{\Pi}{\Pi_{t+1}}\right)^{-\frac{\omega}{\omega-1}} E_t \sum_{j=1}^{\infty} \delta\xi_p\beta \left(\delta\xi_p\beta\right)^{j-1} \lambda_{t+j} y_{t+j} \left(X_{t+1,j-1}\right)^{-\frac{\omega}{\omega-1}} \omega mc_{t+j} \\
&= \lambda_t y_t \omega mc_t + \delta\xi_p\beta \left(\frac{\Pi}{\Pi_{t+1}}\right)^{-\frac{\omega}{\omega-1}} E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j+1} y_{t+j+1} X_{t+1,j}^{-\frac{\omega}{\omega-1}} \omega mc_{t+j+1} \\
&= \lambda_t y_t \omega mc_t + \delta\xi_p\beta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{\omega}{\omega-1}} K_{t+1}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
F_t &= E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} X_{t,j}^{-\frac{1}{\omega-1}} \\
&= \lambda_t y_t + \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{1}{\omega-1}} E_t \sum_{j=1}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j} y_{t+j} X_{t+1,j-1}^{-\frac{1}{\omega-1}} \\
&= \lambda_t y_t + \delta\xi_p\beta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{1}{\omega-1}} E_t \sum_{j=0}^{\infty} \left(\delta\xi_p\beta\right)^j \lambda_{t+j+1} y_{t+j+1} X_{t+1,j}^{-\frac{1}{\omega-1}} \\
&= \lambda_t y_t + \delta\xi_p\beta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{1}{\omega-1}} F_{t+1}
\end{aligned}
$$

Further, the aggregate price index equation can be rewritten as follows:

$$\tilde{p}_t = \left[\frac{1-\xi_p\left(\frac{\Pi}{\Pi_t}\right)^{\frac{1}{1-\omega}}}{1-\xi_p}\right]^{1-\omega}$$

Measure of price dispersion:

$$\begin{aligned} y_t^{agg} &= \int_0^1 y_{t,i}di \\ &= \int_0^1 n_{t,i}di = n_t \end{aligned}$$

Also

$$\begin{aligned} y_t^{agg} &= y_t P_t^{\frac{\omega}{\omega-1}} \int_0^1 P_{t,i}^{-\frac{\omega}{\omega-1}} di \\ &= y_t P_t^{\frac{\omega}{\omega-1}} \mathring{P}_t^{-\frac{\omega}{\omega-1}} \\ &= y_t \mathring{p}_t^{\frac{\omega}{1-\omega}} \end{aligned}$$

with

$$\begin{aligned} \mathring{P}_t^{-\frac{\omega}{\omega-1}} &= \int_0^1 P_{t,i}^{-\frac{\omega}{\omega-1}} di \\ \mathring{P}_t &= \left[\int_0^1 P_{t,i}^{\frac{\omega}{1-\omega}} di\right]^{\frac{1-\omega}{\omega}} \\ \mathring{P}_t &= \left[\int_0^1 P_{t,i}^{\frac{\omega}{1-\omega}} di\right]^{\frac{1-\omega}{\omega}} \\ \mathring{P}_t &= \left[\left(1-\xi_p\right)\tilde{P}_t^{\frac{\omega}{1-\omega}} + \xi_p\left(\Pi\mathring{P}_{t-1}\right)^{\frac{\omega}{1-\omega}}\right]^{\frac{1-\omega}{\omega}} \\ \mathring{p}_t &= \left[\left(1-\xi_p\right)\left(\tilde{p}_t\right)^{\frac{\omega}{1-\omega}} + \xi_p\left(\frac{\Pi}{\Pi_t}\mathring{p}_{t-1}\right)^{\frac{\omega}{1-\omega}}\right]^{\frac{1-\omega}{\omega}} \\ \mathring{p}_t &= \left[\left(1-\xi_p\right)\left(\frac{1-\xi_p\left(\frac{\Pi}{\Pi_t}\right)^{\frac{1}{1-\omega}}}{1-\xi_p}\right)^{\omega} + \xi_p\left(\frac{\Pi}{\Pi_t}\mathring{p}_{t-1}\right)^{\frac{\omega}{1-\omega}}\right]^{\frac{1-\omega}{\omega}} \end{aligned}$$

Equating both y_t^{agg},

$$\begin{aligned} y_t^{agg} &= \int_0^1 y_{t,i}di \\ &= \int_0^1 n_{t,i}di = n_t \end{aligned}$$

So that

$$y_t = n_t \mathring{p}_t^{\frac{\omega}{\omega-1}}$$

A.5. Characterizing the Equilibrium

Government bond market clearing:

$$B_t^G = B_t^M + B_t^H$$

Consolidate household, fiscal and monetary authorities budget constraints:

$$\begin{aligned}
B_{t-1}^G + TR_t + \chi_t W_t \int y_{t,i} &= \frac{B_t^G}{R_t^{gov}} + \tau_t W_t \int n_{t,i} + S_t \\
\frac{B_t^M}{R_t^{gov}} + S_t &= B_{t-1}^M + M_t - M_{t-1} \\
P_t c_t + \frac{B_t^H}{R_t^{gov}} + M_t &= (1-\tau_t) W_t n_t + \Theta_t + TR_t + B_{t-1}^H + M_{t-1}
\end{aligned}$$

Note that

$$n_t = \int n_{t,i}$$

So that

$$c_t = \frac{W_t}{P_t} n_t - \chi_t \frac{W_t}{P_t} \int y_{t,i} + \frac{\Theta_t}{P_t}$$

Note that aggregate intermediate firms profits are:

$$\Theta_t = \int P_{t,i} y_{t,i} - (1-\chi_t) W_t \int n_{t,i}$$

Substitute the demand function and rearrange

$$\frac{\Theta_t}{P_t} = y_t \left(P_t\right)^{\frac{\omega}{\omega-1}-1} \int P_{t,i}^{-\frac{1}{\omega-1}} - (1-\chi_t) \frac{W_t}{P_t} n_t$$

Use the aggregate price index to get

$$\frac{\Theta_t}{P_t} = y_t - (1-\chi_t) \frac{W_t}{P_t} n_t$$

Hence, the resource constraint reads

$$c_t = y_t$$

Thus, given a specification of monetary policy, the equilibrium equations can be written as:

$$\text{Bond Market Clearing:} \quad \frac{B_t^G}{P_t} = \frac{B_t^M}{P_t} + \frac{B_t^H}{P_t}$$

$$\text{Central Bank Budget:} \quad \frac{B_t^M}{R_t^{gov} P_t} + \frac{S_t}{P_t} = \frac{B_{t-1}^M}{P_{t-1}} \frac{1}{\Pi_t} + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{1}{\Pi_t}$$

$$\text{Transfer from CB to Gov:} \quad \frac{S_t}{P_t} = s + \theta_C \left(\frac{B_t^M}{P_t} - b^M \right)$$

$$\text{Government Budget:} \quad \frac{B_{t-1}^G}{P_{t-1}} \frac{1}{\Pi_t} + \frac{TR_t}{P_t} + \chi_t \frac{W_t}{P_t} n_t = \frac{B_t^G}{R_t^{gov} P_t} + \tau_t \frac{W_t}{P_t} n_t + \frac{S_t}{P_t}$$

$$\text{Fiscal Rule} \;: \quad \frac{TR_t}{P_t} = tr - \theta_{TRB,t} \left(\frac{B_{t-1}^G}{P_{t-1}} - b^G \right) - \theta_{TRY} \left(\frac{y_t}{y} - 1 \right)$$

$$\text{Bond Interest Rate} \;: \quad R_t^{gov} = \gamma_t R_t$$

$$\text{Risk Premium:} \quad \gamma_t = \max \left\{ \exp \left(\varkappa \left[\frac{B_t^G}{4 P_t y_t} - \frac{b^G}{4y} \right] \right), 1 \right\}$$

$$\text{Leisure/Labor:} \quad \frac{A c_t}{1 - n_t} = (1 - \tau_t) \frac{W_t}{P_t}$$

$$\text{Euler Equation Bonds:} \quad \beta \frac{\sigma_t}{\sigma_{t-1}} c_t \frac{R_t^{gov}}{\Pi_{t+1}} = \frac{1 - \delta}{\delta \mu_{t+1} \Pi_{t+1}} \left[\frac{B_t^H}{P_t} + \frac{M_t}{P_t} \right] + c_{t+1}$$

$$\text{Recursive Discounting:} \quad \mu_t = 1 + \frac{\sigma_t}{\sigma_{t-1}} \delta \beta \mu_{t+1}$$

$$\text{Feasibility:} \quad c_t = y_t$$

$$\text{Money Demand:} \quad \frac{M_t}{P_t} = \frac{\overline{M_t}}{P_t} - \left[\frac{R_t^{gov} - 1}{R_t^{gov}} \right] \frac{1}{\nu_t c_t}$$

$$\text{Non.lin. Pricing 1:} \quad F_t = \lambda_t y_t + \delta \xi_p \beta \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\frac{1}{\omega - 1}} F_{t+1}$$

$$\text{Non.lin. Pricing 2} \;: \quad K_t = \lambda_t y_t \omega m c_t + \delta \xi_p \beta \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\frac{\omega}{\omega - 1}} K_{t+1}$$

$$\text{Non.lin. Pricing 3} \;: \quad \frac{K_t}{F_t} = \left[\frac{1 - \xi_p \left(\frac{\Pi_t}{\Pi} \right)^{\frac{1}{\omega - 1}}}{1 - \xi_p} \right]^{1 - \omega}$$

$$\text{Inv. Price Dispersion} \;: \quad \mathring{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\Pi_t}{\Pi} \right)^{\frac{1}{\omega - 1}}}{1 - \xi_p} \right)^{\omega} + \xi_p \left(\frac{\Pi}{\Pi_t} \mathring{p}_{t-1} \right)^{\frac{\omega}{1 - \omega}} \right]^{\frac{1 - \omega}{\omega}}$$

$$\text{Marginal Utility} \;: \quad \frac{\sigma_{t-1}}{c_t} = \lambda_t$$

$$\text{Marginal Costs} \;: \quad m c_t = (1 - \chi_t) \frac{W_t}{P_t}$$

$$\text{Subsidy to Firms} \;: \quad \chi_t = 0$$

$$\text{Labor Taxes} \;: \quad \tau_t = \tau$$

$$\text{Production} \;: \quad y_t = n_t \mathring{p}_t^{\frac{\omega}{\omega - 1}}$$

A.6. Steady State

The equilibrium equations can be rewritten in steady state form as follows:

$$\begin{aligned}
\text{Bond Market Clearing} &: \quad b^G = b^M + b^H \\
\text{Central Bank Budget} &: \quad \frac{b^M}{R^{gov}} + s = \frac{b^M}{\Pi} + m - \frac{m}{\Pi} \\
\text{Government Budget} &: \quad \frac{b^G}{\Pi} + tr + \chi wn = \frac{b^G}{R^{gov}} + \tau wn + s \\
\text{Leisure/Labor} &: \quad \frac{Ay}{1-n} = (1-\tau)\, w \\
\text{Gov. Bond Int. Rate:} & \quad R^{gov} = \gamma R \\
\text{Euler Equation Bonds} &: \quad \beta \frac{R^{gov}}{\Pi} = \frac{(1-\delta)(1-\delta\beta)}{\delta\Pi} \left[\frac{b^H}{y} + \frac{m}{y} \right] + 1 \\
\text{Money Demand} &: \quad m = \overline{m} - \left[\frac{R^{gov} - 1}{R^{gov}} \right] \frac{1}{\nu y} \\
\text{Non.lin. Pricing 1} &: \quad F = \frac{\sigma}{1 - \delta \xi_p \beta} \\
\text{Non.lin. Pricing 2} &: \quad K = \frac{\sigma \omega (1-\chi) w}{1 - \delta \xi_p \beta} \\
\text{Non.lin. Pricing 3} &: \quad \frac{K}{F} = 1 \\
\text{Inv. Price Dispersion} &: \quad \mathring{p} = 1 \\
\text{Production} &: \quad y = n \\
\text{Subsidy to Firms:} & \quad \chi = 0
\end{aligned}$$

Solving for the steady state is straightforward:

$$
\begin{aligned}
F &= \frac{\sigma}{1-\delta\xi_p\beta} \\
w &= \frac{1}{\omega(1-\chi)} \\
K &= \frac{\sigma\omega(1-\chi)w}{1-\delta\xi_p\beta} \\
\mathring{p} &= 1 \\
\gamma &= 1 \\
A &= (1-\tau)\,w\frac{1-n}{n} \\
y &= n \\
R^{gov} &= \frac{\Pi}{\beta} + \frac{(1-\delta)\,(1-\delta\beta)}{\delta\beta}\left[\frac{b^H}{y} + \frac{m}{y}\right] \\
R &= \frac{R^{gov}}{\gamma} \\
\overline{m} &= m + \left[\frac{R^{gov}-1}{R^{gov}}\right]\frac{1}{\nu y} \\
\frac{b^M}{y} &= \frac{b^G}{y} - \frac{b^H}{y} \\
\frac{s}{y} &= \left(\frac{1}{\Pi} - \frac{1}{R^{gov}}\right)\frac{b^M}{y} + \frac{\Pi-1}{\Pi}\frac{m}{y} \\
\frac{tr}{y} &= \frac{b^G}{R^{gov}y} + (\tau-\chi)\,w + \frac{s}{y} - \frac{1}{\Pi}\frac{b^G}{y}
\end{aligned}
$$

A.7. Optimal Monetary Policy and the ZLB

Consider a drastically reduced version of our model that collapses to the standard Clarida, Gali and Gertler (1999) model. All variables are in log-deviations from steady state. Expectation operators are omitted for simplicity. u_t and rr_t^n are exogenous and represent price markup and equilibrium real interest rate shocks respectively. The latter is akin to the discount factor shock considered in the main model. The standard New Keynesian Phillips curve and the so-called New IS curve are:

$$
\begin{aligned}
\pi_t &= \beta\pi_{t+1} + \kappa x_t + u_t \\
x_t &= x_{t+1} - (r_t - \pi_{t+1} - rr_t^n)
\end{aligned}
$$

Optimal policy solves the following problem, see also e.g. Levin, Lopez-Salido, Nelson and Yun (2011):

$$
\begin{aligned}
\max_{\pi_t, x_t, r_t, \phi_{1,t}, \phi_{2,t}, \phi_{3,t}} \sum_{t=0}^{\infty} \beta^t [\quad & -\frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) \\
& + \phi_{1,t}\left(\pi_t - \beta\pi_{t+1} - \kappa x_t - u_t\right) \\
& + \phi_{2,t}\left(x_t - x_{t+1} + r_t - \pi_{t+1} - rr_t^n\right) \\
& + \phi_{3,t}\left(r_t + 1/\beta - 1\right)]
\end{aligned}
$$

where the last constraint reflects the lower bound on nominal interest rates. Note that in a zero steady state inflation environment, the steady state real interest rate is identical to the nominal interest rate

$$
R^{\text{real}} = 1 + r = \frac{1}{\beta}.
$$

Thus, in log-deviations, the ZLB becomes binding if

$$
r_t \leq -\left(\frac{1}{\beta} - 1\right).
$$

The first order conditions are:

$$
\begin{aligned}
\pi_t &= \beta\pi_{t+1} + \kappa x_t + u_t \\
x_t &= x_{t+1} - (r_t - \pi_{t+1} - rr_t^n) \\
\pi_t &= \phi_{1,t} - \phi_{1,t-1} - \frac{1}{\beta}\phi_{2,t-1} \\
\lambda x_t &= \phi_{2,t} - \kappa\phi_{1,t} - \frac{1}{\beta}\phi_{2,t-1} \\
0 &= \phi_{2,t} + \phi_{3,t} \\
0 &= \phi_{3,t}\left(r_t + \frac{1}{\beta} - 1\right)
\end{aligned}
$$

Consider a negative shock for rr_t^n. When the ZLB is not binding, $\phi_{3,t} = 0$ and thereby $\phi_{2,t} = 0$. Accordingly, full stabilization of inflation and the output gap is possible since the central bank can set $r_t = rr_t^n$. If the ZLB becomes binding, $\phi_{3,t} > 0$ up to some $t = T$. In this case, $\phi_{2,t} = -\phi_{3,t}$ and full stabilization is not possible. Numerically, its a matter of finding date $t = T$ and then solving two systems of equations, one during the ZLB and one thereafter both being connected via the NK Phillips and New IS curve. For deterministic simulations, this can be done, after some effort, in e.g. Dynare. We proceed likewise for our main model. In difference to the above example, we use the fully non-linear equilibrium equations to obtain optimal policy first order conditions and also simulate the fully non-linear systems of equations.

A.8. Appendix Figures

Figure A1

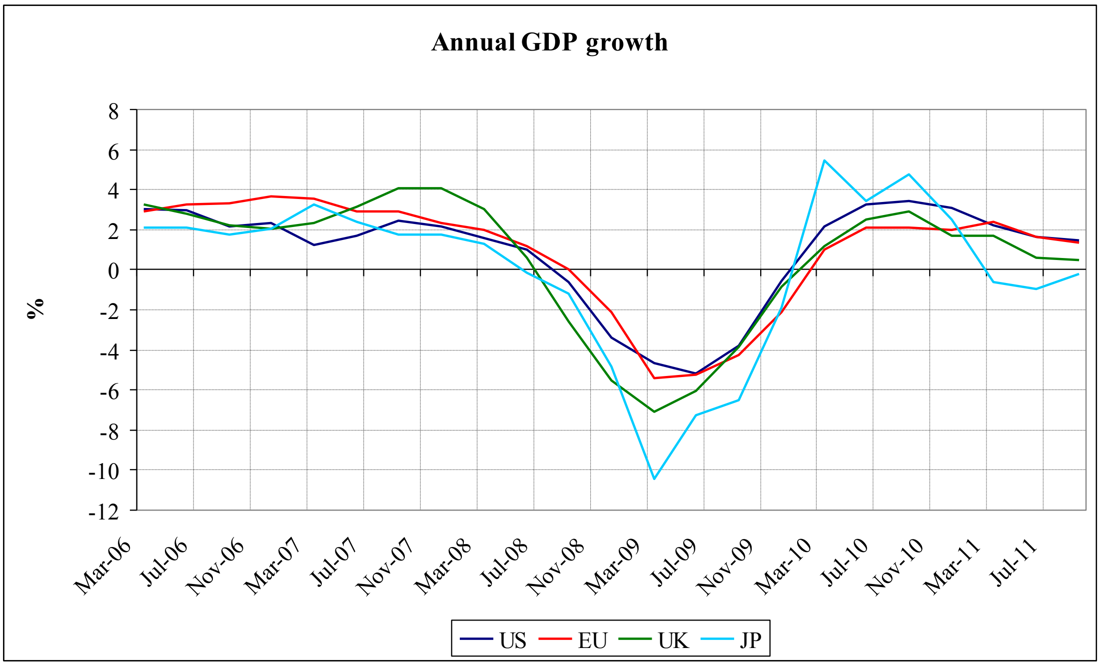


Figure A2

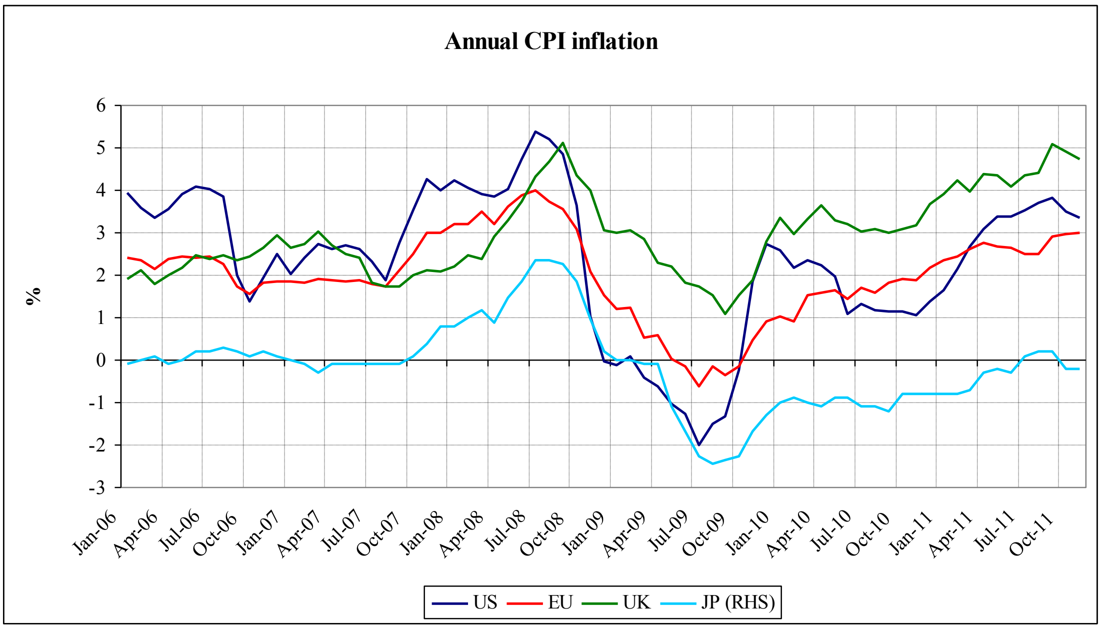


Figure A3

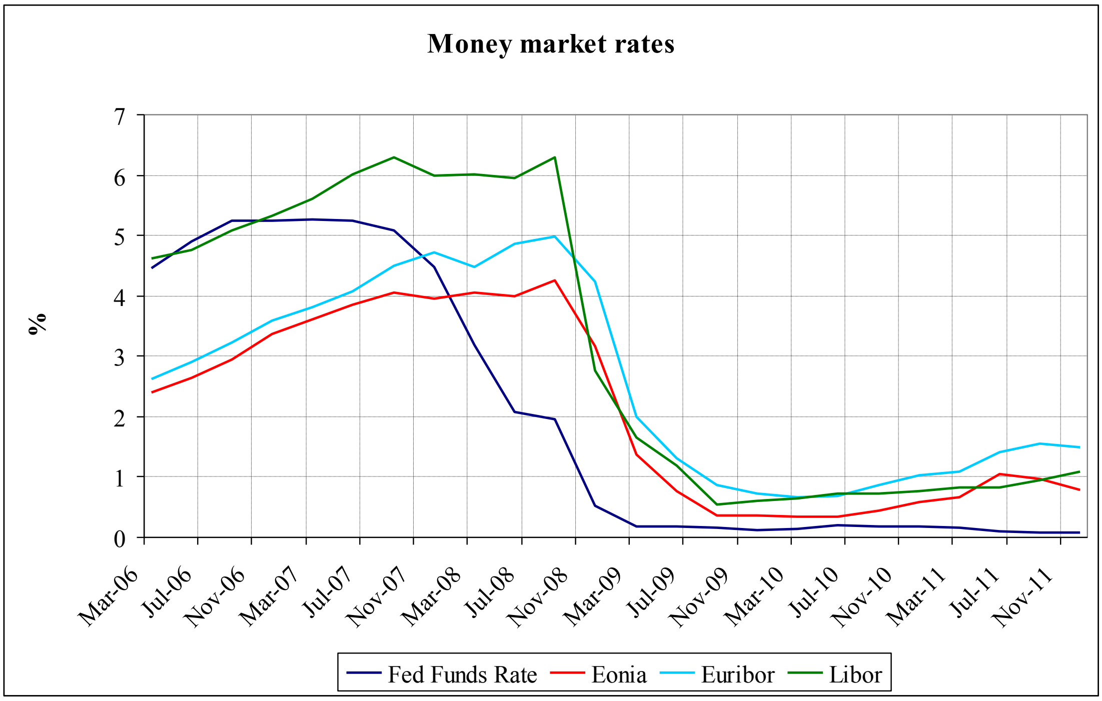


Figure A4

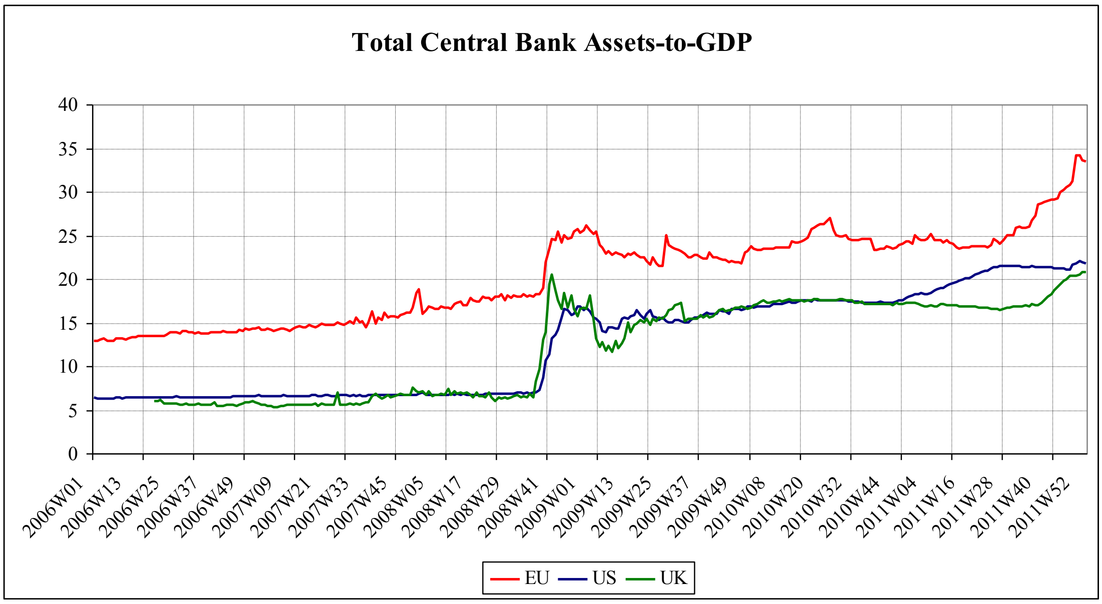
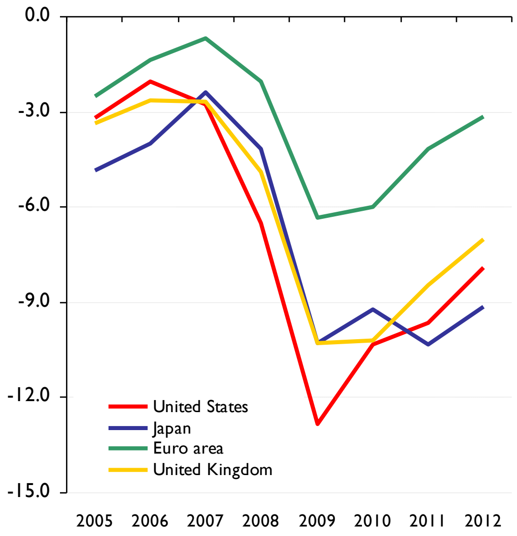


Figure A5

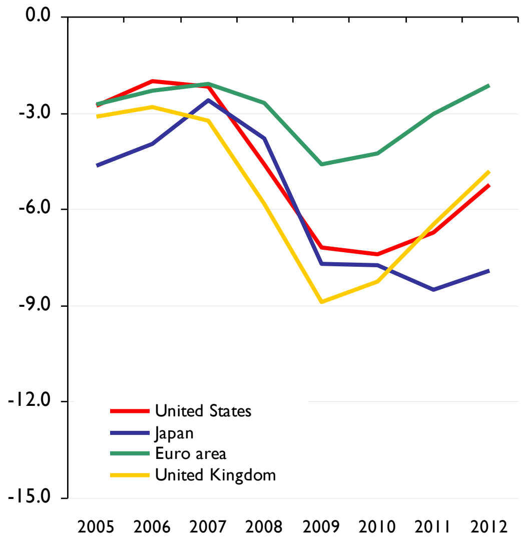
General government budget balance

(2005-2012, percentage of GDP)



General government structural balance

(2005-2012, percentage of GDP)

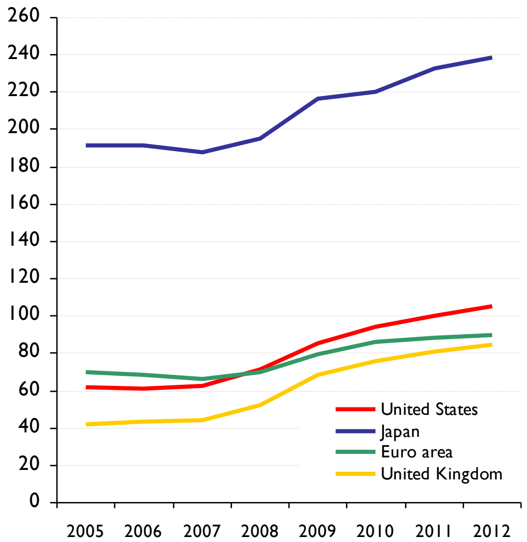


Source: IMF World Economic Outlook September 2011

Figure A6

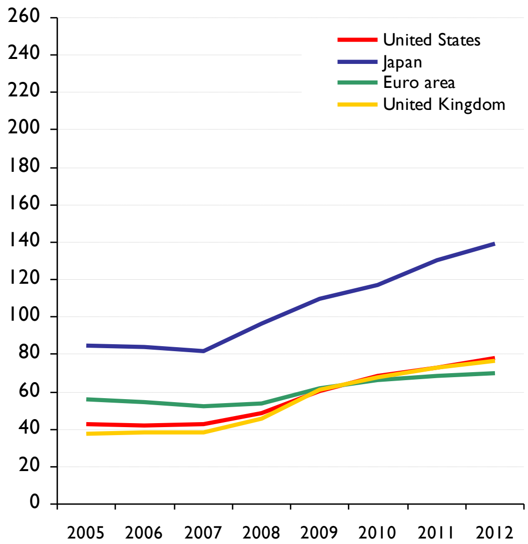
General government gross debt

(2005-2012, percentage of GDP)



General government net debt

(2005-2012, percentage of GDP)



Source: IMF World Economic Outlook September 2011

Figure A7

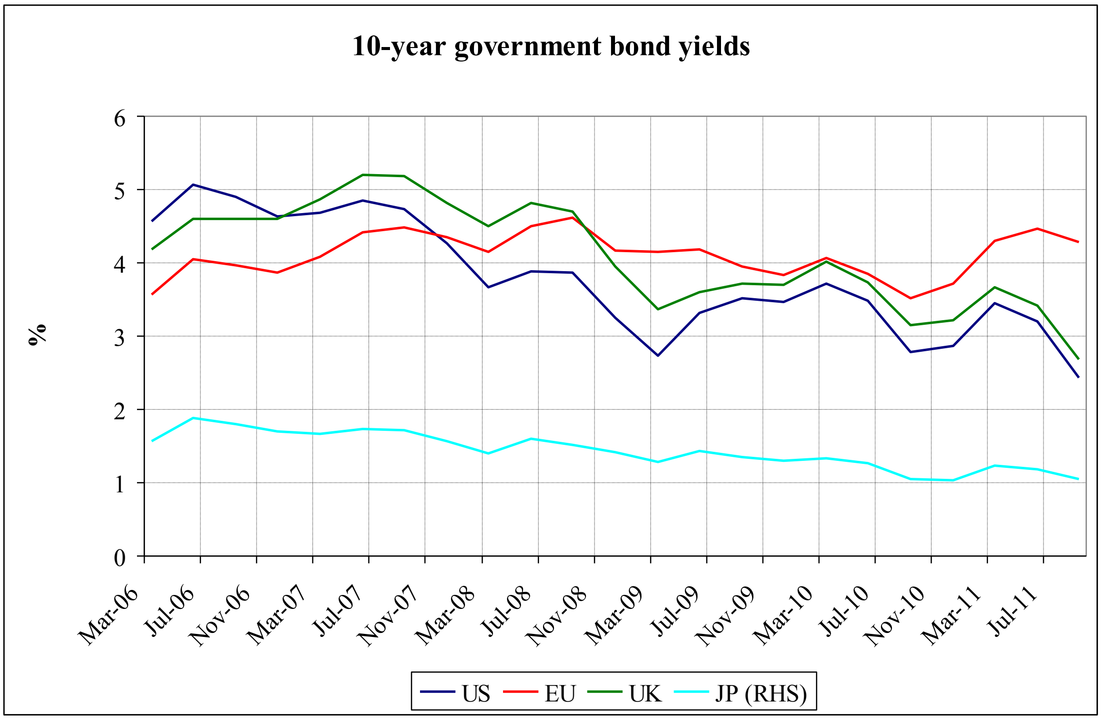
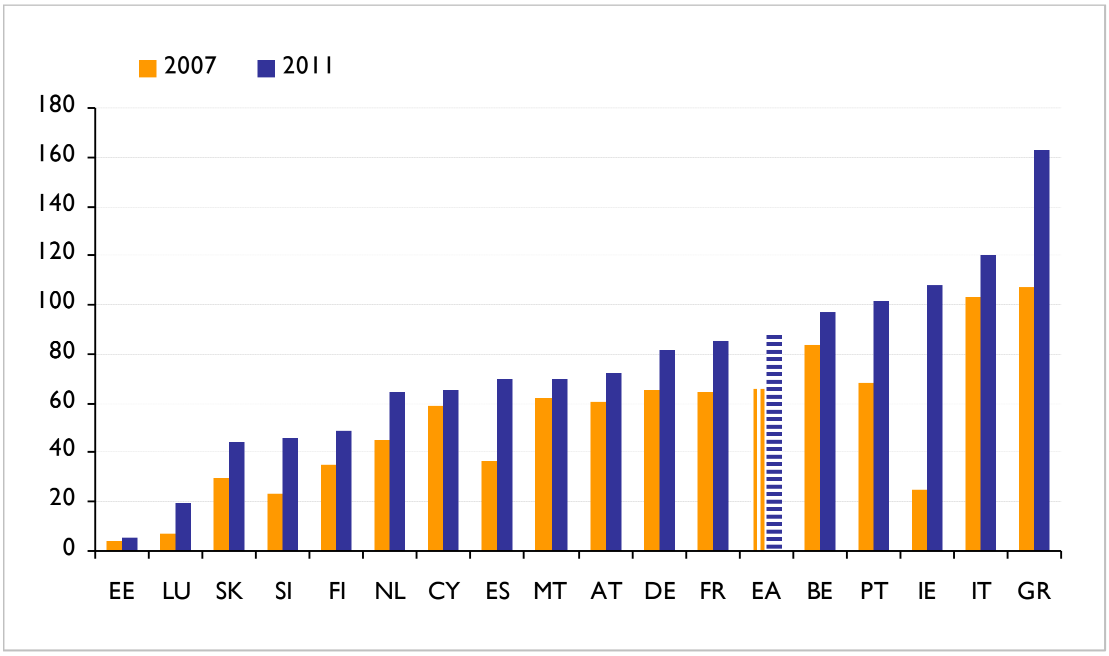


Figure A8

General government gross debt (% of GDP)



Source: European Commission autumn 2011 economic forecast.

Figure A9: OLG, No Sovereign Risk Premium, Responses to Discount Factor Shock

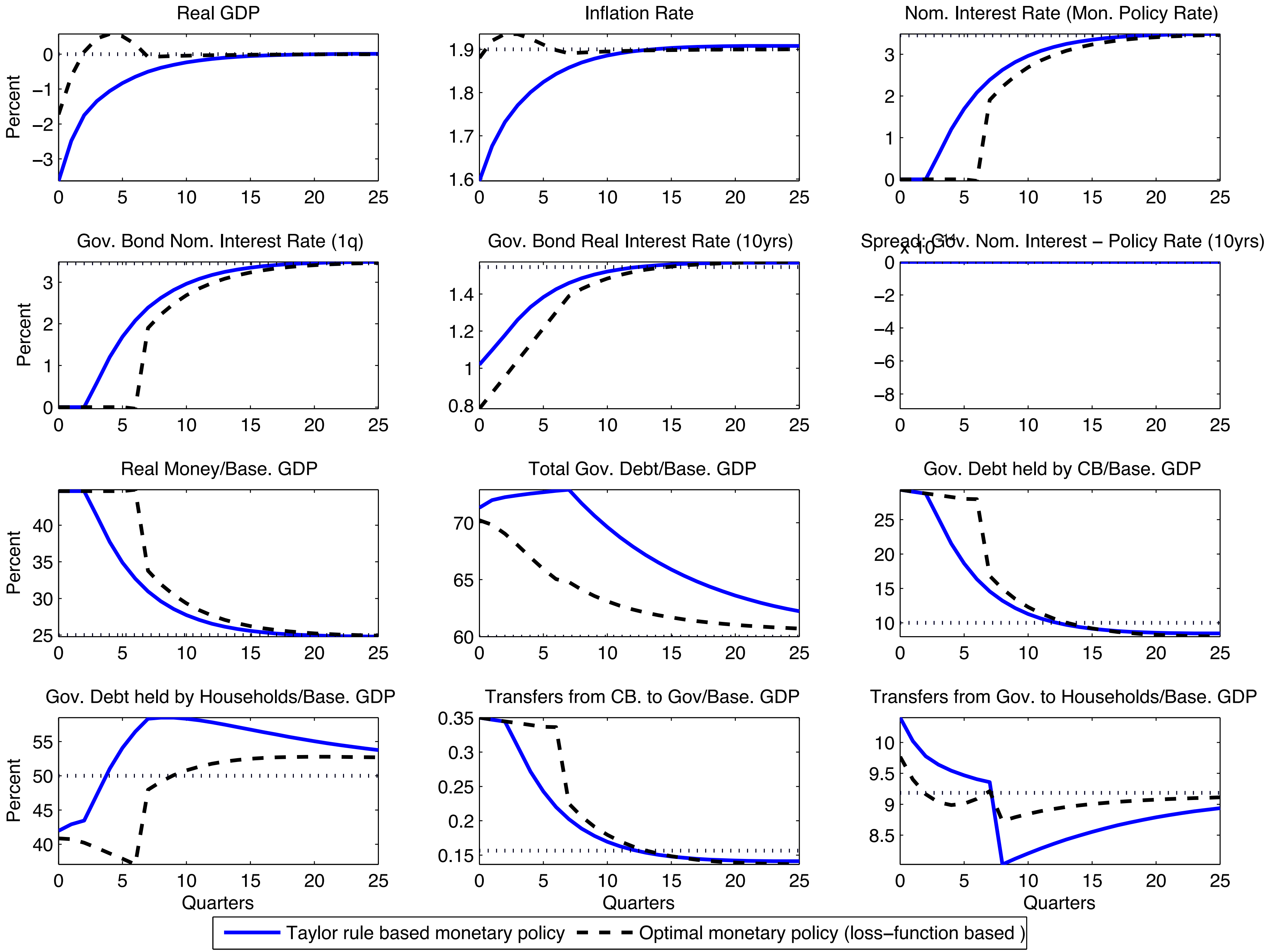


Figure A10: Blanchard–Yaari vs. Inf. Lived Households, Endog. Sov. Risk Premium (χ=0.0165), Resp. to Discount Shock