Federal Reserve Bank of New York
Staff Reports

Decomposing Real and Nominal Yield Curves

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Staff Report No. 570
September 2012
Revised February 2015

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JEL classification: E43, E44, G12

Abstract

We present an affine term structure model for the joint pricing of real and nominal bond yields that explicitly accommodates liquidity risk premia. We estimate the model using a new, computationally efficient procedure that is based on return regressions. The model allows us to address a number of salient questions about the transmission of monetary policy. We show that variations in U.S. nominal term premia are primarily driven by variations in real term premia rather than inflation and liquidity risk premia. Nonetheless, adjusting breakevens for inflation and liquidity risk substantially improves forecasts of inflation. Our estimates imply that the Federal Reserve's large-scale asset purchases lowered Treasury yields primarily by reducing real term premia. Real term premia also account for the positive response of long-term real forward rates to surprise changes in the federal funds target. Applying our model to U.K. data, we find that the inflation risk premium dropped sharply when the Bank of England formally adopted an inflation target.

Key words: TIPS, break-evens, expected inflation, inflation risk premium, affine term-structure model, liquidity risk

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1 Introduction

The evolution of inflation expectations is an important input to monetary policy decisions and is closely watched by financial market participants. Breakeven inflation—the difference between nominal yields from Treasuries and real yields from Treasury inflation-protected securities (TIPS) for a given maturity—reflects inflation expectations, but is subject to two important distortions. First, TIPS are often perceived to be less liquid than Treasuries, especially in times of financial market stress. Second, breakeven inflation incorporates an inflation risk premium, the compensation investors require for bearing inflation risk.

In this paper, we present a Gaussian affine term structure model (ATSM) for the joint pricing of the Treasury and TIPS yield curves that adjusts for the relative illiquidity of TIPS and generates estimates of the inflation risk premium. Our approach has a number of advantages relative to the existing literature. First, we adjust for TIPS liquidity in a transparent way. Specifically, we construct an index of TIPS liquidity using observable measures and include it as a pricing factor in our model. We find that the illiquidity component is sizable for TIPS, especially during the financial crisis. A second advantage of our approach is that it allows for a large number of pricing factors to be included in the model without impairing computational feasibility since our estimation approach is based on linear regressions which are fast to implement and robust to model misspecification. This is important because a relatively large number of factors is needed to jointly explain the time series and cross-section of nominal and real yields with a good degree of fit. Hence, a third advantage is that the pricing errors implied by our model are negligibly small, allowing us to decompose breakeven inflation rates into its components without raising concerns about measurement error.

To estimate the model, we apply a variant of the three step linear regression estimator introduced in Adrian, Crump, and Moench (2013). In the first step, we estimate the dynamics of the pricing factors by running an autoregression. In the second step, we estimate exposures of Treasury and TIPS excess returns with respect to lagged and contemporaneous levels of pricing factors. In the third step, we obtain the parameters governing the factor dynamics under the risk-neutral measure from a cross-sectional regression of the return exposures to the lagged pricing factors onto return exposures to contemporaneous pricing factors. In our empirical application, we employ six factors: three principal components from the cross-section of Treasury yields, two principal components extracted from TIPS yields, and one liquidity factor.

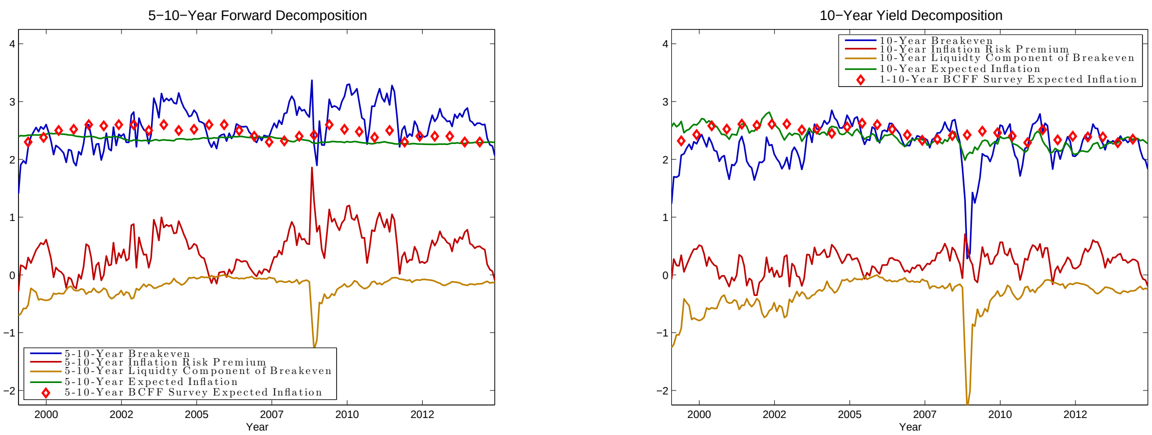

Figure 1: This figure shows the decomposition of breakeven inflation rates into the model-implied expected inflation, the inflation risk premium as well as the liquidity component. The left-hand panel shows this decomposition for 5-10 year forward breakeven inflation whereas the right-hand panel displays the decomposition for the 10 year horizon. Each graph also shows the corresponding consensus CPI forecast from the Blue Chip Financial Forecasts (BCFF) Survey.

An important finding from our model estimates relates to the decomposition of far in the future breakevens into expected inflation, the inflation risk premium, and a liquidity component. It has long been argued by market observers that variations of far in the future forward rates mainly reflect changes in risk and liquidity premia. Our model confirms that conjecture. The left-hand panel of Figure 1 shows that model implied expected 5-10 year forward inflation is stable, while the variation in the forward breakeven rates mainly captures variation in the estimated inflation and liquidity risk premiums. For ten year breakeven inflation, the decomposition is different: both expected inflation and the inflation risk premium vary considerably (the right-hand panel). The liquidity adjustment is quantitatively important for both the 5-10 year and the ten year maturities in the first few years after the inception of the TIPS program and especially during the fall of 2008.

Importantly, our model produces long-term, risk-adjusted inflation expectations that align very closely with surveys of professional forecasters (red diamonds in the charts), despite the fact that we do not incorporate any survey data in the estimation. In fact, both the model implied and the survey expectation for the 5-10 year forward expectation of inflation are centered around a level of about 2.5 percent and exhibit little variation over time. In contrast, breakeven inflation that is unadjusted for risk premiums varies considerably. For the ten year maturity, model-implied expected inflation exhibits somewhat stronger time variation, but is still substantially less volatile than unadjusted breakeven inflation. Moreover, our model-implied expected inflation comoves strongly with survey-based inflation expectations.

Monetary Policy Applications

The model allows us to address a number of salient questions about the transmission of monetary policy. We first analyze the economic underpinnings of the inflation risk premium, and show that model-implied expected inflation is a better forecaster for future inflation than unadjusted breakeven inflation. We next investigate the impact of the Federal Reserve's large-scale asset purchases (LSAPs) on the components of Treasury yields: expected future real short rates, expected inflation, real term premia, inflation risk premia and liquidity premia. The latter three components reflect the compensation investors require for bearing real interest-rate risk, inflation risk and liquidity risk, respectively. We also decompose the response of long-term real forward rates into the expected real yield and the real term premium components. Finally, we apply our model to U.K. nominal and real gilt data, and investigate the impact of the adoption of an inflation target on the inflation risk premium.

Inflation Risk Premium Interpretation

Correlating our estimate of the inflation risk premium with macroeconomic variables we find that the inflation risk premium increases when disagreement among professional forecasters about future inflation rises and when option-implied Treasury volatility increases, and declines when consumer confidence rises and the unemployment rate falls. These correlations indicate that the decomposition of breakeven inflation into expected inflation, liquidity, and the inflation risk premium is economically meaningful, even though the pricing model does not use any macroeconomic variables as inputs.

Inflation Forecasting

Breakeven inflation rates are the primary market based measure of inflation expectations and are therefore of considerable interest to policy makers and market participants. It is natural to ask whether risk-adjusted, model-implied expected inflation is a better forecaster of realized inflation than the unadjusted breakeven inflation rates. We compute our model-implied average expected inflation over the next twelve, 24 and 36 months. We compare these predictions with those implied by actual observed zero-coupon breakeven inflation rates as well as those implied by a simple random walk of inflation. Considering the full sample from 1999:01-2014:11, our model outperforms the unadjusted breakevens at all horizons, with the relative improvement over the unadjusted breakevens ranging from 32% at the twelve month horizon to over 100% at the three year horizon. Our adjusted breakevens also outperform unadjusted breakevens when comparing out-of-sample forecasts.

Conventional Monetary Policy

Our decomposition allows us to study the question of how conventional monetary policy affects the real and nominal yield curves. Based on the identification of monetary policy shocks as in Kuttner (2001) and Bernanke and Kuttner (2005), we find that for the sample period 1999:01-2008:09 both nominal and real forwards significantly comove with unanticipated monetary policy changes, and that for far into the future forwards these effects are exclusively due to changes in real term premia. We also find that the response of breakeven inflation rates to monetary policy shocks is primarily driven by inflation risk premia rather than expected inflation. These results provide support for the notion that an important transmission channel of interest rate policy is via the equilibrium pricing of risk in the economy as proposed by the recent literature on the "risk taking channel" of monetary policy (see Adrian and Shin (2010), Borio and Zhu (2012), and Dell'Ariccia, Laeven, and Marquez (2013)).

Unconventional Monetary Policy

We use our decompositions of the TIPS and Treasury term structures to revisit the effects of the Federal Reserve's large-scale asset purchases and the maturity extension program (MEP). These operations were a major part of the central bank response to the recent financial crisis after the level of the federal funds rate was pushed close to its lower bound. Using an event study methodology similar to Gagnon, Raskin, Remache, and Sack (2011) and

Krishnamurthy and Vissing-Jorgensen (2011), we find that the real and nominal term premium declined due to the LSAPs and the MEP, while the risk-adjusted expectations of real and nominal rates rose slightly. These findings are fully consistent with the "duration risk" and the "preferred habitat" channels of asset purchases, as emphasized by Gagnon, Raskin, Remache, and Sack (2011). In contrast, our results do not support the "signaling channel" of the purchase programs that has been emphasized by Bauer and Rudebusch (2014). In addition, our results suggest that the purchase programs primarily reduced the pricing of real-interest rate risk.

Inflation Targeting

Finally, we demonstrate the robustness of our approach by fitting the United Kingdom's term structure using our regression-based method. As for the U.S., our estimates indicate that volatility in long-horizon breakevens is largely driven by fluctuations in the inflation risk premium while inflation expectations remained relatively stable since the late 1990s. In particular, our model shows a sharp drop in the inflation risk premium in the years following the introduction of a numerical inflation target by the Bank of England in 1992.

The remainder of the paper is organized as follows. Section 2 introduces the joint model for Treasury and TIPS yields. Section 3 discusses estimation. Section 4 summarizes the empirical results. Section 5 presents a number of applications relevant to the transmission of monetary policy. Section 6 reviews the related literature and Section 7 concludes. Detailed derivations are left to an appendix.

2 The Model

2.1 State Variable Dynamics and Pricing Kernel

As is common in Gaussian term structure models[1], we assume that a $K \times 1$ vector of pricing factors evolves under the physical measure ($\mathbb{P}$) according to the autoregression

$$X_{t+1} - \mu_X = \Phi(X_t - \mu_X) + \nu_{t+1} \tag{1}$$

where ν_t are *i.i.d.* Gaussian with $\mathbb{E}_t[\nu_{t+1}] = 0_{K\times 1}$ and $\mathbb{V}_t[\nu_{t+1}] = \Sigma$. We also assume that assets are priced by the stochastic discount factor

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\Sigma^{-1/2}\nu_{t+1}\right), \tag{2}$$

where r_t is the nominal short rate. We follow Duffee (2002) in assuming that the $K \times 1$ price of risk vector λ_t takes the essentially affine form

$$\lambda_t = \Sigma^{-1/2}(\lambda_0 + \lambda_1 X_t). \tag{3}$$

We further define the following objects:

$$\tilde{\mu} = (I_K - \Phi)\mu_X - \lambda_0, \tag{4}$$

$$\tilde{\Phi} = \Phi - \lambda_1. \tag{5}$$

These parameters govern the dynamics of the pricing factors under the risk-neutral or pricing measure ($\mathbb{Q}$) and feature prominently in the recursive pricing relationships derived in the following section.

[1]See Piazzesi (2003) and Singleton (2006) for overviews.

2.2 No-Arbitrage Pricing

2.2.1 Nominal Bonds

In Gaussian affine term structure models the log price, $P_t^{(n)}$, of a risk-free discount bond with remaining time to maturity n, takes the form

$$\log P_t^{(n)} = A_n + B_n' X_t, \tag{6}$$

which implies that

$$r_t = \delta_0 + \delta_1' X_t. \tag{7}$$

Imposing no-arbitrage restrictions gives rise to the following relationship between parameters[2]

$$\begin{aligned} A_n &= A_{n-1} + B_{n-1}'\tilde{\mu} + \frac{1}{2} B_{n-1}' \Sigma B_{n-1} - \delta_0, & (8)\\ B_n' &= B_{n-1}'\tilde{\Phi} - \delta_1', & (9)\\ A_0 &= 0, \qquad B_0 = 0_{K\times 1}. & (10) \end{aligned}$$

Note that log excess one-period holding returns are defined as

$$rx_{t+1}^{(n-1)} = \log P_{t+1}^{(n-1)} - \log P_t^{(n)} - r_t.$$

Plugging in equations (6) and (7) we obtain

$$rx_{t+1}^{(n-1)} = (A_{n-1} - A_n - \delta_0) - (B_n' + \delta_1')\, X_t + B_{n-1}' X_{t+1}.$$

Thus, imposing the recursive equations yields

$$rx_{t+1}^{(n-1)} = \alpha_{n-1} - B_{n-1}'\tilde{\Phi} X_t + B_{n-1}' X_{t+1}, \tag{11}$$

[2] See also Ang and Piazzesi (2003).

where

$$\alpha_{n-1} = -\left(B'_{n-1}\tilde{\mu} + \frac{1}{2}B'_{n-1}\Sigma B_{n-1}\right).$$

As we will see below, this representation of holding period excess returns will be useful for estimation of the model parameters based on predictive return regressions.

2.2.2 Inflation-Indexed Bonds

We expand the ordinary Gaussian ATSM framework to allow for the pricing of inflation-indexed securities jointly with nominal securities so that both yield curves are affine in the state variables. Similar models have been studied before in both continuous and discrete time, and Section 6 discusses the relationship of our model to this literature. Let Q_t be a price index at time t, and let $P_{t,R}^{(n)}$ denote the price at time t of an inflation-indexed bond with face value 1, paying out the quantity $\frac{Q_{t+n}}{Q_t}$ at time $t+n$. The price of such a bond satisfies

$$P_{t,R}^{(n)} = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-r_t - \ldots - r_{t+n-1}\right)\frac{Q_{t+n}}{Q_t}\right], \tag{12}$$

where $\mathbb{E}_t^{\mathbb{Q}}$ denotes the expectation under the pricing measure. Let one period log inflation be $\pi_t = \ln\left(\frac{Q_t}{Q_{t-1}}\right)$, so that

$$\frac{Q_{t+n}}{Q_t} = \exp\left(\sum_{i=1}^{n}\pi_{t+i}\right). \tag{13}$$

We assume that log prices of inflation-indexed bonds are affine in the pricing factors:

$$\log P_{t,R}^{(n)} = A_{n,R} + B'_{n,R}X_t. \tag{14}$$

This implies that one-period inflation is also a linear function of the pricing factors:

$$\pi_t = \pi_0 + \pi_1' X_t, \tag{15}$$

where π_0 is a scalar and π_1 a vector of length K. We can derive pricing recursions for inflation-indexed bonds by rewriting equation (12) in terms of an indexed bond purchased one period ahead:

$$P_{t,R}^{(n)} = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-r_t + \pi_{t+1}\right) P_{t+1,R}^{(n-1)}\right]. \tag{16}$$

Solving this equation and matching coefficients we then find that the coefficients in equation (14) are determined by the following system of difference equations

$$\begin{aligned}
A_{n,R} &= A_{n-1,R} + \left(B_{n-1,R} + \pi_1\right)' \tilde{\mu} + \frac{1}{2}\left(B_{n-1,R} + \pi_1\right)' \Sigma \left(B_{n-1,R} + \pi_1\right) - \delta_{0,R} & (17)\\
B'_{n,R} &= \left(B_{n-1,R} + \pi_1\right)' \tilde{\Phi} - \delta'_1 & (18)\\
A_{0,R} &= 0, \qquad B_{0,R} = 0_{K\times 1} & (19)
\end{aligned}$$

where we have defined the real short rate parameter $\delta_{0,R} = \delta_0 - \pi_0$. Log excess one period holding returns on inflation indexed securities are then given by

$$rx_{t+1,R}^{(n-1)} = \log P_{t+1,R}^{(n-1)} - \log P_{t,R}^{(n)} - r_t. \tag{20}$$

Adding inflation to both sides of equation (20) and combining with equations (15), (17), and (18), we obtain

$$rx_{t+1,R}^{(n-1)} + \pi_{t+1} = \alpha_{n-1,R} - \left(B_{n-1,R} + \pi_1\right)' \tilde{\Phi} X_t + \left(B_{n-1,R} + \pi_1\right)' X_{t+1}, \tag{21}$$

where

$$\alpha_{n-1,R} = -\left(\left(B_{n-1,R} + \pi_1\right)' \tilde{\mu} + \frac{1}{2}\left(B_{n-1,R} + \pi_1\right)' \Sigma \left(B_{n-1,R} + \pi_1\right)\right).$$

Stacking log excess holding period returns on nominal bonds from equation (11) and on inflation-indexed bonds from equation (21) into the vector R, we thus obtain

$$R_{t+1} = \alpha - B\tilde{\Phi} X_t + B X_{t+1},$$

where

$$\alpha = -\left(B\tilde{\mu} + \frac{1}{2}\gamma\right), \quad B = \left(B_1, \ldots, B_{N_N}, B_{1,R} + \pi_1, \ldots, B_{N_R,R} + \pi_1\right)',$$

$$\gamma = \left(B'_1 \Sigma B_1, \ldots, B'_{N_N} \Sigma B_{N_N}, \left(B_{1,R} + \pi_1\right)' \Sigma \left(B_{1,R} + \pi_1\right), \ldots, \left(B_{N_R,R} + \pi_1\right)' \Sigma \left(B_{N_R,R} + \pi_1\right)\right)'. \tag{22}$$

2.3 Expected Inflation

The affine model can be used to compute expected inflation under both the risk-neutral and the physical measure at any horizon. For a given forecast horizon n, model-implied average expected inflation under the pricing measure is given by the breakeven inflation rate, i.e., the difference between the yields on a nominal and an inflation-indexed bond with maturity n:

$$\pi_t^{(n)} = y_t^{(n)} - y_{t,R}^{(n)} = -\frac{1}{n}\left[A_n + B_n' X_t - \left(A_{n,R} + B_{n,R}' X_t\right)\right].$$

Model-implied expected average inflation under the physical measure is then simply obtained by replacing the parameters $\tilde{\mu}$ and $\tilde{\Phi}$ with their risk-adjusted counterparts $(I_K - \Phi)\,\mu_X$ and Φ in equations (8), (9), (17), and (18). The difference between the two measures of inflation expectations constitutes the inflation risk premium — the compensation investors require for bearing inflation risk.

2.4 TIPS Liquidity Effects

Pflueger and Viceira (2013) document that the liquidity of TIPS relative to Treasuries appears to be systematically priced. The relative liquidity of TIPS from their inception until 2003, when the Treasury reaffirmed its commitment to the TIPS program, and in the aftermath of the Lehman bankruptcy in late 2008, which resulted in its considerable TIPS inventory being released into the market have been discussed e.g., in Sack and Elsasser (2004) and Campbell, Shiller, and Viceira (2009). Liquidity premia have also been found in nominal Treasuries, see for example Gurkaynak, Sack, and Wright (2007) and Fontaine and Garcia (2012).

It is straightforward to model the impact of liquidity on nominal and real yields in our framework. Let L_t be a factor capturing systematic changes in liquidity which we assume to be observed. We can simply expand the state space so that the vector of factors X_t includes this liquidity factor. The pricing coefficients remain subject to the recursive restrictions given in equations (8) and (9) as well as (17) and (18) above. In our empirical results documented in Section 4, we will report "liquidity-adjusted" estimates of the inflation risk premium and of expected inflation, i.e., we first subtract the liquidity components associated with the factor L from nominal and inflation-indexed bond yields prior to computing the inflation-related indicators. We discuss the construction of our liquidity factor in Section 4.1.

3 Estimation

In this section, we discuss how the joint pricing model for nominal and inflation-indexed bonds can be estimated with a simple set of linear regressions. We begin by discussing estimation of the risk-neutral VAR parameters $\tilde{\mu}$ and $\tilde{\Phi}$ from a panel of nominal and inflation-indexed bond returns. We then discuss estimation of the parameters (δ_0, δ_1') and (π_0, π_1') which govern the model-implied dynamics of the nominal short rate and inflation.

3.1 Estimation of Risk-neutral VAR Parameters

To estimate our parameters we stack the observed return data as

$$\mathbf{R} = \alpha \iota_T' - B\tilde{\Phi}X_- + BX + E \tag{23}$$

where $\mathbf{R}$ is $N \times T$, X_- and X are $K \times T$ matrices of the stacked X_{t-1}'s and X_t's, respectively, and ι_T is a $T \times 1$ vector of ones. The matrix E is the stacked return measurement errors (or return pricing errors) where the sth column, e_s, satisfies $\mathbb{E}_{s-1}[e_s] = 0$ and $\mathbb{E}_{s-1}[e_s e_s'] = \Sigma_e$. We first run a seemingly-unrelated regression (SUR) of $\mathbf{R}$ on ι_T, X_- and X which yields $\left(\hat{\alpha}_{\text{ols}}, \widehat{B\tilde{\Phi}_{\text{ols}}}, \hat{B}_{\text{ols}}\right)$. Using the estimated residuals, $\hat{E}_{\text{ols}}$, from this regression we obtain $\hat{\Sigma}_e = T^{-1} \cdot \hat{E}_{\text{ols}} \hat{E}_{\text{ols}}'$. Then, our estimator of $\tilde{\Phi}$ is,

$$\hat{\tilde{\Phi}}_{\text{gls}} = -\left(\hat{B}_{\text{ols}}' \hat{\Sigma}_e^{-1} \hat{B}_{\text{ols}}\right) \hat{B}_{\text{ols}}' \hat{\Sigma}_e^{-1} \widehat{B\tilde{\Phi}_{\text{ols}}}.$$

We then run an additional SUR on ι_T and $\left(-\hat{\tilde{\Phi}}_{\text{gls}} X_- + X\right)$ to obtain more efficient estimates of α and B which we label $\hat{\alpha}_{\text{gls}}$ and $\hat{B}_{\text{gls}}$. Finally, we estimate $\tilde{\mu}$ as

$$\hat{\tilde{\mu}}_{\text{gls}} = -\left(\hat{B}_{\text{gls}}' \hat{\Sigma}_e^{-1} \hat{B}_{\text{gls}}\right)^{-1} \hat{B}_{\text{gls}}' \hat{\Sigma}_e^{-1} \left(\hat{\alpha}_{\text{gls}} + \frac{1}{2}\hat{\gamma}_{\text{gls}}\right),$$

where $\hat{\gamma}_{\text{gls}}$ is formed using $\hat{B}_{\text{gls}}$ and $\hat{\Sigma}$ (see equation (22)).

As discussed in Section 2, the parameters $\tilde{\mu}$ and $\tilde{\Phi}$ are related to the market price of risk parameters λ_0 and λ_1 via the relationships $\tilde{\mu} = (I_K - \Phi)\,\mu_X - \lambda_0$ and $\tilde{\Phi} = \Phi - \lambda_1$. Since the pricing factors X are observed and follow the joint vector autoregression given by equation (1), the OLS estimator of μ_X is simply given by the sample mean of the factors X and the OLS estimator of Φ is obtained by regressing the demeaned observations of X on their one period lags equation by equation (see e.g., Lütkepohl (2007)). We stack the estimated innovations into the matrix $\hat{V}$ and construct an estimator of the state variable variance-covariance matrix $\hat{\Sigma} = T^{-1} \cdot \hat{V}\hat{V}'$. Alternative estimators of μ and Φ could be used such as bias-corrected versions (Bauer, Rudebusch, and Wu (2012); see Section 4.3 for further discussion). Given estimates $\hat{\mu}_X$ and $\hat{\Phi}$, we then obtain estimates of the market price of risk parameters via

$$\hat{\lambda}_0 = \left(I_K - \hat{\Phi}\right)\hat{\mu}_X - \hat{\tilde{\mu}}_{\text{gls}}, \tag{24}$$

$$\hat{\lambda}_1 = \hat{\Phi} - \hat{\tilde{\Phi}}_{\text{gls}}. \tag{25}$$

Asymptotic standard errors for these estimators are provided in Appendix B. Our estimation approach is in the spirit of Adrian, Crump, and Moench (2013) who propose a three-step least-squares estimator for the parameters λ_0 and λ_1 in an affine model for nominal bonds and show that it is consistent and asymptotically normal under standard distributional assumptions. The estimator may be viewed as a generalization of the Fama and MacBeth (1973) procedure allowing for time varying prices of risk (see also Adrian, Crump, and Moench (2014)). As we use principal components as factors in our model, we appeal to the literature on estimation of dynamic factor models which demonstrates that under weak conditions the principal components may be treated as if they were observed data in subsequent regressions (e.g., Bai and Ng (2006)).

Other than Adrian, Crump, and Moench (2013), to the best of our knowledge, all estimation approaches for Gaussian ATSMs use yields instead of returns to estimate the model parameters. In addition, it is typically assumed that the *yield* fitting errors are serially uncorrelated. However, estimated yield pricing errors from ATSMs show highly-persistent autocorrelation across a wide range of model specifications and estimation methods (see Hamilton and Wu (2011) and Adrian, Crump, and Moench (2013)). Instead, we assume that *return* fitting errors are serially uncorrelated. This implies that the only source of predictability in excess returns is generated by the factors, which we view as a desirable property of a term-structure model.

3.2 Estimation of Short Rate and Inflation Parameters

Before we can compute the recursive bond pricing parameters for nominal bonds given in equations (8) and (9), we need to estimate the parameters (δ_0, δ_1') governing the nominal short rate. Since the nominal short rate is directly observed, this is simply achieved by performing an OLS regression of the short rate onto a constant and the vector of pricing factors as in Adrian, Crump, and Moench (2013).

As can be seen from equations (17) and (18), for inflation-indexed bonds we also need to estimate the parameters π_0 and π_1 which govern the evolution of inflation as a function of the state variables. We show in Appendix A that inflation-indexed bond yields are linear and quadratic functions of these two parameters, respectively. Therefore the information in the time series and cross-section of inflation-indexed bonds can be used to estimate them. In Appendix A we provide explicit expressions for real yields as linear-quadratic functions of π_0 and π_1 (given estimates for $\tilde{\mu}$ and $\tilde{\Phi}$) which may be used for numerical optimization. In our empirical application we use the sum of squared real yield fitting errors as the criterion function.

4 Empirical Results

In this section, we provide results from the estimation of our joint term structure model. We first discuss the data sources as well as our choice of pricing factors. We next characterize the fit of the preferred model specification for both the Treasury and the TIPS curves and present results on the estimated prices of risk. Finally, we decompose breakeven inflation into its constituents: inflation expectations, the inflation risk premium, and a liquidity component.

4.1 Data and Factor Construction

We obtain zero coupon bond yields from the Gurkaynak, Sack, and Wright (2007, 2010) datasets (GSW hereafter) which are available at a daily frequency on the Board of Governors of the Federal Reserve's research data page.[3] The GSW real and nominal yield curves are based on fitted Nelson-Siegel-Svensson curves, the parameters of which are published along with the estimated zero coupon curve. We use these parameters to back out the cross-section of real and nominal zero-coupon yields for maturities up to 10 years for TIPS and Treasuries, using end-of-month values from 1999:01 to 2014:11 for a total of $T = 191$ monthly observations. In the estimation, we use a cross-section of $N_N = 11$ one-month holding period returns for nominal Treasuries with maturities $n = 6, 12, 24, \ldots, 120$ months and $N_R = 9$ excess returns on TIPS with maturities $n = 24, \ldots, 120$ months.[4] We use the one-month Treasury yield from GSW as the nominal risk free rate. The price index Q_t used to calculate TIPS payouts is seasonally unadjusted CPI-U, which is available from the Bureau of Labor Statistics website.

We have discussed in Section 2.4 that liquidity considerations play an important role in the pricing of TIPS securities. We therefore explicitly model the relative liquidity in the TIPS and nominal Treasury market. To this end, we construct a composite factor of relative TIPS liquidity as the equal weighted average of two indicators.

The first is the average absolute TIPS yield curve fitting error from the Nelson-Siegel-Svensson model of Gurkaynak, Sack, and Wright (2010) which we obtain from the Board of Governors of the Federal Reserve.[5] Because large fitting errors may imply stress in the market and investors' inability to take advantage of mispricing, this measure is a good proxy for the poor relative liquidity of TIPS. Notably, the series shows a sharp spike in late 2008 after the failure of Lehman Brothers which severely compromised market functioning for inflation-indexed securities for a period of time.

The second liquidity indicator that we use is the 13-week moving average of the ratio of

[3]See `http://www.federalreserve.gov/econresdata/researchdata.htm`

[4]We start at a maturity of two years because short maturities tend to be distorted by the "carry adjustment" due to the CPI indexation lag. In fact, GSW do not publish maturities below 24 months.

[5]The absolute yield curve fitting errors first appeared as a measure of liquidity in Fleming (2000); Hu, Pan, and Wang (2013) first used the measure in an asset pricing context.

primary dealers' nominal Treasury transaction volumes relative to TIPS transaction volumes. We obtain this series from the Federal Reserve Bank of New York's FR2004 "Weekly Release of Primary Dealer Positions, Transactions, and Financing." This second indicator captures the liquidity of TIPS relative to nominal Treasuries in the first few years after the introduction of the TIPS program particularly well. To construct our composite liquidity indicator, we first standardize both series and then compute their equal weighted average. To ensure the positivity of the index we further add to each observation the negative of the time series minimum of the equal weighted average of the two standardized series. We enforce the positivity of the index so that illiquidity can only result in higher, not lower, yields.

Following Adrian, Crump, and Moench (2013) and various other authors (e.g., Joslin, Singleton, and Zhu (2011), Wright (2011)), we use principal components extracted from yields as pricing factors in our model. Specifically, we use two sets of principal components. First, we extract K_N principal components from nominal Treasury yields of maturities $n = 3, \ldots, 120$ months. We then obtain additional factors as the first K_R principal components from the residuals of regressions of TIPS yields of maturities $n = 24, \ldots, 120$ months on the K_N nominal principal components as well as the liquidity factor. This orthogonalization step is done so as to reduce the unconditional collinearity among the pricing factors. In sum, we then have a total of $K = K_N + K_R + 1$ model factors. In the next subsection we discuss our choices of K_N and K_R.

4.2 Model Selection

In order to price the time series and cross-section of nominal and real Treasury bonds with a high level of precision, one needs to use model factors that span the same space as that spanned by both sets of yields. It has often been documented (see e.g., Scheinkman and Litterman (1991), Garbade (1996, Chapter 16)) that the cross-sectional variation of nominal Treasury yields is almost fully captured by three principal components which are commonly referred to as level, slope, and curvature. For parsimony, we therefore choose $K_N = 3$ nominal pricing factors which we estimate as the first three principal components of the nominal Treasury yield curve.[6]

Relatively little is known about the factor structure of the joint cross section of TIPS and Treasury yields. While Gurkaynak, Sack, and Wright (2010) document that three principal components suffice to explain almost all of the cross-sectional variation in nominal yields, TIPS yields, or breakeven inflation rates separately, they do not study the extent to which the principal components extracted from the three sets of yields are cross-correlated. Moreover, they do not explicitly consider the role of liquidity as another source of comovement across TIPS yields. We choose the number K_R of real pricing factors in the following simple way. First, we regress TIPS yields on the liquidity factor and the first three principal components of the nominal yield curve. We then extract principal components from the residuals of these regressions. A variance decomposition shows that about 90 percent of the cross-sectional variation is explained by the first and that more than 99 percent are accounted for by the first two principal components. We therefore choose to use $K_R = 2$ principal components extracted from these orthogonalized real yields residuals as additional pricing factors. Combined with the liquidity factor and the three nominal factors, our model therefore has a total of six pricing factors. We will see in Section 5.2 that this parsimonious model does very well at predicting inflation out-of-sample.

Figure 2 provides time series plots of all six model factors. The first three factors are the typical level, slope, and curvature of the nominal Treasury yield curve. The fourth and fifth factor represent the first two principal components extracted from the components of TIPS yields that are orthogonal to the nominal principal components and the liquidity factor. These two components do not have a clear-cut economic interpretation. The sixth factor is our composite liquidity indicator. As the chart shows, the illiquidity of the TIPS market relative to nominal Treasuries was high in the early years of the TIPS program, then receded markedly when the Treasury reaffirmed its commitment to the TIPS program, and shot up in the wake of the Lehman bankruptcy. Given this set of pricing factors as well as the excess holding period Treasury and TIPS returns, we then estimate the parameters of our model

[6] Cochrane and Piazzesi (2005, 2008), Duffee (2011), Joslin, Priebsch, and Singleton (2012), Adrian, Crump, and Moench (2013) and others show that factors with negligible contemporaneous effects on the yield curve can have strong predictive power for future excess returns. Consistently with that literature, Section 4.3 presents findings that the additional factors in our joint model significantly affect excess returns on nominal bonds.

according to the procedure laid out in Section 3.

4.3 Parameter Estimates and Model Fit

In this section, we show that our preferred model fits both yield curves precisely and gives rise to substantial time variation in the prices of risk. We start by discussing some specification tests for our choice of pricing factors. Specifically, we assess the relative importance of each of the model factors in explaining cross-sectional variation of nominal Treasury returns, TIPS returns, and their joint cross-section. Table 1 provides Wald statistics and their associated p-values for tests of whether the risk factor exposures associated with individual pricing factors are jointly different from zero. Not surprisingly, we find that nominal Treasury returns are significantly exposed to all three principal components extracted from nominal Treasury yields. However, nominal Treasury returns do not comove with the two principal components extracted from orthogonalized TIPS yields and only little with the liquidity factor, as indicated by the associated Wald statistics in the last three rows of the first column of Table 1. Hence, the cross-sectional variation of nominal Treasuries in our model is largely captured by the nominal principal components. In contrast, TIPS returns comove strongly with innovations to all six pricing factors of the model, including the liquidity factor. Moreover, considering the joint cross-section of nominal Treasury and TIPS returns, we see that all six model factors are associated with significant risk exposures. This is important as the full column rank of the matrix B is required for the identification of the market price of risk parameters (see the discussion in Adrian, Crump, and Moench (2013)).

We next assess the fit of the model for Treasury and TIPS yields and study the estimates of the price of risk parameters. Table 2 reports the time series properties of the yield pricing errors for the nominal Treasury curve implied by the model. We see that the average yield pricing errors are very small, not exceeding two and a half basis points in absolute value. They also exhibit little variability as the standard deviations of Treasury yield fitting errors are of the order of not more than nine basis points across all maturities. Table 3 reports analogous results for yield and return fitting errors of TIPS. Our model fits TIPS yields as well as nominal Treasuries as the maximum average yield pricing errors for TIPS across the maturity spectrum is about one basis point in absolute value. The variability of TIPS pricing errors is also similar to that of nominal Treasuries, ranging from about six to eight basis points with the exception being the two-year maturity which is considerably more volatile.

Consistent with the relationship between yield and return pricing errors discussed in Adrian, Crump, and Moench (2013), pricing errors for both nominal and TIPS yields display a notable degree of serial correlation while the corresponding return pricing errors show little serial correlation. This implies that the predictability in excess holding period returns on nominal Treasuries and TIPS is fully captured by the pricing factors in our model. Moreover, as term premia represent expected excess returns on bonds, this finding gives a first indication that our model provides reasonable estimates of term premia.

Figures 3 and 4 provide a number of different visual diagnostics of the time series fit of the model for both curves. In particular, the upper two panels of these figures show the observed and model-implied time series of yields at different maturities for the Treasury and TIPS curve, respectively. Consistent with the results in Tables 2 and 3 discussed above, the two lines are almost visually indistinguishable in all of these charts. The lower two panels in the figures plot the observed and model-implied excess holding period returns. We also superimpose the model-implied expected excess returns to demonstrate that there is substantial time variation in the estimated compensation for risk.

The upper two panels of Figures 5 and 6 provide plots of unconditional first and second moments of both yield curves as observed and fitted by the model. The charts visually demonstrate that the model fits both moments very well for both curves. The lower left panels of Figures 5 and 6 provide plots of the estimated yield loadings $-\frac{1}{n}B_n$ for Treasuries and $-\frac{1}{n}B_{n,R}$ for TIPS. These allow us to interpret the model factors according to their respective loadings on different sectors of both yield curves. In line with the previous literature, the first principal component clearly represents a level factor for both the Treasury and the TIPS term structure. Similarly, the second nominal principal component represents a slope factor featuring positive loadings on short maturities and negative loadings on long maturities in both curves. The charts further reveal that the two real factors as well as the liquidity factor have essentially no impact on nominal yields and affect TIPS yields somewhat more for intermediate than for longer maturities.

The lower right panels of both figures display the corresponding excess return loadings $B_n\lambda_1$ for Treasuries and $B_{n,R}\lambda_1$ for TIPS. These show that the slope factor is an important driver of expected excess returns on nominal Treasuries, consistent with the evidence presented in e.g., Campbell and Shiller (1991) and Adrian, Crump, and Moench (2013). In addition, the second principal component extracted from the components of TIPS yields that are orthogonal to nominal yields and the liquidity factor also shows a strong impact on expected excess returns on nominal bonds. A look at the lower-right panel of Figure 6 reveals that a different set of factors is largely responsible for the dynamics of expected excess returns on TIPS. In particular, the liquidity factor as well as the level and curvature of the nominal yield curve are important drivers of expected excess returns on TIPS.

Table 4 provides the estimated market price of risk parameters for our model as well as the associated t-statistics. The last two columns of the table show Wald statistics for the tests that the corresponding rows of $\Lambda = [\lambda_0,\ \lambda_1]$ and λ_1 are jointly different from zero. The first test allows us to assess whether a given risk factor is priced in the cross-section of Treasury and TIPS returns. The second enables us to test whether the price of risk associated with a given factor features significant time variation. All but the fourth and sixth factor are significantly priced in the joint cross-section. Moreover, the prices of risk of the level, slope, curvature, as well as the second real factor feature significant time variation.

Given the pricing factors and the estimated model parameters, we can decompose breakeven inflation rates at any horizon into its three constituents: expected inflation, the inflation risk premium, as well as a liquidity premium. In Figure 1 in the introduction we showed the time series of these components for both the ten year breakeven inflation as well as the 5-10 year forward breakeven inflation rate. These charts highlight the two main conclusions from our model. First, while expected inflation explains some of the variation of average inflation over the next ten years, it is stable at long forward horizons. This implies that the bulk of the variability of long-term forward breakeven inflation rates is driven by inflation risk premia. Second, while liquidity effects have played only a minor role from around 2004 through the first half of 2008, they have strongly contributed to the dynamics of breakeven rates in the early years of the TIPS program and in the recent financial crisis. In particular, our model largely attributes the collapse of long-term forward breakeven inflation rates in the crisis to liquidity effects rather than changes in underlying inflation expectations. Figure 7 provides a more detailed picture of term premia for the 5-10 year forward horizon. The upper-left chart shows the decomposition of the 5-10 year Treasury forward into risk-adjusted future nominal short rates, the nominal forward term premium and the liquidity premium. The latter is essentially zero, but featured a small uptick in the financial crisis. While the nominal 5-10 year forward rate had fallen to levels below three percent in 2011 and 2012 in response to the large-scale asset purchase programs by the Federal Reserve, it increased sharply in the spring and summer of 2013. This episode is sometimes referred to as the "taper tantrum" as the sudden rise in term premiums and yields was largely attributed to speculation about a tapering of asset purchases by the Federal Reserve. In the past year long-term nominal forward rates have declined substantially but remain above the low levels seen in 2012. Our model indicates that the sudden rise and subsequent decline of long-term forward rates is primarily driven by movements in long-term nominal forward premiums but that expectations of long-term nominal short rates have remained steady around two percent.

The upper-right chart provides the equivalent decomposition of the 5-10 year TIPS yield into a risk-adjusted expected real short rate, the real term premium and the liquidity component of real yields. The liquidity component amounted to about one percent at the beginning of our sample, but was close to zero between 2004 and mid-2008 when it spiked up to about two percent. The estimated real term premium declined from about three percent to around one percent in 2011 and 2012 and spiked up sharply during the taper tantrum episode in 2013. As for much of the sample, it is currently estimated to be higher than the corresponding real forward rate, implying that risk-adjusted expected real short rates are negative at the 5-10-year forward horizon.

The lower-left chart in Figure 7 superimposes the two estimated forward term premia, as well as their difference, the forward inflation risk premium. The chart shows that at low and intermediate frequencies real term premia account for the bulk of variations of nominal term premia. Instead, inflation risk premia only capture a relatively small share of the variation in nominal term premia. Hence, the compensation investors require for bearing real interest rate risk — the risk that real short rates don't evolve as they expected — is the main driving force behind movements in nominal term premia according to our model.

While both the estimated nominal and real long-term forward term premia have consistently been positive throughout the sample period, the forward inflation risk premium has been negative at times. However, the fact that it has been positive on average is consistent with the notion that—adjusted for liquidity risk—investors require compensation for holding nominal instead of inflation-protected long-term Treasury securities. Finally, the lower-right chart visualizes the decomposition of liquidity-adjusted 5-10 year forward breakevens into risk-adjusted expected average future inflation over the next five to ten years and the inflation risk premium. According to our model, the inflation risk premium accounts for most of the variability of forward breakeven rates while average expected future inflation has been very stable around 2.5 percent and has only declined slightly in recent years. Importantly, the persistence of long-term inflation expectations implied by our model is consistent with corresponding survey measures of inflation expectations. For example, the 5-10 year forward consensus inflation forecast from the Blue Chip Financial Forecasts survey which is observed twice a year and also shown in the chart has also hovered around 2.5 percent since the late 1990s.

In sum, the estimates implied by our model suggest that movements in long-term forwards mainly reflect changes in term premia rather than investors' expectations. In formal terms, this indicates that the pricing factors of our model mean revert at horizons between five and ten years out under the physical measure.

As discussed in Section 3, we could alternatively use a bias-corrected estimator for the VAR which would affect the estimated degree of persistence. Wright (2013) shows that bias-adjusted estimates of the VAR in ATSMs imply implausibly volatile nominal short rate expectations and are not well aligned with survey forecasts. We prefer the OLS estimator based on the evidence in Wright (2013) along with the results presented in this and the next section which show that our estimated term premia have reasonable properties and that risk-adjusted inflation expectations implied by our model are fully consistent with available survey data.

5 Monetary Policy Applications

In this section, we use our model to extract quantities of interest to policy makers and market participants. We start by studying the dynamics of our estimated inflation risk premium relative to relevant financial and economic time series. We then document that our breakeven inflation rates adjusted for the inflation risk premium provide a better predictor of the level of future inflation than observed breakevens. Next, we use our model to assess the channels through which large scale asset purchase programs by the Federal Reserve have impacted nominal and real Treasury yields. We then decompose the reaction of the nominal and real forward curves to unexpected changes in monetary policy. Finally, we apply our methodology to an analysis of the real and nominal term structures of U.K. gilt yields.

5.1 Interpreting the Estimated Inflation Risk Premium

It is difficult to directly interpret dynamics of the inflation risk premium implied by our model as it is a linear combination of the factors which do not all have a clear-cut economic meaning. Instead, we correlate our estimated inflation risk premium with a number of observable macroeconomic and financial variables the choices of which are motivated by economic theory. In particular, we consider 1) the three-month swaption implied Treasury volatility from Merrill Lynch (SMOVE); 2) the cross-sectional standard deviation of individual inflation forecasts four quarters ahead from the BCFF survey (DISAG); 3) the unemployment rate (UNEMP); 4) consumer confidence as measured by the Conference Board survey (CONF); 5) year-over-year core CPI inflation (CPI); and 6) the nominal trade-weighted exchange value of the U.S. Dollar (DOLLAR).

Figure 8 plots the estimated 5-10 year inflation risk premium along with each of these series. The top left chart shows that the IRP co-moves strongly with the SMOVE index at medium frequencies. This suggests that the expected volatility of Treasury securities at least to some extent reflects movements in the required compensation for bearing inflation risk demanded by investors. The top right chart documents that the IRP also correlates strongly with forecaster disagreement about future inflation. This implies that market participants command higher inflation compensation at times when there is broad disagreement about the inflation outlook. The estimated IRP also exhibits positive co-movement with the unemployment rate and negative co-movement with consumer confidence (middle two graphs). This is consistent with the notion that risk premia are countercyclical which likely also explains the negative relationship between the IRP and core inflation shown in the bottom left. Finally, there is a marginally negative relationship between the IRP and the trade-weighted exchange value of the dollar.

Table 5 provides estimation results from univariate and multivariate regressions of the 5-10 year IRP on these six variables (top panel). In addition we also provide results for the 5-10 year forward nominal (middle panel) and real (bottom panel) forward term premium. Note that as the inflation risk premium is the simple difference between the nominal and the real term premium, the coefficients in the regression on observable factors in the top panel equals the difference between the coefficients in the bottom two panels. It should also be emphasized that given the high degree of persistence in a number of these series, the reported t-statistics and R^2 need to be interpreted with caution. We therefore focus our discussion on assessing the degree of (partial) correlations between these variables. In order to do so we standardize the variables such that we can interpret the regression coefficients in relative terms. Finally, because the nominal and real term premia trend down over our sample we include a linear time trend in all regressions.

The results of these regressions for the IRP are consistent with the conclusions drawn from Figure 8. Looking at the nominal and real term premia separately we note that among the explanatory variables we have considered, the SMOVE index of implied Treasury volatility is most strongly correlated with the estimated nominal 5-10 year forward nominal term premium. This is consistent with the findings in Adrian, Crump, and Moench (2013) and corroborates the notion that the estimated term premium reflects the risk of holding Treasury securities. Since variation in nominal term premia is primarily driven by variation in real term premia it is not surprising that the SMOVE index also has the strongest relationship for the real term premium. In all, this analysis shows that the estimated term premia have an economically meaningful interpretation. We do, however note that the variability of the inflation risk premium is large, and may be difficult to reconcile with theories of time-varying risk premiums based on consumption asset pricing models. However, the tight linkage between the risk premia and volatility suggests that intermediary asset pricing theories that feature value-at-risk constraints might be a more promising economic foundation, as such theories directly link pricing of risk to the equilibrium level of volatility.

5.2 Inflation Forecasting

Breakeven inflation rates are the primary market based measure of inflation expectations and are therefore of considerable interest to policy makers and market participants alike. However, breakeven inflation dynamics may be influenced by changes in bond market liquidity as well as inflation risk premia. Since our model allows us to separate these effects, it is natural to assess its performance in terms of predicting future inflation. We compute our model-implied average expected inflation over the next 12, 24 and 36 months as described in Section 2.3. We compare these predictions with those implied by actual observed zero-coupon breakeven inflation rates as well as those resulting from a simple random walk forecast for monthly CPI inflation.

We compare both the in-sample (upper panel) and the out-of-sample (lower panel) predictive power in terms of root mean squared errors (RMSE) in Table 6. In-sample, our model forecast strongly outperforms the unadjusted breakevens at all horizons, with the relative improvement ranging from 32 percent at the one year horizon to 102 percent at the three year horizon. [7] Our adjusted breakevens also outperform an in-sample simple random walk forecast substantially across all forecast horizons, with gains ranging from about 45 percent at the one year to about 83 percent at the three year horizon.

Figure 9 displays the in-sample inflation forecasts implied by the different models along with future average inflation over different forecast horizons. These charts show that our model forecasts for inflation are generally smoother than observed breakevens and visibly closer to actual inflation especially in the recent crisis period. Moreover, consistent with the above discussion, the random walk forecast actually rises through the crisis period when market-based measures of inflation were sharply declining.

[7] Note that the actual breakeven inflation rates at the twelve month horizon are subject to substantial volatility resulting from issues related to the fitting of zero coupon TIPS yields by the Nelson-Siegel-Svensson methodology. Consequently, unadjusted breakeven forecasts at these horizons should be interpreted with caution.

We assess the out-of-sample inflation prediction performance by recursively reestimating our model each month beginning in January 2004. The results, shown in the lower panel of Table 6, document that the risk-adjusted breakevens implied by our model continue to outperform the random walk at all forecast horizons, with RMSE gains ranging from 18 percent at the one year horizon to 2 percent at the three year horizon. More importantly, the model also beats the unadjusted breakevens with forecast improvements ranging from 4 percent one year out to 11 percent three years out. It is important to emphasize that we estimate π_0 and π_1, the parameters governing the model-implied inflation process, only through the information contained in the cross-section and time series of inflation-indexed bonds.

In sum, the results in this section show that using our model to adjust breakeven inflation for liquidity and inflation risk premia substantially increases their predictive value for future realized inflation.

5.3 Assessing the Effects of Conventional Monetary Policy

Our model allows us to decompose changes in the nominal and real term structures into expected future real short rates and expected future inflation, real term premia and inflation risk premia, as well as a liquidity premium. We can thus use the model to parse out the differential effects of policy changes on the various components of the yield curve. To do so we require a daily decomposition of nominal and real yields. While estimation of our model is done at the monthly frequency we can obtain daily decompositions because our factors are observed at a daily frequency (see Adrian, Crump, and Moench (2013) for further details).[8]

In this section, we consider the effects of surprise changes in conventional U.S. monetary policy which until late 2008 primarily consisted in adjusting the target federal funds rate. Specifically, we use the surprise component of federal funds rate futures on FOMC announcement days which has been used widely used in previous studies (see, e.g., Kuttner (2001) and Bernanke and Kuttner (2005)).[9] Other studies have used alternative proxies of

[8]Because the relative volume component of our liquidity factor is available at a weekly frequency, we assume it to be constant throughout each week.

[9]We use the daily rate changes implied by the first (and second) Federal funds futures contract in the

monetary policy shocks. Nakamura and Steinsson (2013) estimate a significant response of real yields to near-term monetary policy surprises using the heteroskedasticity-based identification approach proposed by Rigobon (2003) and Rigobon and Sack (2004). In another paper, Hanson and Stein (2014) use the two-day change of the two-year nominal Treasury yield on days of announcements by the Federal Open Market Committee (FOMC) as a gauge of near and medium term monetary policy surprises. They find that long-term real forward rates show a sizable reaction to this measure of monetary policy shocks and conjecture that this is driven by movements in term premia rather than changes in long-term expected real forward rates.

In our analysis, we regress changes in the various components of the one-year forward curve onto the policy shock measure on days of FOMC announcements. We restrict ourselves to the sample period 1999:01-2008:09 in order to capture the effects of surprises about conventional monetary policy and to avoid conceptual problems with the fed funds futures surprise in the zero lower bound period at the end of our sample. The results are provided in Figure 10. The upper two charts show the regression coefficients of liquidity-adjusted nominal and real one-year forward rates on the policy shocks along with the 95% confidence interval. At the 2-3 year forward horizon, both nominal and real yields feature similarly sizable and statistically significant responses to the policy shock of about 70-80 basis points in reaction to a 100 basis point surprise change in the federal funds rate. Interestingly, while the response of nominal forward rates declines strongly with increasing maturity, real forward rates show sizable and significant reactions to policy shocks even at very long maturities of above ten years. These results are quantitatively and qualitatively consistent with Hanson and Stein (2014) despite the fact that we use a different policy shock measure and study a somewhat different sample period.

Which components of forward rates account for these strong reactions to policy shocks? We can directly assess this question using our model-based decompositions. The middle and bottom panels of Figure 10 display the decomposition of these forward rates into expectations and risk premium components. The middle panel shows that a substantial portion of the real forward responses at maturities up to about five years is due to changes in risk-

first two-thirds (last third) of the month.

adjusted expected future short rates. Hence, our results are consistent with some degree of monetary non-neutrality at short and intermediate horizons. These findings are consistent with Nakamura and Steinsson (2013) who obtain near-term monetary policy surprises from a heteroskedasticity-based identification and study the decomposition using our model-based yield decomposition. In contrast, the response of long-horizon real forwards to monetary policy shocks is almost exclusively attributed to real term premia.

The responses of risk-adjusted average forward inflation expectations and the inflation risk-premium to the two shocks are provided in the bottom panels of Figure 10. They show a statistically significant but economically negligible increase of inflation expectations, but a sizeable negative response of the inflation risk premium, particularly at intermediate horizons. Hence, a surprise increase in the federal funds rate compresses the compensation investors require to bear inflation risk, especially in the medium term. This also explains why intermediate maturity nominal forward rates react less to policy shocks than their corresponding real counterparts.

In sum our results thus provide evidence for the "risk taking channel" of monetary policy, which implies that an important transmission channel of interest rate policy is via the equilibrium pricing of risk in the economy (see Borio and Zhu (2012), Adrian and Shin (2010), and Dell'Ariccia, Laeven, and Marquez (2013)). In fact, according to our analysis longer-term real and nominal Treasury yields are primarily affected by monetary policy shocks through a repricing of real interest rate and inflation risk.

5.4 Assessing the Effects of Unconventional Monetary Policy

Beginning in early 2009 the Federal Reserve has instituted a series of purchases of longer-maturity Treasury securities. These are commonly referred to as large-scale asset purchases (LSAPs). In addition, in the latter half of 2011, the Federal Reserve also carried out a maturity extension program (MEP) which consisted of purchases of longer-maturity nominal Treasury securities and sales of shorter-maturity nominal Treasury securities. These operations, alongside forward guidance about the future path of monetary policy, were a major part of the Federal Reserve's response to the recent financial crisis once the federal funds rate was pushed close to the zero lower bound. A number of recent papers have provided estimates of the market impact of these purchases (and sales) using an event study methodology (see, for example, Gagnon, Raskin, Remache, and Sack (2011) and Krishnamurthy and Vissing-Jorgensen (2011)). Here we take a similar approach, relying on our model estimates to decompose the nominal and real yield curves into their constituent components and showing their one-day changes around major announcement dates pertaining to the LSAPs and the MEP. We use the dates selected and discussed in Appendix A of Woodford (2012) to identify reactions to major announcements relevant to the balance sheet actions of the Federal Reserve. These dates are comprised of eight days pertaining to LSAP1, four days pertaining to LSAP2, and two days pertaining to the MEP. In addition, we include two days pertaining to LSAP3 announcements, (September 13 and December 12, 2012).

We study the effects of these announcements on the components of real and nominal Treasury yields for maturities of two and ten years. We chose these two maturities to capture the impact of the programs on medium-term and long-term Treasury yields. We regress the one-day changes in yields and their components on announcement date dummy variables which take on the value of one for the dates chosen. The reported coefficients are rescaled so they can be interpreted as the cumulated change in each component on the set of LSAP announcement dates.

Table 7 displays the results for the two-year maturity. In total, the two-year nominal and real yields declined by about 70 and 120 basis points, respectively, on the 16 days relating to these balance sheet actions. Decomposing these changes, we see that the two-year nominal and real term premia dropped more sharply on these days, by a total of about 75 basis points and 144 percentage points, respectively. Consequently, the inflation risk premium – the difference between these two series – rose by about 70 basis points. Parsing the changes for the different programs in the bottom panel of Table 7 separately, we see that the bulk of these yield curve changes is attributed to LSAP1. Meanwhile, risk-adjusted expected average nominal and real short-term interest rates over the following two years rose only modestly and by similar magnitudes around these announcement dates. Accordingly, model-implied average expected inflation over the next two years was essentially unchanged by the announcements.

Table 8 shows the same set of results for the ten-year maturity. The nominal and real ten-year Treasury yields cumulatively fell by about 100 and 132 basis points on these announcement days, respectively. Nominal and real term premia, however, both declined by 108 and 161 basis points on all announcement days. Hence, the yield declines are more than accounted for by declines of real term premia which were only partly offset by an increase in the inflation risk premium. According to our estimates, the purchase programs thus primarily achieved their effects on yields through a reduction of uncertainty about future real rate prospects. Consistent with this interpretation, both the risk-adjusted nominal and real expected average future short rates over the next ten years increased somewhat on these days. Note that our finding that expected average nominal and real short rates rose on days of purchase program announcements are in contrast with the results of Bauer and Rudebusch (2014) who argue that the yield effects of purchases are due to a signaling effect about future monetary policy. Instead, our findings are consistent with the notion that the reduction in duration risk leads to a compression in term premia with the bulk accounted for by a reduction in real term premia.

In sum, these results suggest that the risk premium channel was a key transmission mechanism for the various purchase programs conducted by the Federal Reserve since 2009. Gagnon, Raskin, Remache, and Sack (2011) find a similar result in their study of the effects of LSAP1. While these authors only consider nominal term premia, our results suggest that the purchase programs reduced real term premia resulting from uncertainty surrounding future real interest rates by more than nominal term premia.

5.5 The U.K. Term Structure

To explore the robustness of our model, we estimate an additional specification using government debt curves from the United Kingdom. The British Treasury first issued inflation-indexed debt in 1981. The inflation indexed gilt market is very liquid, with about £300bn of "linkers" currently outstanding comprising nearly a quarter of all U.K. government debt.

Unfortunately, we do not observe equivalent measures of liquidity of the U.K. inflation-indexed bond market. We therefore do not include a liquidity factor in our model of the

U.K. term structures. As for the U.K., we estimate a model specification that features three principal components of nominal yields and two principal components of orthogonalized real yields as pricing factors.

We estimate our U.K. specification using end-of-month zero-coupon yields provided by the Bank of England. The sample is constructed from underlying bond data by a cubic spline interpolation methodology and extends from 1985:01 to 2012:12 for a total of $T = 336$ observations. We compute excess returns on $N = 11$ nominal maturities of $n = 6, 12, 24, \ldots, 120$ months and $N_R = 8$ real maturities of $n = 60, 66, \ldots, 120$ months. The short rate is the one-month nominal yield. The price series used to calculate Q_t for the United Kingdom, RPI, is available from the U.K. Office of National Statistics website. Average RPI inflation during this sample period is 2.48%, slightly higher than U.S. CPI-U inflation.

Table 9 summarizes the cross-sectional fit of the model. The average yield pricing error for nominal maturities is about five basis points for the three-year and about 2 basis points for the ten-year maturity. Moreover, the standard deviation of the pricing errors is less than four basis points for all maturities. Pricing errors for inflation-indexed yields are even smaller, with an average of .1 basis points and a standard deviation of 7.7 basis points at the five-year maturity and an average of 1.3 basis points and standard deviation of 4.8 basis points at the ten-year maturity. Similar to the estimates for U.S. data, the autocorrelation of yield pricing errors is high whereas the autocorrelation of return pricing errors is much closer to zero. Malik and Meldrum (2014) present estimates of the U.K. nominal term structure using the estimation method of Adrian, Crump, and Moench (2013) and report pricing errors of similar magnitude.

The good model fit is visualized in the upper two panels of Figure 11 which plot the observed ten-year rates for U.K. nominal and real yields against the model-implied fit. The lower left panel of Figure 11 plots the estimated real and nominal term premiums, as well as the inflation risk premium at the 10 year horizon. The trends in the estimated nominal term premium are generally consistent with the term premium estimation by Bianchi, Mumtaz, and Surico (2009) and Malik and Meldrum (2014), fluctuating around 1-2% in the first part of the sample but dropping to mostly negative values in the latter part of the sample.

Interestingly, the estimated inflation risk premium declines steadily since the introduction of the inflation target in the U.K. in 1992 (see King (1997a,b) for descriptions of the inflation targeting regime in the U.K.). The inflation risk premium shows a further drop around the years 1997 and 1998 when the Bank of England was granted independence. Hence, our model attributes a substantial fraction of the decline in breakeven inflation rates during that period to a reduction of inflation risk premia, indicating that investors' pricing of inflation risk changed as a direct result of the new monetary policy regime.

6 Related Literature

The literature extracting inflation expectations from the joint pricing of TIPS and Treasury yield curves is growing rapidly. Early papers on the topic include Chen, Liu, and Cheng (2010), Grishchenko and Huang (2013), and Hördahl and Tristani (2010).

A number of papers have pointed out that TIPS have been less liquid than off the run Treasury securities until about 2003 (see Sack and Elsasser (2004) and Dudley, Roush, and Ezer (2009)) which generated a liquidity premium in breakeven inflation. Pflueger and Viceira (2013) document that the liquidity of TIPS relative to Treasuries appears to be systematically priced. D'Amico, Kim, and Wei (2014) model this type of liquidity as a latent factor in an ATSM. In contrast, we use an observable factor to adjust TIPS yields for their relative liquidity.

Christensen, Lopez, and Rudebusch (2010) report estimates from an ATSM model with three nominal factors (level, slope and curvature) and one real factor (level). Their model is parsimonious, as the prices of risk are restricted so as to be consistent with a Nelson and Siegel (1987) yield curve (see Christensen, Diebold, and Rudebusch (2011) for the relation between the ATSM models and the Nelson-Siegel curve). The parsimony of the approach of Christensen, Lopez, and Rudebusch (2010) can be viewed as an alternative to our regression-based approach to overcome the computational challenges of estimating ATSMs. However, the strong parameter restrictions imposed in their setup are likely rejected in our model. Furthermore, we find that our specification using five yield factors and a liquidity factor generates smaller pricing errors than specifications with fewer factors, while preserving good out-of-sample performance. Furthermore, our model explicitly adjusts for the relative liquidity of TIPS, while Christensen, Lopez, and Rudebusch (2010) do not.

Haubrich, Pennacchi, and Ritchken (2012) present a model that uses inflation swaps, actual inflation, and survey inflation in addition to the TIPS and Treasury yield curves. Similar to an earlier paper by Adrian and Wu (2009), Haubrich, Pennacchi, and Ritchken (2012) allow for heteroskedasticity explicitly by estimating a GARCH model for the yield processes. Prices of risk are restricted to be functions of these estimated second moments. While the model of Haubrich, Pennacchi, and Ritchken (2012) combines the different data sources elegantly, the resulting inflation risk premium differs sharply from our estimates. In fact, the inflation risk premium is close to constant over time, implying that movements of far in the future breakeven forward rates reflect changes in inflation expectations. In contrast, in our model, the inflation risk premium varies substantially over time, while far in the future expected inflation is essentially constant. We view our finding as a desirable feature, implying that variations in 5-10 year forward breakevens mainly reflect changes in risk and liquidity premia — consistent with the survey evidence.

Chernov and Mueller (2012) present an ATSM model where a "hidden factor" is extracted from inflation surveys. They show that this hidden inflation survey factor is a significant price of risk factor. While we do not incorporate any survey inflation expectations, our framework would allow the introduction of their hidden factor as an unspanned factor in a straightforward manner (see Adrian, Crump, and Moench (2013) for details on how to incorporate unspanned factors in the regression based estimation of ATSMs). Such unspanned factors would affect the pricing of risk, but not the cross sectional fit of the yield curve (i.e., it would change the $\mathbb{P}-$dynamics, but not the $\mathbb{Q}-$dynamics).

Haubrich, Pennacchi, and Ritchken (2012) and Grishchenko and Huang (2013) also incorporate survey inflation expectations in their estimates of the inflation risk premium. However, those papers consider the inflation forecasts as true probability assessments, while Chernov and Mueller (2012) consider the forecasts of inflation to be subjective and possibly different from the ATSM implied inflation estimate. In contrast, we do not use any survey information in our estimation, but find model-implied inflation expectations that track survey forecasts closely.

A number of recent papers (Grishchenko, Vanden, and Zhang (2012), Christensen, Lopez, and Rudebusch (2014), Kitsul and Wright (2013) and Fleckenstein, Longstaff, and Lustig (2014)) have used the fact that final payouts on TIPS include an embedded option[10] to extract risk-neutral deflation probability forecasts. In unreported results we follow a similar approach as in Grishchenko, Vanden, and Zhang (2012) and find that with our parameter estimates the value of the option is generally small but was sizable during the financial crisis.

A commonality among the alternative approaches by D'Amico, Kim, and Wei (2014), Christensen, Lopez, and Rudebusch (2010), Haubrich, Pennacchi, and Ritchken (2012), and Chernov and Mueller (2012) is their use of maximum likelihood estimation, requiring a Kalman filter for the extraction of unobserved factors. As discussed earlier, such estimates are computationally costly, and convergence to a global maximum is generally difficult to verify. In contrast, our approach relies on linear regressions, which is numerically fast, computationally robust, and allows the straightforward extension of the model to include additional factors.

We follow the approach of many other recent papers that are pricing TIPS and use the zero coupon yield curves of Gurkaynak, Sack, and Wright (2007) and Gurkaynak, Sack, and Wright (2010) to fit Gaussian ATSMs. Prices of zero-coupon bonds have cleaner theoretical behavior than coupon-bearing securities, and fitting a zero curve adjusts for the differences in duration due to the coupon structures of various issuances. However, our estimation methodology does not rely on the availability of zero coupon bonds, and can instead be estimated from holding period returns of Treasuries and TIPS (see Adrian, Crump, and Moench (2013) who present an example of estimating the Treasury yield curve from Treasury returns that are extracted from coupon bearing yields).

7 Conclusion

We present a joint Gaussian affine term structure model for the cross section of TIPS and nominal Treasury securities that has a number of desirable features relative to the existing

[10] At maturity, the value of the bond will be the greater of the nominal principal ($100,000) or the principal adjusted for cumulative CPI-U inflation since issuance. TIPS coupon payments are not subject to this optionality.

literature. We adjust for the relative liquidity of TIPS by using the absolute pricing errors and the outstanding volume of TIPS relative to Treasuries in a model-consistent way. Our estimation approach is regression based, allowing straightforward implementation, and making the model easily adaptable to questions that require additional conditioning variables. Relative to other models in the literature, our pricing errors are small, providing decompositions of breakeven inflation into an inflation risk premium, expected inflation, and a liquidity premium with little fitting error. Importantly, we find that the volatility of far into the future forward breakeven inflation rates is primarily due to variations in risk and liquidity premia, while long-term inflation expectations are relatively stable.

Our estimated inflation risk premium is highly correlated with observable macroeconomic and financial variables such as disagreement about future inflation amongst professional forecasters, consumer confidence and measures of option-implied Treasury volatility. Furthermore, breakevens adjusted for risk and liquidity premia outperform unadjusted breakevens and a random walk in predicting realized inflation out-of-sample. When we use our estimates to investigate the impact of the Federal Reserve's large-scale asset purchases on the term structure of interest rates, we find that their primary effect is attributable to a decline in real term premia while they resulted in small increases of the inflation risk premium and risk neutral real and nominal short rates. These results are consistent with the "duration risk" and the "preferred habitat" channels of asset purchases, but not with the "signaling channel." In addition, surprises about conventional interest rate policy are associated with statistically and economically significant changes in the real term premium and the inflation risk premium. These results provide support for the notion that one transmission channel of interest rate policy is via the equilibrium pricing of risk in the economy.

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Appendix

A Estimation of π

Given the structure of the model, one can estimate π_0 and π_1 by using information contained in the entire cross-section of TIPS yields. Equation (18) implies that

$$\begin{aligned}(B_{n,\mathrm{R}}+\pi_1)' &= \pi_1'\tilde{\Phi}^n - (\delta_1-\pi_1)'\sum_{\ell=0}^{n-1}\tilde{\Phi}^\ell \\ &= \pi_1'\tilde{\Phi}^n - \delta_1'\sum_{\ell=0}^{n-1}\tilde{\Phi}^\ell + \pi_1'\sum_{\ell=0}^{n-1}\tilde{\Phi}^\ell \\ &= -\delta_1'\psi_n + \pi_1'\psi_{n+1},\end{aligned}$$

so that $B_{n,\mathrm{R}}' = -\delta_1'\psi_n + \pi_1'(\psi_{n+1}-I_K)$ where $\psi_n = \sum_{\ell=0}^{n-1}\tilde{\Phi}^\ell$. Similarly, equations (17) and (19) imply that

$$\begin{aligned}A_{n,\mathrm{R}} &= \sum_{i=1}^{n}(A_{i,\mathrm{R}}-A_{i-1,\mathrm{R}}) \\ &= -\delta_1'\sum_{i=1}^{n}\psi_{i-1}\tilde{\mu} + \pi_1'\sum_{i=1}^{n}\psi_{i-1}\tilde{\mu} - n\cdot\delta_0 + n\cdot\pi_0 \\ &+ \frac{1}{2}\sum_{i=1}^{n}(\delta_1'\psi_{i-1})\Sigma(\delta_1'\psi_{i-1})' + \frac{1}{2}\sum_{i=1}^{n}(\pi_1'\psi_{i-1})\Sigma(\pi_1'\psi_{i-1})' - \sum_{i=1}^{n}(\pi_1'\psi_{i-1})\Sigma(\delta_1'\psi_{i-1})'\end{aligned}$$

Collecting terms yields

$$\begin{aligned}A_{n,\mathrm{R}} = &\left[-\delta_1'\sum_{i=1}^{n}\psi_{i-1}\tilde{\mu} + \frac{1}{2}\sum_{i=1}^{n}(\delta_1'\psi_{i-1})\Sigma(\delta_1'\psi_{i-1})' - n\cdot\delta_0\right] \\ &+ \pi_1'\left[\sum_{i=1}^{n}\psi_i\tilde{\mu} - \sum_{i=1}^{n}\psi_i\Sigma\psi_{i-1}'\delta_1\right] \\ &+ \pi_1'\left[\frac{1}{2}\sum_{i=1}^{n}\psi_i\Sigma\psi_i'\right]\pi_1 \\ &+ n\cdot\pi_0.\end{aligned}$$

Thus, real yields may be written as

$$y_{t,\mathrm{R}}^{(n)} = -\frac{1}{n}\left(A_{n,\mathrm{R}} + B_{n,\mathrm{R}}'X_t\right),$$

where

$$\begin{aligned}-\frac{1}{n}A_{n,\mathrm{R}} = &\left[\delta_1'\frac{1}{n}\sum_{i=1}^{n}\psi_{i-1}\tilde{\mu} - \frac{1}{2}\delta_1'\left\{\frac{1}{n}\sum_{i=1}^{n}\psi_{i-1}\Sigma\psi_{i-1}'\right\}\delta_1 + \delta_0\right] \\ &+ \pi_1'\left[-\left\{\frac{1}{n}\sum_{i=1}^{n}\psi_i\right\}\tilde{\mu} + \left\{\frac{1}{n}\sum_{i=1}^{n}\psi_i\Sigma\psi_{i-1}'\right\}\delta_1\right] \\ &- \pi_1'\left[\frac{1}{2}\left\{\frac{1}{n}\sum_{i=1}^{n}\psi_i\Sigma\psi_i'\right\}\right]\pi_1 \\ &- \pi_0,\end{aligned}$$

and

$$-\frac{1}{n}B_{n,\mathrm{R}}' = \delta_1'\left\{\frac{1}{n}\psi_n\right\} - \pi_1'\left\{\frac{1}{n}(\psi_{n+1}-I_K)\right\}.$$

Thus,

$$\begin{aligned}
y_{t,\mathrm{R}}^{(n)} &= -\pi_0 + \left[\delta_1' \left\{\frac{1}{n}\psi_n\right\} X_t + \delta_1' \frac{1}{n} \sum\nolimits_{i=1}^{n} \psi_{i-1}\tilde{\mu} - \frac{1}{2}\delta_1' \left\{\frac{1}{n}\sum\nolimits_{i=1}^{n} \psi_{i-1}\Sigma\psi_{i-1}'\right\}\delta_1 + \delta_0\right] \\
&+ \pi_1' \left[-\left\{\frac{1}{n}\left(\psi_{n+1} - I_K\right)\right\} X_t - \left\{\frac{1}{n}\sum\nolimits_{i=1}^{n} \psi_i\right\}\tilde{\mu} + \left\{\frac{1}{n}\sum\nolimits_{i=1}^{n} \psi_i \Sigma \psi_{i-1}'\right\}\delta_1\right] \\
&- \pi_1' \left[\frac{1}{2}\left\{\frac{1}{n}\sum\nolimits_{i=1}^{n} \psi_i \Sigma \psi_i'\right\}\right]\pi_1.
\end{aligned}$$

is a linear-quadratic function of π_0 and π_1. Denote this function by $g\left(\pi_0, \pi_1; n, t\right)$. We then solve for the estimated π_0 and π_1 via,

$$\left(\hat{\pi}_0, \hat{\pi}_1'\right)' = \arg\min_{\pi_0, \pi_1} \sum_{i=1}^{N_R} \sum_{t=1}^{T} \left(y_{t,\mathrm{R}}^{(n_i)} - g\left(\pi_0, \pi_1; n_i, t\right)\right)^2.$$

B Asymptotic Standard Errors

Here we provide standard errors for our estimators under standard regularity conditions. Let $\hat{\alpha}_{\mathrm{ols}}$, $\hat{A}_{0,\mathrm{ols}} = -\widehat{B\tilde{\Phi}}_{\mathrm{ols}}$, and $\hat{B}_{\mathrm{ols}}$ be the appropriately partitioned OLS estimators of equation (23). In addition, define $\hat{A}_{\mathrm{ols}} = \left[\hat{A}_{0,\mathrm{ols}} \;\vdots\; \hat{B}_{\mathrm{ols}}\right]$ and partition the OLS variance matrix as,

$$\mathcal{V}_{\alpha,A,\mathrm{ols}} = \begin{pmatrix} \mathcal{V}_{\alpha,\mathrm{ols}} & \mathcal{C}_{\alpha,A,\mathrm{ols}} \\ \mathcal{C}_{\alpha,A,\mathrm{ols}}' & \mathcal{V}_{A,\mathrm{ols}} \end{pmatrix}.$$

Then, $\sqrt{T}\mathrm{vec}\left(\hat{\tilde{\Phi}}_{\mathrm{gls}} - \tilde{\Phi}\right) \to_d \mathcal{N}\left(0, \mathcal{V}_{\tilde{\Phi}}\right)$ where $\mathcal{V}_{\tilde{\Phi}} = \mathcal{H}_{\tilde{\Phi}} \mathcal{V}_{A,\mathrm{ols}} \mathcal{H}_{\tilde{\Phi}}'$ and

$$\mathcal{H}_{\tilde{\Phi}} = -\left(\left[I_{\mathrm{K}} \;\vdots\; -\tilde{\Phi}'\right] \otimes \left(B'\Sigma_e^{-1}B\right)^{-1} B'\Sigma_e^{-1}\right).$$

Recall that our VAR may be written as

$$\left(X - \mu_X\right) = \Phi\left(X - \mu_X\right) + V,$$

and define $\mu = \left(I_K - \Phi\right)\mu_X$. Then,

$$\begin{bmatrix} \sqrt{T}\left(\hat{\mu} - \mu\right) \\ \sqrt{T}\mathrm{vec}\left(\hat{\Phi} - \Phi\right) \end{bmatrix} \to_d \mathcal{N}\left(0, \mathcal{V}_{\mu,\Phi,\mathrm{ols}}\right),$$

where

$$\begin{pmatrix} \mathcal{V}_{\mu,\mathrm{ols}} & \mathcal{C}_{\mu,\Phi,\mathrm{ols}} \\ \mathcal{C}_{\mu,\Phi,\mathrm{ols}}' & \mathcal{V}_{\Phi,\mathrm{ols}} \end{pmatrix} = \mathrm{plim}_{T\to\infty}\left(\begin{pmatrix} 1 & \iota_T' X_-'/T \\ X_-\iota_T/T & X_- X_-'/T \end{pmatrix}^{-1} \otimes \Sigma\right).$$

This implies that

$$\sqrt{T}\mathrm{vec}\left(\hat{\lambda}_1 - \lambda_1\right) \to_d \mathcal{N}\left(0, \mathcal{V}_{\tilde{\Phi}} + \mathcal{V}_{\Phi,\mathrm{ols}}\right).$$

Define $\mathcal{X} = \left[X' \;\vdots\; X_-'\right]'$ along with

$$\begin{aligned}
\Omega &= \mathrm{plim}_{T\to\infty} T^{-1} \cdot \mathcal{X} M_\iota \mathcal{X}', \qquad \Omega_1 = \mathrm{plim}_{T\to\infty} T^{-1} \cdot \mathcal{X} M_\iota X_-, \\
\Omega_2 &= \mathrm{plim}_{T\to\infty} T^{-1} \cdot \mathcal{X} \iota_T, \qquad \Omega_3 = \mathrm{plim}_{T\to\infty} T^{-1} \cdot X_- \iota_T.
\end{aligned}$$

We have that $\sqrt{T}\left(\hat{\tilde{\mu}}_{\text{gls}}-\mu\right)\to_d \mathcal{N}\left(0,\mathcal{V}_{\tilde{\mu}}\right)$ where

$$\begin{aligned}\mathcal{V}_{\tilde{\mu}} &= \mathcal{H}_{\tilde{\mu},\alpha}\mathcal{V}_{\alpha,\text{ols}}\mathcal{H}'_{\tilde{\mu},\alpha}+\mathcal{H}_{\tilde{\mu},A}\mathcal{V}_{A,\text{ols}}\mathcal{H}'_{\tilde{\mu},A}+\left(\mathcal{H}_{\tilde{\mu},\alpha}\mathcal{C}_{\alpha,A,\text{ols}}\mathcal{H}'_{\tilde{\mu},A}\right)+\left(\mathcal{H}_{\tilde{\mu},\alpha}\mathcal{C}_{\alpha,A,\text{ols}}\mathcal{H}'_{\tilde{\mu},A}\right)' \\ &\quad +\mathcal{H}_{\tilde{\mu},\Sigma}\left(\left(I_{K^2}+\kappa_{K,K}\right)\left(\Sigma\otimes\Sigma\right)\right)\mathcal{H}'_{\tilde{\mu},\Sigma},\end{aligned}$$

$\kappa_{m,n}$ is the $(m,n)-$commutation matrix,

$$\begin{aligned}\mathcal{H}_{\tilde{\mu},\alpha} &= -\left(B'\Sigma_e^{-1}B\right)^{-1}B'\Sigma_e^{-1}, \\ \mathcal{H}_{\tilde{\mu},A} &= -\left(\tilde{\mu}'\otimes\left(B'\Sigma_e^{-1}B\right)^{-1}B'\Sigma_e^{-1}\right)\mathcal{H}_B+\left(\varrho_1\otimes\left(B'\Sigma_e^{-1}B\right)^{-1}B'\Sigma_e^{-1}A'_B\left(I_N\otimes\Sigma\right)\right)\mathcal{H}_B \\ &\quad +\left(B'\Sigma_e^{-1}B\right)^{-1}B'\Sigma_e^{-1}\mathcal{H}_\alpha, \\ \mathcal{H}_{\tilde{\mu},\Sigma} &= -\frac{1}{2}\left(\varrho_1\otimes\left(B'\Sigma_e^{-1}B\right)^{-1}B'\Sigma_e^{-1}C'_B\right), \\ \mathcal{H}_\alpha &= \left(\left(T^{-1}\Omega_2\right)'\otimes I_n\right)-\left(\left(T^{-1}\left[-\tilde{\Phi}\ \vdots\ I_{\text{K}}\right]\Omega_2\right)'\otimes I_N\right)\mathcal{H}_B+\left(\Omega'_3\otimes B\right)\mathcal{H}_{\tilde{\Phi}}, \\ \mathcal{H}_B &= \left(\left[\left[-\tilde{\Phi}\ \vdots\ I_{\text{K}}\right]\Omega\left[-\tilde{\Phi}\ \vdots\ I_{\text{K}}\right]'\right]^{-1}\left[-\tilde{\Phi}\ \vdots\ I_{\text{K}}\right]\Omega\otimes I_n\right) \\ &\quad -\left(\left[\left[-\tilde{\Phi}\ \vdots\ I_{\text{K}}\right]\Omega\left[-\tilde{\Phi}\ \vdots\ I_{\text{K}}\right]'\right]^{-1}\left[-\tilde{\Phi}\ \vdots\ I_{\text{K}}\right]\Omega_1\left[I_{\text{K}}\ \vdots\ -\tilde{\Phi}'\right]\otimes B\left(B'\Sigma_e^{-1}B\right)^{-1}B'\Sigma_e^{-1}\right),\end{aligned}$$

A_B is the $NK\times N$ partitioned diagonal matrix with B_i as the ith diagonal entry, $C_B=\left(\text{vec}\left(B_1B'_1\right)\cdots\text{vec}\left(B_NB'_N\right)\right)'$, and ϱ is the $(K+1)\times 1$ vector with first element equal to 1 and all other elements equal to 0.

Thus,

$$\sqrt{T}\left(\hat{\lambda}_0-\lambda_0\right)\to_d \mathcal{N}\left(0,\mathcal{V}_{\tilde{\mu}}+\mathcal{V}_{\mu,\text{ols}}\right).$$

Finally the asymptotic covariance between $\sqrt{T}\hat{\tilde{\mu}}$ and $\sqrt{T}\hat{\tilde{\Phi}}$ is

$$\mathcal{H}_{\tilde{\mu},\alpha}\mathcal{C}_{\alpha,A,\text{ols}}\mathcal{H}_{\tilde{\Phi}}+\mathcal{H}_{\tilde{\mu},A}\mathcal{V}_{A,\text{ols}}\ \mathcal{H}_{\tilde{\Phi}},$$

and the asymptotic covariance between $\sqrt{T}\hat{\lambda}_0$ and $\sqrt{T}\hat{\lambda}_1$ is

$$\mathcal{H}_{\tilde{\mu},\alpha}\mathcal{C}_{\alpha,A,\text{ols}}\mathcal{H}_{\tilde{\Phi}}+\mathcal{H}_{\tilde{\mu},A}\mathcal{V}_{A,\text{ols}}\mathcal{H}_{\tilde{\Phi}}+\mathcal{C}_{\mu,\Phi,\text{ols}}.$$

C Tables and Figures

Table 1: **Signficance of the Columns of B**

This table provides Wald statistics and their associated p-values for tests of whether the risk factor exposures of a subset of returns associated with individual pricing factors are jointly different from zero. W_{B_N} denotes Wald statistics for the risk exposures of all nominal Treasury returns to a given factor, W_{B_R} are the Wald statistics for the risk exposures of all TIPS returns to a given factor, and W_B are the corresponding Wald statistics for the joint cross-section of returns. p-values are given in parentheses. The sample period is 1999:01-2014:11.

	W_{B_N}	W_{B_R}	W_B
β_{F1}	2493866.867	1945930.739	4439797.606
$p-$val	(0.000)	(0.000)	(0.000)
β_{F2}	88891.833	58765.299	147657.132
$p-$val	(0.000)	(0.000)	(0.000)
β_{F3}	3239.381	10772.638	14012.019
$p-$val	(0.000)	(0.000)	(0.000)
β_{F4}	3.645	109708.711	109712.355
$p-$val	(0.979)	(0.000)	(0.000)
β_{F5}	15.236	7082.117	7097.353
$p-$val	(0.172)	(0.000)	(0.000)
β_{F6}	60.925	527279.433	527340.358
$p-$val	(0.000)	(0.000)	(0.000)

Table 2: **Treasuries: Fit Diagnostics**

This table summarizes the time series properties of the pricing errors implied by our benchmark model. "Mean", "std", "skew", and "kurt" refer to the sample mean, standard deviation, skewness, and kurtosis of the errors; $\rho(1), \rho(6)$ denote their autocorrelation coefficients of order one and six. Panel 1 reports properties of the yield pricing errors and Panel 2 reports properties of the excess return pricing errors. The sample period is 1999:01-2014:11.

	n = 12	n = 24	n = 36	n = 60	n = 84	n = 120
Panel 1: Yield Pricing Errors						
mean	-0.016	-0.013	-0.023	-0.018	0.008	0.008
std	0.039	0.086	0.080	0.044	0.026	0.045
skew	-1.007	-1.939	-1.629	-0.548	0.480	-1.336
kurt	9.779	11.453	8.338	2.785	4.800	7.634
$\rho(1)$	0.704	0.854	0.898	0.842	0.855	0.855
$\rho(6)$	0.257	0.285	0.372	0.586	0.484	0.357
Panel 2: Return Pricing Errors						
mean	-0.000	-0.031	-0.074	-0.012	0.057	-0.052
std	0.482	1.143	1.245	1.329	1.182	2.804
skew	0.506	0.654	0.377	0.008	-0.596	-0.163
kurt	5.938	18.378	21.896	5.135	11.160	14.886
$\rho(1)$	0.086	0.172	0.333	-0.268	-0.049	0.103
$\rho(6)$	0.168	0.052	-0.055	0.044	0.009	0.029

Table 3: **TIPS: Fit Diagnostics**

This table summarizes the time series properties of the pricing errors implied by our benchmark model. "Mean", "std", "skew", and "kurt" refer to the sample mean, standard deviation, skewness, and kurtosis of the errors; $\rho(1), \rho(6)$ denote their autocorrelation coefficients of order one and six. Panel 1 reports properties of the yield pricing errors and Panel 2 reports properties of the excess return pricing errors. The sample period is 1999:01-2014:11.

	n = 24	n = 36	n = 60	n = 84	n = 120
Panel 1: Yield Pricing Errors					
mean	-0.010	-0.007	0.002	0.009	-0.006
std	0.186	0.027	0.067	0.055	0.066
skew	-2.296	0.408	1.794	0.713	-0.855
kurt	10.391	3.073	6.824	3.951	4.899
$\rho(1)$	0.818	0.731	0.865	0.906	0.861
$\rho(6)$	0.515	0.427	0.609	0.689	0.385
Panel 2: Return Pricing Errors					
mean	-0.000	0.008	0.017	0.007	-0.104
std	2.862	0.787	2.081	2.029	4.204
skew	-0.212	0.648	0.222	-0.051	0.978
kurt	11.903	4.625	8.034	6.466	10.421
$\rho(1)$	-0.095	-0.009	-0.166	-0.144	0.016
$\rho(6)$	-0.037	0.076	-0.057	-0.004	-0.078

Table 4: **Market Prices of Risk**

This table summarizes the estimates of the market price of risk parameters λ_0 and λ_1 for the benchmark specification. t-statistics are reported in parentheses. The standard errors are based on the limiting variance provided in the Appendix. Wald statistics for tests of the rows of Λ and of λ_1 being different from zero are reported along each row, with the corresponding p-values in parentheses below.

	λ_0	$\lambda_{1.1}$	$\lambda_{1.2}$	$\lambda_{1.3}$	$\lambda_{1.4}$	$\lambda_{1.5}$	$\lambda_{1.6}$	W_Λ	W_{λ_1}
X_1	-0.026	-0.008	-0.035***	-0.018	-0.006	0.028**	0.004	17.632**	17.619***
	(-0.129)	(-0.654)	(-2.862)	(-1.513)	(-0.524)	(2.486)	(0.304)	(0.014)	(0.007)
X_2	-0.045	0.011	-0.056**	-0.012	-0.066***	0.059***	0.018	21.050***	21.050***
	(-0.031)	(0.457)	(-2.327)	(-0.496)	(-2.891)	(2.632)	(0.727)	(0.004)	(0.002)
X_3	0.176	0.003	0.059	-0.116***	-0.104**	-0.000	-0.107**	23.312***	23.312***
	(0.021)	(0.075)	(1.354)	(-2.739)	(-2.523)	(-0.005)	(-2.327)	(0.002)	(0.001)
X_4	0.344	0.103	0.109	0.033	0.013	0.030	-0.271	2.071	2.071
	(0.002)	(0.649)	(0.644)	(0.195)	(0.082)	(0.184)	(-1.365)	(0.956)	(0.913)
X_5	0.123	0.030	0.145**	-0.047	0.092	-0.282***	-0.012	34.168***	34.168***
	(0.006)	(0.514)	(2.381)	(-0.789)	(1.616)	(-4.975)	(-0.182)	(0.000)	(0.000)
X_6	0.361	0.112	0.123	0.127	0.049	-0.085	-0.311**	6.607	6.606
	(0.003)	(1.078)	(1.108)	(1.162)	(0.477)	(-0.793)	(-2.396)	(0.471)	(0.359)

Table 5: **5-10 Year Forward Term Premium Regressions**

This table displays results from linear regressions of the five to ten year inflation risk premium as well as the nominal and real term premiums implied by our model on various observable macroeconomic and financial indicators. These are 1) the three-month swaption implied Treasury volatility from Merrill Lynch (SMOVE); 2) the cross-sectional standard deviation of individual inflation forecasts four quarters ahead from the Blue Chip Financial Forecasts survey (DISAG); 3) the unemployment rate (UNEMP); 4) consumer confidence as measured by the Conference Board survey (CONF); 5) year-over-year core CPI inflation (CPI); and 6) the nominal trade-weighted exchange value of the U.S. Dollar (DOLLAR). All variables are standardized and t-statistics are in parentheses. All regressions include a linear time trend but the coefficient and t-statistic is omitted from the table. The sample period is 1999:01-2014:11.

				5-10Y Nominal TP			
SMOVE	0.52						0.61
	(12.46)						(9.50)
DISAG		0.31					0.01
		(6.40)					(0.19)
UNEMP			0.44				0.16
			(6.61)				(1.84)
CONF				-0.50			0.33
				(-7.48)			(3.21)
CPI					-0.33		-0.18
					(-6.27)		(-3.45)
DOLLAR						-0.08	-0.08
						(-1.57)	(-2.21)
Adj. R^2	0.65	0.48	0.48	0.51	0.47	0.37	0.71
				5-10Y Real TP			
SMOVE	0.30						0.40
	(8.95)						(7.15)
DISAG		0.16					-0.01
		(4.49)					(-0.27)
UNEMP			0.21				0.08
			(4.21)				(1.08)
CONF				-0.26			0.24
				(-4.98)			(2.70)
CPI					-0.15		-0.08
					(-3.68)		(-1.77)
DOLLAR						0.00	-0.01
						(0.02)	(-0.27)
Adj. R^2	0.74	0.66	0.66	0.67	0.65	0.63	0.75
				5-10Y IRP			
SMOVE	0.22						0.21
	(10.79)						(7.36)
DISAG		0.15					0.02
		(6.70)					(0.94)
UNEMP			0.23				0.08
			(7.74)				(2.01)
CONF				-0.25			0.09
				(-8.24)			(1.94)
CPI					-0.18		-0.10
					(-8.03)		(-4.28)
DOLLAR						-0.08	-0.08
						(-3.56)	(-4.42)
Adj. R^2	0.45	0.28	0.32	0.35	0.34	0.17	0.61

Table 6: **Inflation Forecasting**

This table compares the root mean squared error of three models for predicting future inflation. The first uses the model-implied inflation expectations derived in Section 2.3. These represent breakeven inflation rates adjusted for liquidity and risk premia. The second method takes unadjusted TIPS breakevens as a predictor of future inflation. The third is a simple random walk forecast, i.e. it takes average realized inflation over the prior n months as a prediction of average inflation over the next n months. Forecasts are performed over horizons from 12 to 36 months, and forecasting errors are computed using overlapping observations. The first panel reports in-sample results for the full sample from 1999:01-2014:11. The second panel reports out-of-sample results, using a five-year "learning period," forecasting over the period 2004:01-2014:11.

	12m	24m	36m
	Panel A: In Sample		
Model Forecast	1.361	0.806	0.562
Breakevens	1.797	1.246	1.137
Random Walk Forecast	1.972	1.340	1.026
	Panel B: Out of Sample		
Model Forecast	1.882	1.348	1.077
Breakevens	1.957	1.375	1.198
Random Walk Forecast	2.214	1.437	1.101

Table 7: **LSAP Event Regressions: 2-year Yield Decompositions**

The upper panel gives a regression estimate of the coefficient β and corresponding t-statistics from the equation $\Delta\tilde{y}_t^{(2)} = \beta\mathbb{I}(LSAP_t) + \varepsilon_t^{(2)}$ where $\Delta\tilde{y}_t^{(2)}$ is the one-day change in the two-year yield or component of the two-year yield and $\mathbb{I}(LSAP_t)$ is an indicator function for an LSAP announcement date. The lower panel gives the coefficients from the regression $\Delta\tilde{y}_t^{(2)} = \beta_1\mathbb{I}(LSAP1_t) + \beta_2\mathbb{I}(LSAP2_t) + \beta_{MEP}\mathbb{I}(MEP_t) + \beta_3\mathbb{I}(LSAP3_t) + \varepsilon_t^{(2)}$. Reported results are product of estimated coefficient and number of event dates. t-statistics are provided in parentheses.

	y_N	TP_N	$E[i]$	y_R	TP_R	$E[r]$	Liq^R	BEI	IRP	$E[\pi]$	Liq^{BEI}
					All LSAP Events						
LSAP	-0.72	-0.75	0.04	-1.20	-1.44	-0.07	0.31	0.48	0.65	0.11	-0.28
t-stat	(-3.30)	(-6.37)	(0.17)	(-3.71)	(-6.11)	(-0.32)	(1.62)	(1.71)	(5.21)	(0.44)	(-1.62)
					Individual LSAP Events						
LSAP 1	-0.74	-0.58	-0.16	-0.90	-1.10	-0.10	0.31	0.16	0.49	-0.05	-0.28
LSAP 2	-0.04	-0.07	0.02	-0.18	-0.20	0.01	0.01	0.14	0.13	0.02	-0.01
MEP	0.09	-0.04	0.14	0.11	0.01	0.12	-0.01	-0.02	-0.05	0.02	0.01
LSAP 3	-0.03	-0.06	0.04	-0.23	-0.14	-0.09	0.01	0.20	0.08	0.13	-0.01

Table 8: **LSAP Event Regressions: 10-year Yield Decompositions**

The upper panel gives a regression estimate of the coefficient β and corresponding t-statistics from the equation $\Delta\tilde{y}_t^{(10)} = \beta\mathbb{I}(LSAP_t) + \varepsilon_t^{(10)}$ where $\Delta\tilde{y}_t^{(10)}$ is the one-day change in the ten-year yield or component of the ten-year yield and $\mathbb{I}(LSAP_t)$ is an indicator function for an LSAP announcement date. The lower panel gives the coefficients from the regression $\Delta\tilde{y}_t^{(10)} = \beta_1\mathbb{I}(LSAP1_t) + \beta_2\mathbb{I}(LSAP2_t) + \beta_{MEP}\mathbb{I}(MEP_t) + \beta_3\mathbb{I}(LSAP3_t) + \varepsilon_t^{(10)}$. Reported results are product of estimated coefficient and number of event dates. t-statistics are provided in parentheses.

	y_N	TP_N	$E[i]$	y_R	TP_R	$E[r]$	Liq^R	BEI	IRP	$E[\pi]$	Liq^{BEI}
All LSAP Events											
LSAP	-1.00	-1.08	0.09	-1.32	-1.61	0.14	0.15	0.32	0.52	-0.06	-0.14
t-stat	(-4.08)	(-4.93)	(0.68)	(-7.25)	(-7.13)	(1.43)	(1.62)	(2.12)	(3.46)	(-1.14)	(-1.62)
Individual LSAP Events											
LSAP 1	-0.95	-0.91	-0.04	-1.07	-1.26	0.04	0.15	0.12	0.34	-0.08	-0.14
LSAP 2	-0.00	-0.02	0.02	-0.15	-0.18	0.02	0.00	0.15	0.16	-0.01	-0.00
MEP	-0.06	-0.14	0.08	-0.03	-0.09	0.07	-0.01	-0.03	-0.05	0.01	0.01
LSAP 3	0.01	-0.01	0.03	-0.07	-0.08	0.00	0.00	0.08	0.06	0.02	-0.00

Table 9: **UK Bonds: Fit Diagnostics**

This table summarizes the time series properties of the pricing errors implied by our alternative estimation using UK term structure data. "Mean" and "std" refer to the sample mean and standard deviation of yield pricing errors; $\rho^y(1)$ and $\rho^{rx}(1)$ denote autocorrelation coefficients of order one for yield pricing errors and excess return pricing errors. The sample period is 1985:01-2012:12.

	n = 36	n = 60	n = 84	n = 120
Nominal Yield Pricing Errors:				
mean	-0.050	-0.037	-0.040	-0.018
std	0.040	0.029	0.029	0.031
$\rho^y(1)$	0.899	0.947	0.944	0.827
$\rho^{rx}(1)$	0.279	0.359	0.173	-0.069
Real Yield Pricing Errors				
mean		0.001	0.007	-0.013
std		0.077	0.015	0.048
$\rho^y(1)$		0.930	0.938	0.949
$\rho^{rx}(1)$		0.055	0.463	0.107

Figure 2: **Pricing Factors: Observed Time Series**

This figure plots the time series of the factors of our model. These are the first three principal components extracted from the cross-section of end-of-month observations of nominal Treasury yields of maturities $n = 3, \ldots, 120$ months. The fourth and fifth factors are the first two principal components extracted from the cross-section of orthogonalized real yields of maturities $n = 24, \ldots, 120$ months, the residuals from regressing real yields on the first three principal components of the nominal Treasury yield curve as well as the liquidity factor. The sixth factor is the liquidity factor described in Section 4.1.

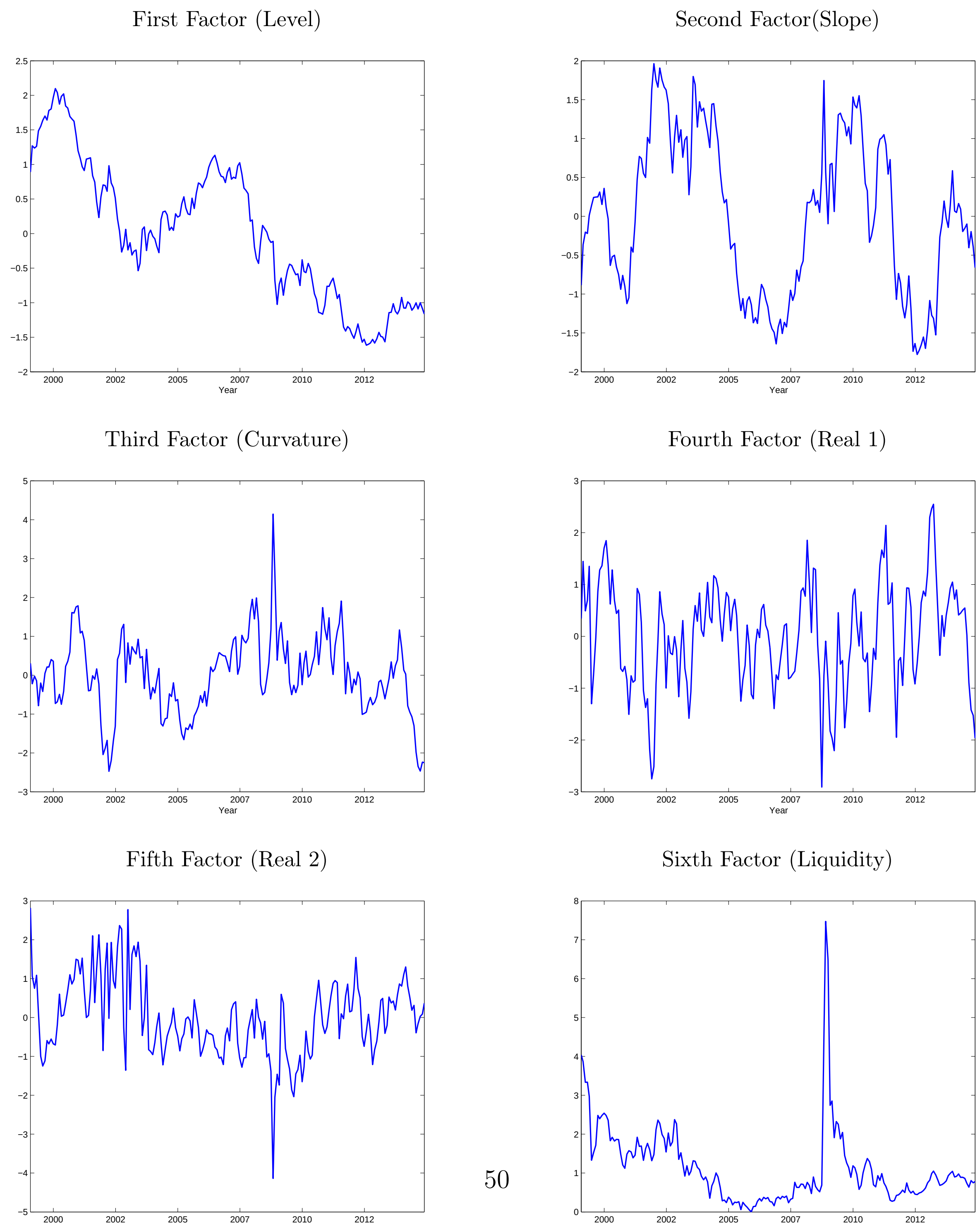


Figure 3: **Treasuries: Observed and Model-Implied Time Series**

This figure provides time series plots of observed and model-implied Treasury yields and excess returns. The upper panels plot zero-coupon Treasury yields at two-year and ten-year maturities and the bottom panels plot excess holding period returns at two-year and ten-year maturities. The observed yields and returns are plotted by solid blue lines, whereas dashed green lines correspond to model-implied yields and returns. Dashed red lines in the lower panels are model-implied expected excess holding period returns.

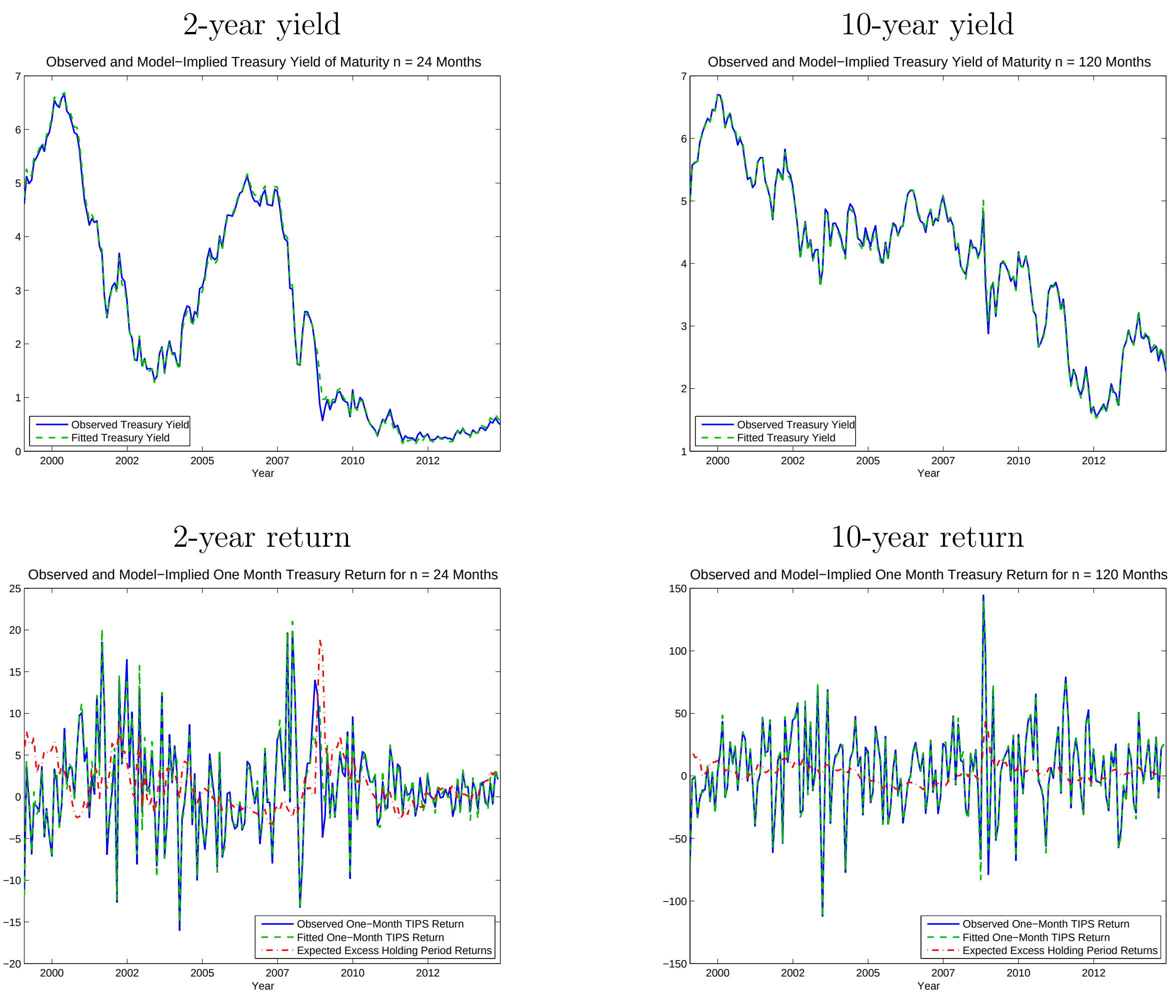


Figure 4: **TIPS: Observed and Model-Implied Time Series**

This figure provides time series plots of observed and model-implied TIPS yields and excess returns. The upper panels plot zero-coupon TIPS yields at five-year and ten-year maturities and the bottom panels plot excess holding period returns at five-year and ten-year maturities. The observed yields and returns are plotted by solid blue lines, whereas dashed green lines correspond to model-implied yields and returns. Dashed red lines in the lower panels are model-implied expected excess holding period returns.

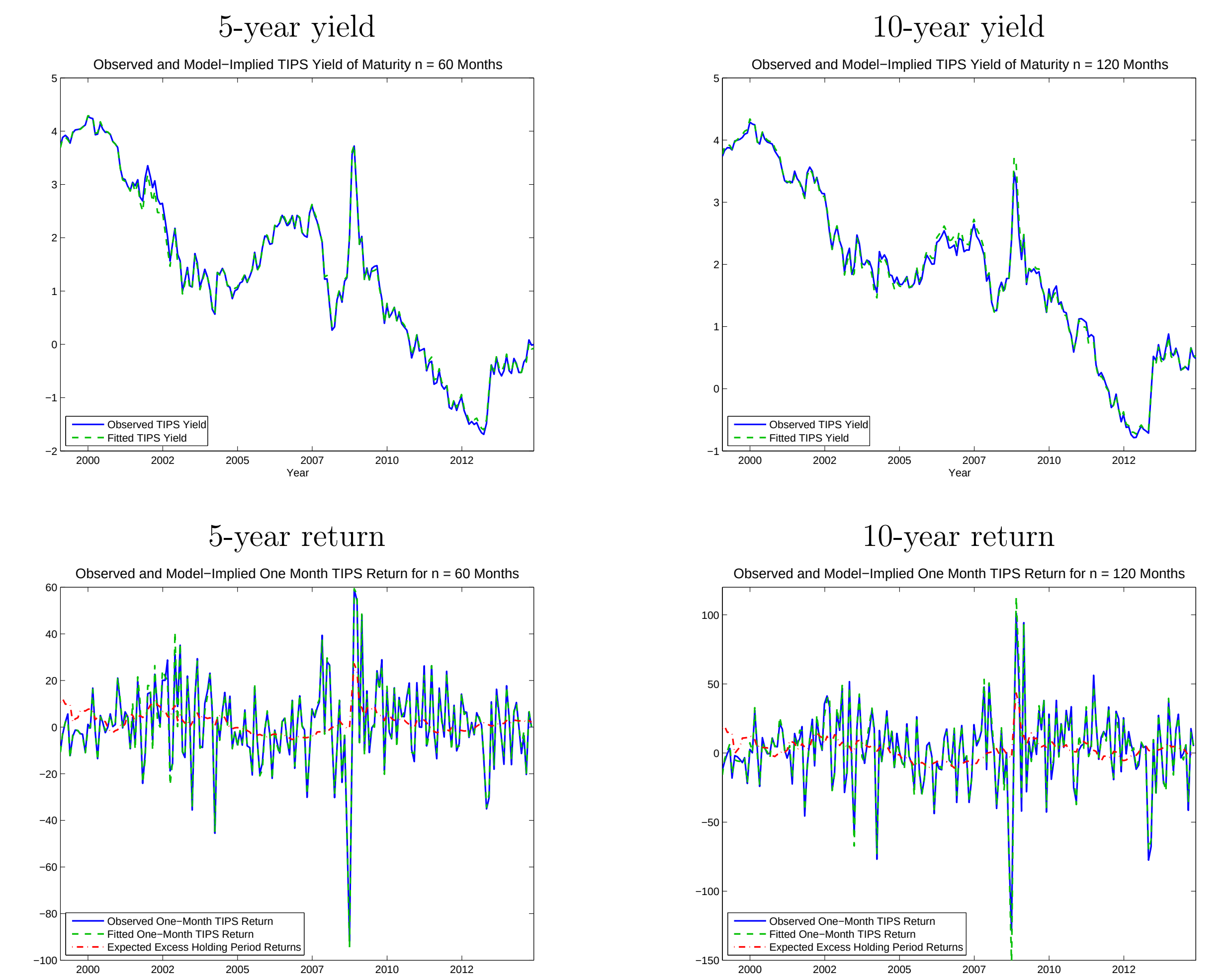


Figure 5: **Treasuries: Cross Sectional Diagnostics**

This figure provides graphs exhibiting the cross-sectional fit and interpretation of the factors as drivers of Treasury yields. The upper two panels plot the unconditional means and standard deviations of observed yields against those implied by the model. The lower left panel plots the implied yield loadings $-\frac{1}{n}B_n$. These coefficients can be interpreted as the response of the n-month yield to a contemporaneous shock to the respective factor. The lower right panel plots the expected return loadings $B_n'\lambda_1$. These coefficients can be interpreted as the response of the expected one-month excess holding return on an n-month bond to a contemporaneous shock to the respective factor.

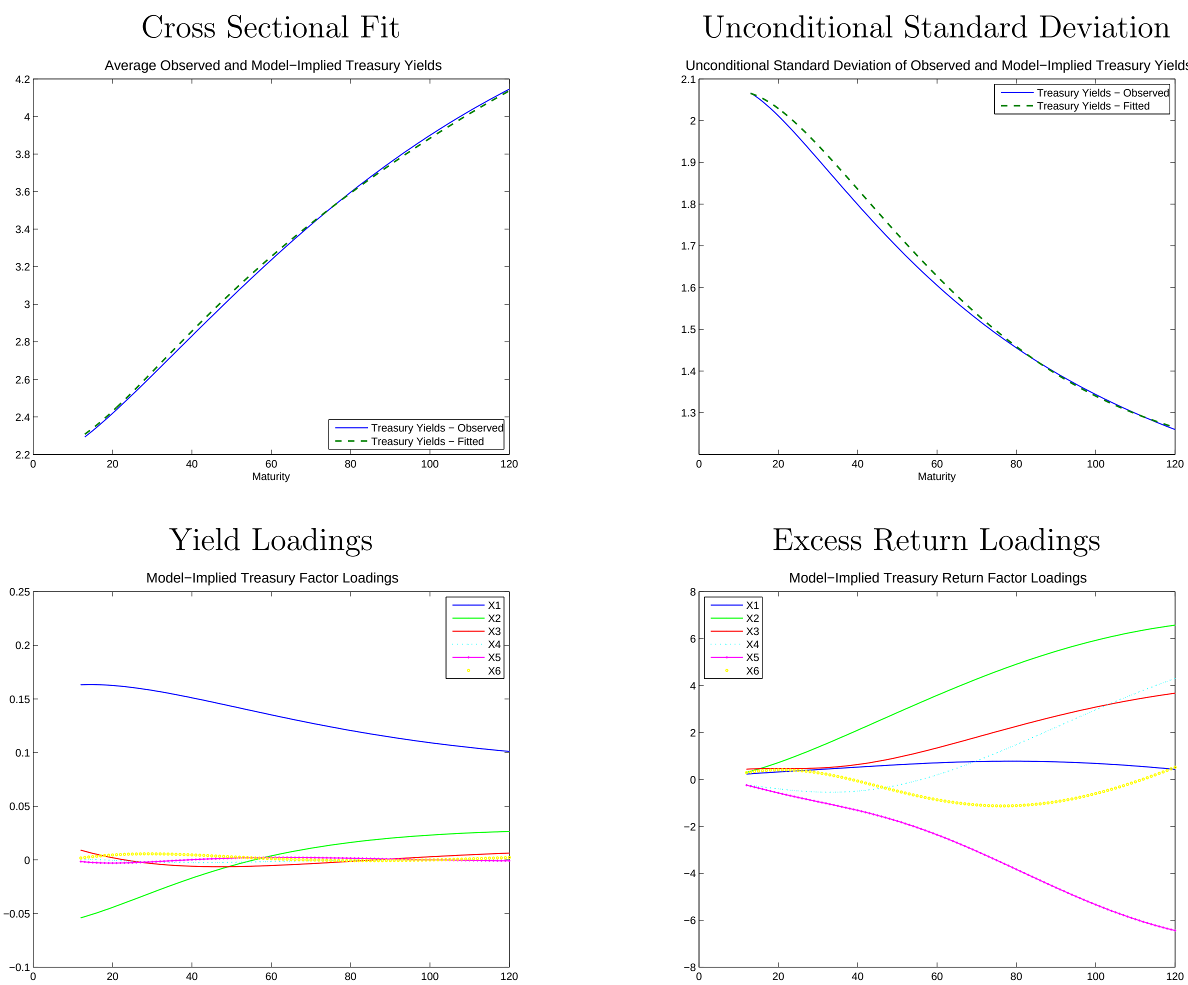


Figure 6: **TIPS: Cross Sectional Diagnostics**

This figure provides graphs exhibiting the cross-sectional fit and interpretation of the factors as drivers of TIPS yields. The upper two panels plot unconditional means and standard deviations of yields against those implied by the model. The lower left panel plots the implied yield loadings $-\frac{1}{n}B_{n,R}$. These coefficients can be interpreted as the response of the n-month yield to a contemporaneous shock to the respective factor. The lower right panel plots the expected return loadings $B'_{n,R}\lambda_1$. These coefficients can be interpreted as the response of the expected one-month excess holding return on an n-month bond to a contemporaneous shock to the respective factor.

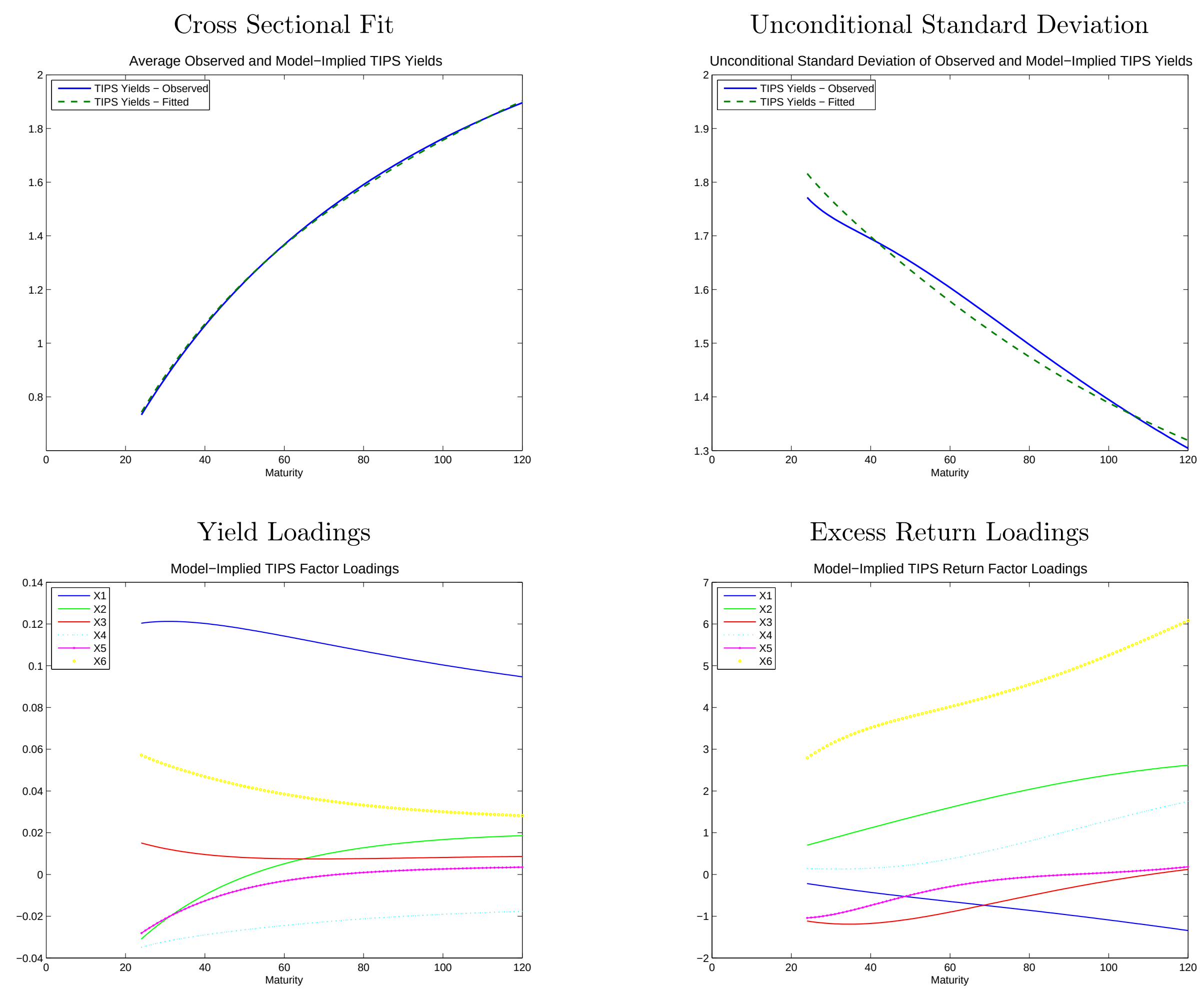


Figure 7: **Treasury and TIPS Term Premia**

This figure provides plots of the Treasury term premium and TIPS term premium across maturities. The upper panels plot decompositions of 5-10 year forward Treasury and TIPS rates into the expected future short rates and the respective nominal and real term premium. A black dotted line represents the liquidity component of each series. The lower left panel plots the 5-10 year forward Treasury and TIPS term premia together. The difference between the measures is the inflation risk premium. The lower right panel plots the decomposition of liquidity-adjusted breakevens into expected inflation in red and the inflation risk premium in black.

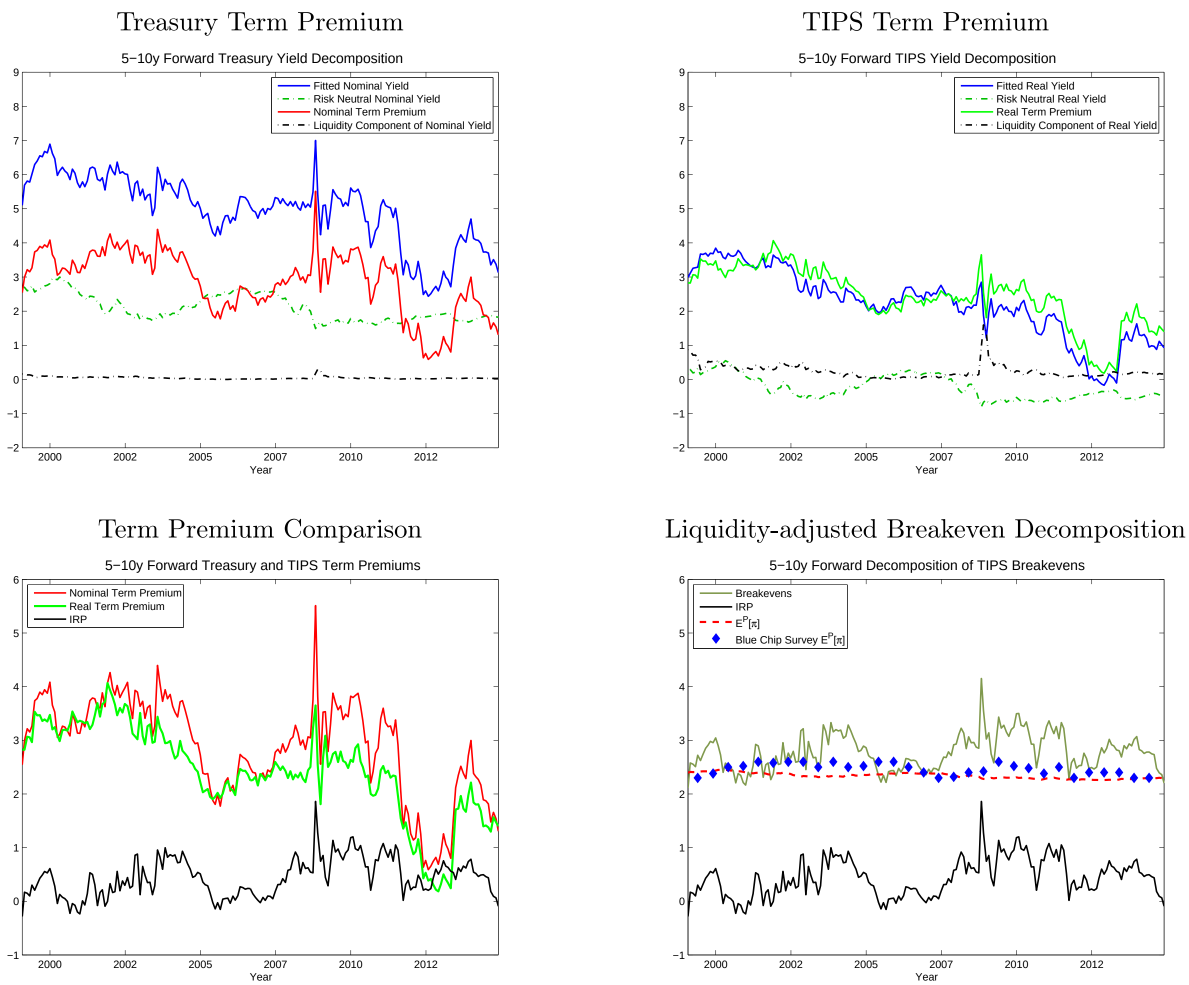


Figure 8: **Interpretation of Inflation Risk Premium**

This figure plots the 5-10 year forward inflation risk premium generated by our model against several observable variables. We consider 1) the three-month swaption implied Treasury volatility from Merrill Lynch (SMOVE); 2) the cross-sectional standard deviation of individual inflation forecasts four quarters ahead from the Blue Chip Financial Forecasts survey (DISAG); 3) the unemployment rate (UNEMP); 4) consumer confidence as measured by the Conference Board survey (CONF); 5) year-over-year core CPI inflation (CPI); and 6) the nominal trade-weighted exchange value of the U.S. Dollar (DOLLAR).

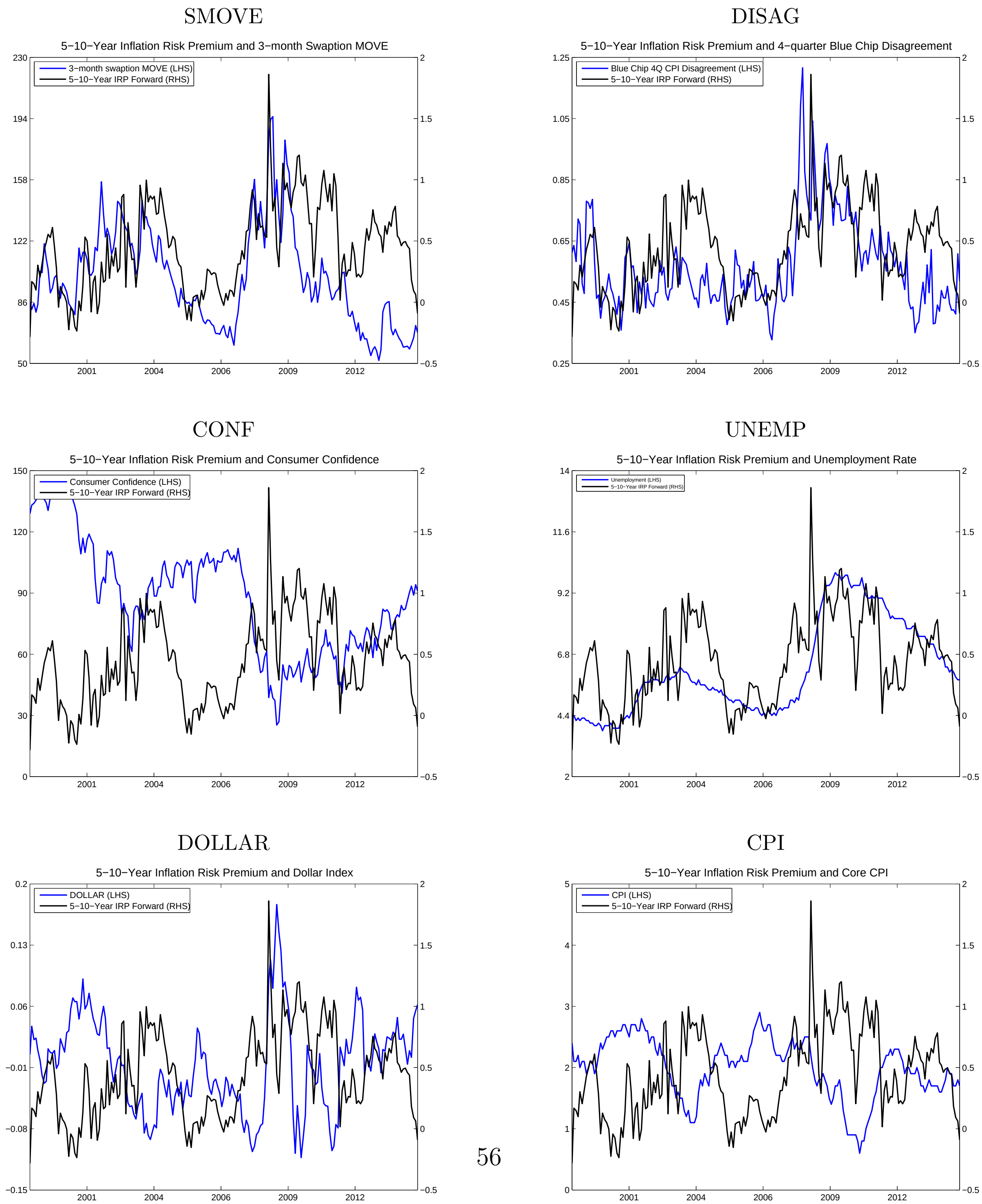
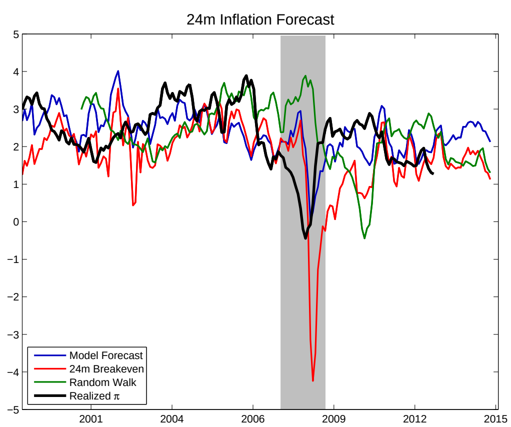


Figure 9: **Inflation Forecasting**

This figure plots the results of the inflation forecasting exercises; each panel displays the result at a different forecasting horizon. The first method, plotted in blue, is the forecast generated by our model, which represents the breakevens adjusted for liquidity and inflation risk premia. The second method, plotted in red, takes unadjusted zero-coupon Treasury-TIPS breakevens as estimates of future inflation. The final forecasting approach, plotted in green, is a simple random walk forecast. This method takes realized inflation over the previous n months as a forecast of average n-month future inflation. Realized CPI inflation is plotted in black. The crisis period of 2007:09-2009:05 is shaded in gray.

24 Months



36 Months

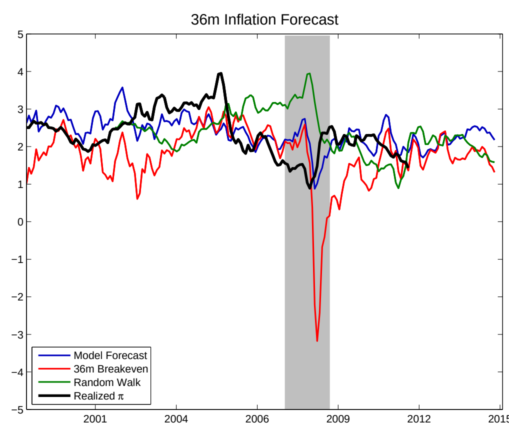


Figure 10: **Long-Term Forward Regressions: Fed Funds Shocks**

This figure plots the coefficient $\beta^{(n)}$ from the equation $\Delta f_t^{(n)} = \alpha^{(n)} + \beta^{(n)} z_t + \varepsilon_t^{(n)}$, where $\Delta f_t^{(n)}$ is a one-day change in forwards and z_t is a fed funds shock around FOMC announcement days. 95% confidence intervals are given by the gray bands. The sample period is 1999:01- 2008:09.

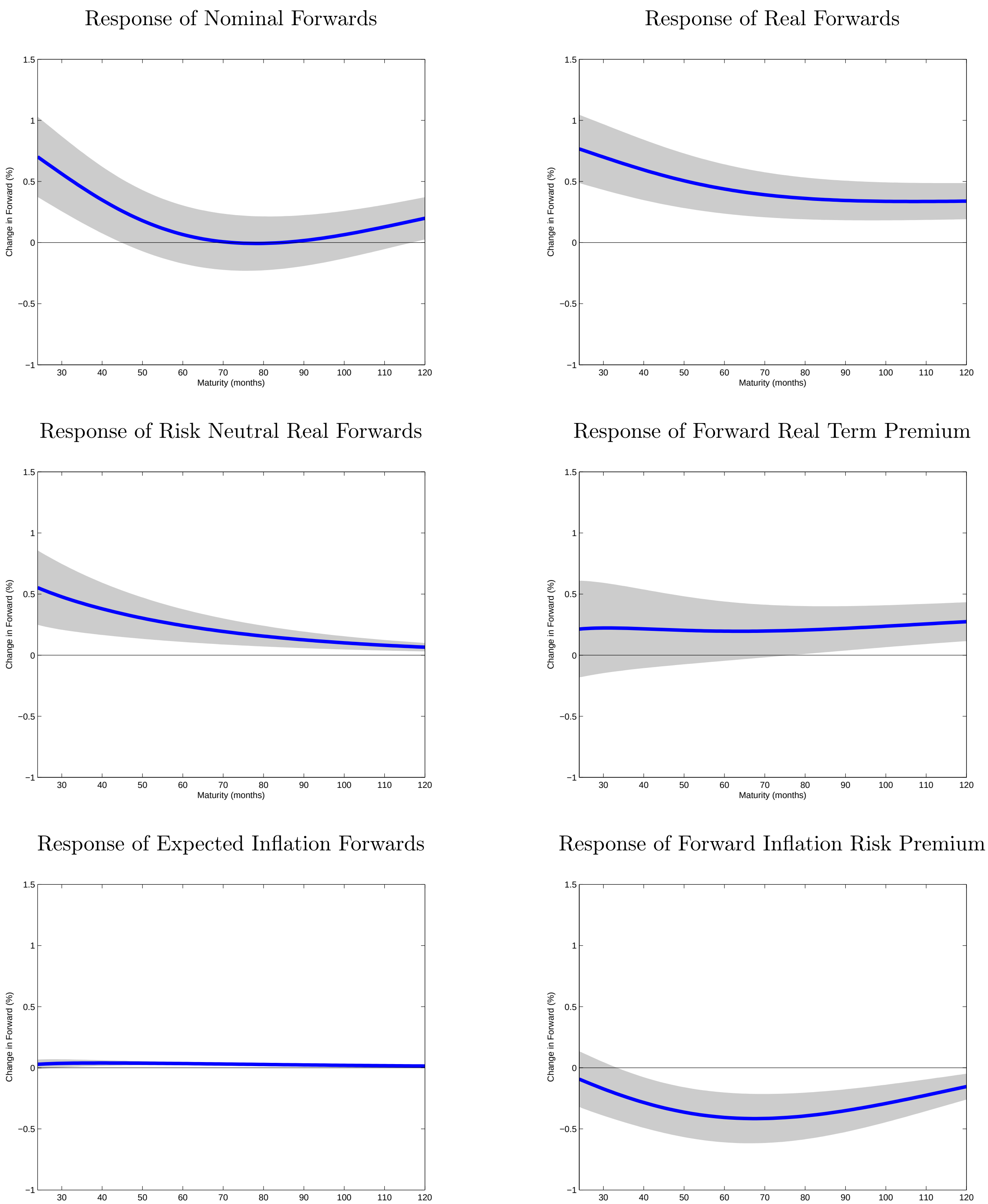


Figure 11: **UK Yield Curve Specification**

This figure plots the estimates implied by the United Kingdom specification estimating the model using data from UK zero-coupon yields provided by the Bank of England. The upper panels plot the yields of zero coupon Gilts and Linkers at ten-year maturities. The observed yields are plotted by solid blue lines, whereas dashed red lines correspond to model-implied yields. The lower left panel plots the ten year nominal and real term premia together. The difference between the two measures is the inflation risk premium. The lower right panel plots the decomposition of zero-coupon breakevens into expected inflation in red and the inflation risk premium in black.