Liquidity Regulation, the Central Bank and the Money Market

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Abstract

As reliance on excessively short-term wholesale funding has been one of the major causes for the 2007-2009 financial crisis, recent advances in global liquidity regulation try to curb the excessive reliance on short-term wholesale funding without being clear on how such an approach will affect the overall equilibrium on money markets. In particular, liquidity regulation may interfere with the central bank's influence on short-term money market rates. This paper tries to fill the gap in understanding the interaction between the money market, the central bank, and the regulator. Importantly, it shows that the existence of a central bank can be welfare-improving when the market equilibrium is driven by collateral constraints and asymmetric information. Regulation can be welfare-improving in the presence of an externality and also in case of collateral constraints, but reduces activity on the unsecured market. This implies that in case of collateral constraints the regulator can lead to a complete crowding out of the unsecured market which leads to an increased central bank intermediation need.

Keywords: regulation, Basel III, Pigovian tax, externality, interbank lending, central bank

JEL-Codes: E41; E42; E43; E58; G01; H12; H23; L51

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1 Introduction

By now, it has become a widely acknowledged fact that maturity mismatch in banks' funding structure has been one of the major causes of the 2007-2009 financial crisis. Even though regulatory efforts at the international level address this issue with rigour, the optimal design of liquidity regulation receives less attention, let alone interactions with other regulatory measures, central banks' monetary policy implementation or the sense or non-sense of its implementation during an ongoing crisis.

The connection between debt maturity choice and risk-taking is not novel, and it is not novel to the 2007-2009 crisis. The fact that the choice of risk in a portfolio is closely related to the degree of asymmetric information in a market and the possibility of a separating equilibrium has already been nicely shown by e.g. Flannery (1986) or Diamond (1991). Our model relates to this strand of literature in the sense that asymmetric information is the cause of a sub-optimal outcome. Berger et al. (2005) test these models to confirm that for low-risk firms funding maturity tends to increased when the degree of asymmetric information is lower.

Risk-taking has been closely connected to a bank's funding structure. The Basel Committee has identified this link as a crucial cause of the 2007-2009 financial crisis. As a result, Basel III banking regulation contains an extensive framework for liquidity regulation that requires banks to hold a short-term liquidity buffer (the *Liquidity Coverage Ratio*) and prove a more stable long-term funding base (the *Net Stable Funding Ratio*).[1] While it is clear that such a form of regulation may achieve its objective even though it may not achieve the most efficient outcome (Perotti and Suarez 2011), implications for central banks' monetary policy – which by definition are closely related to the management of liquidity and exerting an influence on market liquidity – or the money market equilibrium during or after the crisis are not at the centre of attention.

Most of the literature looking at the interaction between liquidity regulation and central bank operations discuss the central bank's function as a lender of last resort and the effect of it on market efficiency. A notable earlier work is Repullo (2005) who studies risk-taking by borrowers when the central bank acts as a lender of last resort. In contrast to Cao and Illing (2009), the existence of a lender of last resort does not induce excessive risk-taking in Repullo's model. In Cao and Illing, the existence of a lender of last resort must be matched with ex ante liquidity regulation to avoid excessive risk-taking.

The reasons why short-term wholesale funding was widespread before the 2007-2009 financial crisis have been extensively analysed (Taylor and Williams 2008; Eisenschmidt and Tapking 2009, Brunnermeier and Oehmke 2010). The origins of the 2007-2009 financial crisis indeed lie in the excessive reliance on very short-term wholesale funding for long-term projects (Brunnermeier 2008), commonly referred to as maturity mismatch, which can dry up very quickly during a crisis (Brunnermeier and Pedersen 2008) and which can also be the

[1] Appendix A.1 provides details on these measures.

result of asymmetric information (Huang and Ratnovski 2011).

We model the short-term interbank market, both secured and unsecured, under asymmetric information - in a baseline scenario, when collateral for borrowing in the secured market is scarce, and in the presence of external effects. While even in the presence of asymmetric information the market outcome is optimal when there is abundant collateral available and there are no external effects, the market outcome can be suboptimal in case of collateral shortage or in the presence of an externality.

We show that the intermediation of a central bank in the interbank market can be welfare-improving in the presence of collateral constraints when there is asymmetric information. The central bank acts as a mediator on the interbank market, via a corridor system. It provides a deposit facility and a lending facility, where it lends funds against central bank eligible collateral. When the central bank accepts a wider set of collateral than the market, its intermediation can improve social welfare compared to the market outcome. Our model also emphasises that the presence of the central bank cannot reduce inefficiencies caused by the presence of external effects. Such a situation makes the case for the intervention of a regulator.

We show that the regulator can internalise the externality by taxing risky behaviour. Our model thereby reflects recent regulatory developments in liquidity regulation. However, in the presence of collateral constraints the regulator faces the trade-off of constraining the unsecured market activity to attenuate the effects of the externality while the regulator should not shut down the unsecured market completely as it is necessary for refinancing due to collateral constraints. For this case, our analysis shows that the presence of both the regulator and the central bank can lead to a freeriding of the regulator at the cost of the central bank to shut down the unsecured market, which leads to a complete crowding out of market activity by central bank funding. This implies that the existence of a regulator and a central bank may actually lead to a suboptimal outcome.

Our paper is structured as follows. Section 2 introduces the basic model setup, characterised by asymmetric information, and the notation for three cases: baseline model with neither collateral constraints nor externality, collateral constraints, and the presence of an externality. Section 3 gives the normative analysis for the social planner, compares the normative and the positive outcome and establishes the case for an intervention in the interbank market. We show in section 4 that a central bank can remove the inefficiencies caused by collateral constraints and asymmetric information, while we show in section 5 that a regulator can remove the inefficiencies caused by the externality. Section 6 then highlights that the combination of a central bank and a regulator may not always be welfare-improving compared to the case with either a central bank or a regulator. Section 7 concludes.

2 The basic model

The basic set-up consists of an interbank market that has both an unsecured and a secured segment for collateralised borrowing. To keep the model general, we interpret collateralised borrowing as a loan backed by a fixed collateral amount that covers the outstanding debt (plus interest) and that can be seized by the lender if the borrower defaults on the loan. We do not assume additional possibilities for litigation. However, modelling unsecured versus collateralised borrowing can also be interpreted as different forms of limited liability.

There are two types of banks on the interbank market, (wholesale) borrowers and (wholesale) lenders, who can form an agreement to finance a project. Borrowers can ask for the amount I which they can invest in two types of investment: either a safe project which returns A with certainty, or a risky project which returns θ with probability p_i and 0 with probability $1-p_i$, where $I, \theta, A > 0$. Note that we do not impose any restrictions on the amount I such that it can also be interpreted as a refinancing requirement. We assume that all agents are risk-neutral, implying that they maximise their expected payout.

We introduce asymmetric information by assuming the probability $p_i \in [0,1]$ to be borrower-specific. The probability p_i can thus be interpreted as the borrower's type. The borrowers' type is distributed along the interval $[0,1]$ according to the probability distribution function f. A borrower i will know about their type p_i, but the lenders cannot observe p_i. Both borrowers and lenders know the distribution f of types in the population.

The borrower always needs to invest the full amount I, i.e. he cannot partition his resources to invest in both types of investment.[2] However, we allow for the option to combine borrowing on the secured and on the unsecured market, i.e. the borrower can borrow a share ρ of the funding on the secured market and a share $1-\rho$ on the unsecured market. Borrowers have thus two options to access funding: they can use collateral for collateralised borrowing on the secured market, but they can also access the unsecured market directly without using their collateral.

All borrowers have to use the unsecured market to some extent since we assume that collateral is scarce and that the constraints are the same for all borrowers. Otherwise, all borrowers would prefer to use the secured market. We show this in Appendix B, section A.1. Borrowers can only borrow the share $\lambda < 1$ of the total loan I on the secured market and have to borrow the rest on the unsecured market. We assume that borrowers need to borrow the total amount I in order to be able to invest. Put differently, all borrowers will have to resort to the unsecured market to some extent.

Let R^s be the interest rate on the secured market and R^u be the interest rate on the unsecured market. Given the fact that borrowers and lenders have the choice between the secured and the unsecured market, and that the lender will need to be compensated for the additional risk borne when lending unsecured, $R^u \geq R^s$.

[2] This assumption can easily be relaxed, but keeps the model more parsimonious.

Corollary 1 *If the equilibrium market interest rate on the secured market is R^s, then $A \geq R^s I$ or $\theta \geq R^s I$ is a necessary condition for market activity.*

If we assume that there is a safe store of assets (that does not bear interest), there is always the risk-free alternative of not conducting any investment or lending activity. In this case, it is clear that $R^s \geq 1$, since the lender always has the alternative to keep their funds.[3]

Corollary 2 *If there is a safe store of assets, then the equilibrium market interest rate on the secured market is $R^s \geq 1$.*

To keep the model simple, we assume that the lender can always claim the collateral in case the investment does not pay off and that a loan must be overcollateralised, such that the lender does not bear any risk when lending secured. Assuming perfect market functioning, the secured interest rate will then be equal to 1 whenever there is a safe store of assets as alternative.

For a borrower to have an incentive to invest in the safe project, $A \geq R^s I$ must hold, and in order to have an incentive to invest in the risky project, $\theta \geq R^s I$ must hold. In order to have an incentive to invest in the risky project instead of the safe project, $\theta \geq A$ (and $\theta > A$ if $p_i < 1$) must hold.

Corollary 3 *A necessary condition for investment to take place in the risky project instead of the safe project is $\theta \geq A$ (and $\theta > A$ if $p_i < 1$ for all borrowers).*

2.1 Strategies for borrowers

The borrower can either borrow partly on the secured and partly on the unsecured market, or he can borrow only on the unsecured market. It will always be optimal for the borrower to borrow as much as possible on the secured market, such that the payoff options reduce to borrowing the share λI on the secured market and the share $(1-\lambda)I$ on the unsecured market and investing either safe or risky or borrowing the whole amount of I on the unsecured market and investing either safe or risky.[4] Table 1 presents the corresponding payoff structure.

The borrower's return when investing in the safe asset and borrowing as much as possible on the secured market is $\Pi_B^{s\lambda}(safe) = A-(R^s\lambda+R^u(1-\lambda))I$. The borrower's expected return when investing in the risky asset and borrowing as much as possible on the secured market is $\Pi_B^{s\lambda}(risky) = (\theta - (R^s\lambda + R^u(1-\lambda))I)p_i + (-R^s\lambda I)(1-p_i) = (\theta - R^u(1-\lambda)I)p_i - R^s\lambda I$.

The trade-off for the borrower is therefore driven by the share they can borrow on the secured market and their probability to be successful when investing

[3] Including collateral liquidation costs in the model would lead to a secured rate which is slightly higher than 1, because the lender would have to take these collateral liquidation costs into account when setting the appropriate secured interest rate. As this does not change the basic structure of the model, we ignore these costs.

[4] As shown in Appendix B, section A.1, the borrower would prefer to borrow fully on the secured market if this was possible.

Table 1: Payoff structure for the borrower in the collateral-constrained case

market	safe investment	risky investment
secured	$A - (R^s\lambda + R^u(1-\lambda))I$	if project successful: $\theta-(R^s\lambda+R^u(1-\lambda))I$
		if project unsuccessful: $-R^s\lambda I$
unsecured	$A - R^u I$	if project successful: $\theta - R^u I$
		if project unsuccessful: 0.

risky:

$$\begin{aligned}
\Pi_B^{s\lambda}(safe) &= A - (R^s\lambda + R^u(1-\lambda))I \\
\Pi_B^{u}(safe) &= A - R^u I \\
\Pi_B^{s\lambda}(risky) &= (\theta - R^u(1-\lambda)I)p_i - R^s\lambda I \\
\Pi_B^{u}(risky) &= (\theta - R^u I)p_i + (0)(1-p_i) = (\theta - R^u I)p_i.
\end{aligned}$$

From these payoffs, six strategies follow for the borrower. First, the borrower can borrow a share λ on the secured market and the share$1-\lambda$ on the unsecured market and invest in the safe asset. Second, the borrower can borrow a share λ on the secured market and the share$1-\lambda$ on the unsecured market and invest in the risky asset. Third, the borrower can borrow a share $\rho < \lambda$on the secured market and the share$1-\rho$ on the unsecured market and invest in the safe asset. Fourth, the borrower can borrow a share ρ on the secured market and the share$1-\rho$ on the unsecured market and invest in the risky asset. Fifth, the borrower can borrow fully on the unsecured market and invest safe. Sixth, the borrower can borrow fully on the unsecured market and invest risky. We will evaluate these strategies in turn.

Assume first that the borrower borrows the maximum share λ on the secured market. $\Pi_B^{s\lambda}(safe) > \Pi_B^{u}(safe)$ always holds. The optimal strategy for the borrower thus depends on the individual value of p_i. For very low p_i, borrowers will borrow on the secured market as much as possible and invest in the safe asset as long as $A > (R^s\lambda + R^u(1-\lambda))I)$. For very high p_i, borrowers will borrow on the secured market as much as possible and invest in the risky asset, since $R^s < R^u$ as long as $(\theta - R^u(1-\lambda)I)p_i - R^s\lambda I > 0$.

In follows that a borrower is indifferent between the two pure strategies when $A-(R^s\lambda + R^u(1-\lambda))I = (\theta - R^u(1-\lambda)I)p_\lambda^T - R^s\lambda I$, where p_λ^T denotes the borrower-specific probability of success of the indifferent borrower. Put differently,

$$p_\lambda^T := \frac{A - R^u(1-\lambda)I}{\theta - R^u(1-\lambda)I}.$$

Lemma 1 *A borrower that borrows a share λ on the secured market and the rest on the unsecured market will choose the safe asset whenever $p_i \leq p_\lambda^T$ and the risky asset otherwise.*

Note that the share of borrowers choosing the safe asset is lower than in the case with full information as long as $\theta > A > R^u I(1-\lambda)$ and $\lambda < 1$, also refer to Appendix B, section A.1.

Assume second that the borrower borrows the share $\rho < \lambda$ on the secured market. However, since $R^s < R^u$, borrowing a share smaller than the maximum share on the secured market if the borrower borrows on the secured market at all is never optimal.

Lemma 2 *A borrower will never borrow a share $\rho < \lambda$ on the secured market.*

Next, consider the option of fully borrowing on the unsecured market. This option is only profitable for the borrower if this enables him to shift some of the risk regarding the payoff from the risky project to the lender. Borrowing fully on the unsecured market and investing in the risky asset is preferable if and only if $(\theta - R^u I)p_\lambda^T > A - (R^s\lambda + R^u(1-\lambda))I$. Some calculations show that this is equivalent to $p_\lambda^T < p^Y$, with $p^Y := \frac{R^s}{R^u}$.[5] $\frac{R^s}{R^u}$ denotes the threshold value for borrowing secured versus unsecured. If the individual success probability p_i is very high,

Below the threshold value $p^Y < p_\lambda^T$ it is never optimal to borrow risky. Therefore, for a borrower with $p^Y < p_\lambda^T$ will always invest safe and borrow the share λ on the secured market.

Lemma 3 *A borrower with $p^Y < p_i < p_\lambda^T$.*

Proposition 1 *If $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$, i.e. $p_\lambda^T > p^Y$ the borrower will always borrow on the secured market as much as possible. He will invest in the safe asset whenever $p_i \leq p_\lambda^T$ and in the risky asset whenever $p_i > p_\lambda^T$.*

If $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} < \frac{R^s}{R^u}$, i.e. $p_\lambda^T > p^Y$ the borrower will borrow as much as possible on the secured market. The borrower then invests in the safe asset if $p_i \in [0, p_\lambda^Z]$ and in the risky asset if $p_i \in [p^Y, 1]$.

If $p_i \in [p_\lambda^Z, p^Y]$, the borrower will fully borrow on the unsecured market and invest in the risky asset.

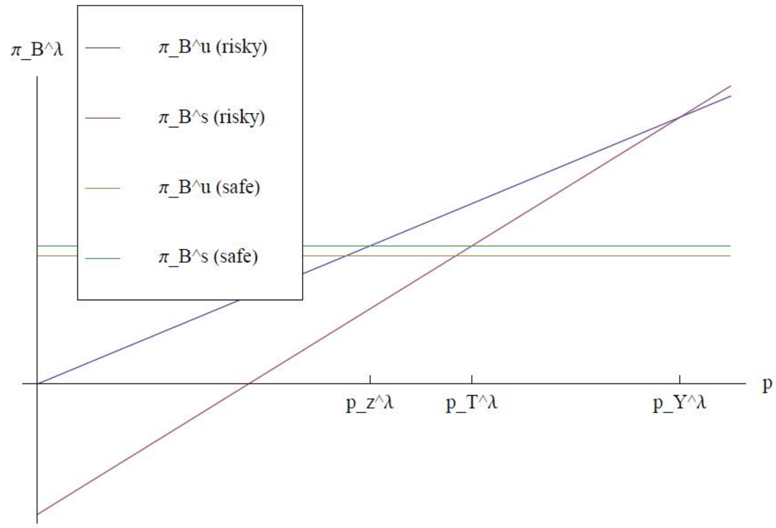
2.2 Strategies for lenders

2.3 Equilibrium

When all borrowers have to borrow a certain share of resources on the unsecured market, the fact that a borrower borrows on the unsecured market will not reveal his type. More generally, the share λ can therefore be interpreted as the degree of uncertainty in the market. Therefore, the resulting equilibrium has to include some pooling.

[5]Given that $p_\lambda^T < p^T$, $p^T < p^Y$ implies $p_\lambda^T < p^Y$. If $p^Y < p^T$, there can be situations where $p_\lambda^T < p^Y < p^T$. The equation reduces to the known equation for p^T and p^Y for $\lambda = 1$.

Figure 1: Borrower payoff structure for the collateral-constrained case



2.4 Payoff structure for lenders

The payoff for the lender is the net profit from lending, i.e. the difference between the profit when lending, R^u or R^s times the investment I, and the opportunity cost of lending, which is just I, because we assumed that the resources do not lose their value when they are not lent. When the borrower invests in the safe asset, the lender receives the profit from lending with certainty, while they only receive the profit with probability p_i if the borrower invests in the risky asset and have a loss otherwise. As a consequence, Table 2 illustrates the corresponding payoff functions.

Table 2: Payout structure for the lender

market	safe investment	risky investment
secured	$(R^s - 1)I$	$(R^s - 1)I$
unsecured	$(R^u - 1)I$	if project successful: $(R^u - 1)I$
		if project unsuccessful: $-I$

The lender's (certain) profit when lending on the secured market is $\Pi_L^s = (R^s - 1)I$. The lender's profit when lending on the unsecured market is $\Pi_L^u(risky) = (R^u I)p_i + (-I)(1 - p_i)$ if the borrower takes a risky investment, and $\Pi_L^u(safe) = (R^u - 1)I$ otherwise.

As the lender neither knows p_i nor if the borrower will invest risky or safe, the lender will have to consider the expected return from lending on the unsecured market. This depends on the expectation of p depending on the known distribution function f, taking into account whether the borrower invests in the safe/risky asset, as well as whether the borrower with such a p will borrow on the secured/unsecured market. In other words, the lender has to form a belief about the borrower's type p_i conditional on whether the borrower participates in a certain (i.e. secured or unsecured) market. Let $q \in \{0; 1\}$ be an indicator that is 1 whenever a transaction takes place on the unsecured market.

The lender's expected profit when lending on the unsecured market is thus given by a conditional expectation, namely the expected return conditional on the borrower borrowing on the unsecured market. This is the agreed payout $(R^u - 1)I$ if the borrower invests in the safe asset, or if they invest in the risky asset and are successful, and it is $-I$ if the borrower invests in the risky asset and is unsuccessful. This means that we have to write the lender's expected return on the unsecured market as a function of conditional expectations on the borrower borrowing on the unsecured market (i.e. given that the transaction takes place on the unsecured market, or $q = 1$):

$$\Pi_L^u = (R^u - 1)I\phi((\mathcal{A} \cup \mathcal{B})|q = 1) - I\phi(\mathcal{C}|q = 1)$$

where the terms $\phi(\mathcal{A})$ denotes the probability of the case that a borrower invests safe, $\phi(\mathcal{B})$ denotes the probability that a borrower invests risky, but is successful and $\phi(\mathcal{C})$ denotes the probability that a borrower invests risky, but is unsuccessful. As a consequence, $\phi(\mathcal{A} \cup \mathcal{B} \cup \mathcal{C}) = 1$ or $\mathbf{1}_{\mathcal{A}} + \mathbf{1}_{\mathcal{B}} + \mathbf{1}_{\mathcal{C}} = 1$. $\phi((\mathcal{A} \cup \mathcal{B})|q = 1)$ and $\phi(\mathcal{C}|q = 1)$ denote the respective conditional probabilities. The complexity arises from the fact that the share which the borrower borrows on the unsecured market is determined by his type.

Intuitively, the borrower has an incentive to borrow unsecured instead of secured, because the unsecured market allows the borrower to shift the risk of being unsuccessful when investing in the risky project to the lender. For a borrower with a relatively low p_i, investing risky may not be an option, because the expected return from borrowing risky is too low compared to the safe return when investing safe. Such borrowers would always prefer the secured market because of lower interest rates on the secured market.

Without asymmetric information about the borrower's type, the higher rate on the unsecured market should compensate the lender for carrying the risk of failure. However, the lender cannot perfectly distinguish between borrowers on the unsecured market. Therefore, we have to establish the lenders' conditional expectation about the probability of a borrower to invest in the safe or in the risky asset conditional on the borrower borrowing secured or unsecured.

To simplify the notation for this conditional expectation, we define several functions on the interval $[0, 1]$: The probability to invest in the safe asset $\mathbf{1}_{\mathcal{A}}$ is 1 if the borrower of type p_i invests in the safe asset and 0 otherwise. The probability of investing in the risky asset $\mathbf{1}_{\mathcal{B} \cup \mathcal{C}}$ is 1 if the borrower of type p_i invests in the risky asset and 0 otherwise. The probability of an investor investing in the risky asset and being successful $\mathbf{1}_{\mathcal{B}}$ is 1 if the borrower of type p_i invests in the risky asset and is successful and 0 otherwise. Analogously, $\mathbf{1}_{\mathcal{C}}$ is 1 if the borrower of type p_i invests in the risky asset and is unsuccessful and 0 otherwise.

Note that $\mathbf{1}_{\mathcal{B}}$ and $\mathbf{1}_{\mathcal{C}}$ can only be observed after the investments are realised. All other variables are deterministic functions of p_i, i.e. they are known if p_i is known, given the rational, profit-maximising behaviour of the borrower.

Using this notation, the expected return of the lender on the unsecured market, which is conditional on the borrower borrowing on the unsecured market, can be written as

$$\begin{aligned}\Pi_L^u &= E((R^u-1)I(\mathbf{1}_{\mathcal{A}}+\mathbf{1}_{\mathcal{B}})-I\mathbf{1}_{\mathcal{C}}|q=1)\\ &= (R^u-1)I\phi(\mathcal{A}\cup\mathcal{B}|q=1)-I\phi(\mathcal{C}|q=1).\end{aligned}$$

We can reformulate the above conditional expectation into an unconditional expectation by introducing some more notation: We define ψ as the function giving the share of funding that a borrower of type p_i will borrow on the unsecured market. We define id as the identity function. Using some basic mathematical identities, the lender's expected profit when lending on the unsecured market is then given by :

$$\Pi_L^u = \frac{E(R^u I\psi(\mathbf{1}_{\mathcal{A}}+(\mathbf{1}-\mathbf{1}_{\mathcal{A}})id)}{E(\psi)} - I \tag{1}$$

where the expected value is calculated with respect to the measure induced on $[0,1]$ by the density f.

The numerator is the sum of two components: The expected value in case of a safe borrower (where the payout probability is 1; it is multiplied by the share that is borrowed on the unsecured market) and the expected value in case of a risky borrower (where the payout probability is p_i, which is taken up by including the function id in the formula; it is again multiplied by the share that is borrowed on the unsecured market). The denominator is the total share of funding obtained on the unsecured market. Finally, I has to be subtracted to calculate the lender's net profit.

The behaviour of the lender determines the interest rates R^u and R^s. If the lender is indifferent between lending on the secured or on the unsecured market, the resulting rates R^u and R^s constitute an equilibrium.

This choice between the secured and the unsecured market only makes sense if there is a borrower that is willing to borrow on the unsecured market. Thus, we assume that we are in the second case, with $\frac{A}{\theta} < \frac{R^s}{R^u}$.

The lender's certain profit when lending on the secured market is

$$\Pi_L^s = (R^s-1)I.$$

We recall Equation 1, where ψ gives the share of funding that a borrower of type p_i will borrow on the unsecured market.

We have seen above that in the basic case no borrower will borrow on the unsecured market to invest in the safe asset. Thus, $\psi\mathbf{1}_{\mathcal{A}}$ will always be 0. We have also seen that precisely the borrowers in the range $p_i \in [p^Z, p^Y]$ will borrow on the unsecured market, i.e. ψ will be equal to 1 for $p \in [p^Z, p^Y]$, and to 0 else. The lender's expected payout when lending on the unsecured market becomes

$$\begin{aligned}\Pi_L^u &= \frac{E(R^u I\psi((\mathbf{1})id)}{E(\psi)} - I \\ &= R^u I E(p|p \in [p^Z, p^Y]) - I.\end{aligned}$$

Thus, the lender will lend on the unsecured market if

$$R^u I E(p|p \in [p^Z, p^Y]) - I > R^s I - I,$$

which is equivalent to

$$E(p|p \in [p^Z, p^Y]) > R^s/R^u.$$

For any $p \in [p^Z, p^Y)$, we know that $p < p^Y = R^s/R^u$. Thus, the conditional expectation above can never be greater than R^s/R^u, and it never makes sense for the lender to lend on the unsecured market.

One can also interpret this result in the sense that the lender will, in this case, set the rate on the unsecured market prohibitively high, i.e. $R^u \geq \frac{R^s\theta}{A}$. This pushes all borrowers onto the secured market and will avoid any moral hazard behaviour.

We have a *pooling equilibrium*: all borrowers borrow on the secured market, and their type cannot be distinguished by their market behaviour.

In case there are *collateral constraints*, it is not only the borrowers with certain p_i that would borrow on the unsecured market. In principle, all borrowers will borrow (at least a certain share of their funding) on the unsecured market.

In this case, when calculating the expected profit on the unsecured market, the lender will have to consider both the case where the borrower invests in the safe asset and the case where the borrower invests in the risky asset.

We recall that the lender's expected profit when lending on the unsecured market is given by Equation 1 (where the expected value is calculated with respect to the measure induced on $[0,1]$ by the density f):

$$\Pi_L^u = \frac{E(R^u I\psi(\mathbf{1}_\mathcal{A} + (\mathbf{1} - \mathbf{1}_\mathcal{A})id)}{E(\psi)} - I,$$

where ψ gives the share of funding that a borrower of type p_i will borrow on the unsecured market.

We have to distinguish two cases: $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$ and $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} < \frac{R^s}{R^u}$.

First, we consider the case with $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$, i.e. $p_\lambda^T > p^Y$.

In this case, all borrowers will borrow as much as possible on the secured market, and they will invest in the risky asset if and only if $p_i > p_\lambda^T$. The lender will thus set the unsecured rate based on the belief that every borrower borrows a share $(1-\lambda)$ on the unsecured market. The unsecured rate will be somewhat higher than the secured rate, since losses will be passed on to the lender in case of a non-successful risky investment.

We see this formally in the following calculation:

$$\begin{aligned}
\Pi_L^u &= \frac{E(R^u I\psi(\mathbf{1}_{\mathcal{A}} + (\mathbf{1} - \mathbf{1}_{\mathcal{A}})id)}{E(\psi)} - I \\
&= \frac{E(R^u I(1-\lambda)(\mathbf{1}_{\mathcal{A}} + (\mathbf{1} - \mathbf{1}_{\mathcal{A}})id)}{(1-\lambda)} - I \\
&= R^u I E(1_{safe} + (\mathbf{1} - \mathbf{1}_{\mathcal{A}})id) - I \\
&= R^u I \left[\int_0^{p_\lambda^T} 1 f dp + \int_{p_\lambda^T}^1 p f dp \right] - I \\
&= R^u I - R^u I \int_{p_\lambda^T}^1 (1-p) f dp - I.
\end{aligned}$$

The last equation holds because we know that $\int_0^1 f dp = 1$ (as f is a probability density).

The lender's payout on the secured market is

$$\Pi_L^s = R^s I - I.$$

Setting the two equal, we obtain

$$\begin{aligned}
R^s I - I &= R^u I - R^u I \int_{p_\lambda^T}^1 (1-p) f dp - I \\
R^s &= R^u \left(1 - \int_{p_\lambda^T}^1 (1-p) f dp \right)
\end{aligned}$$

as equilibrium condition for the secured/unsecured rate.

Next, we consider the case with $\frac{A - R^u(1-\lambda)I}{\theta - R^u(1-\lambda)I} < \frac{R^s}{R^u}$, i.e. $p_\lambda^T < p^Y$.

In the collateral-constrained case, ψ is equal to 1 for the risky borrowers that borrow fully on the unsecured market (which are the ones with $p \in [p_\lambda^Z, p^Y]$). It is equal to $1 - \lambda$ for the borrowers that borrow as far as possible on the secured market. We recall that borrowers with $p_i < p_\lambda^Z$ invest in the safe asset, while the others invest in the risky asset.

Thus, the formula becomes

$$\Pi_L^u = \frac{R^u I(\int_0^{p_\lambda^Z} (1-\lambda) f dp + \int_{p_\lambda^Z}^{p^Y} 1 p f dp + \int_{p^Y}^1 (1-\lambda) p f dp)}{\int_0^{p_\lambda^Z} (1-\lambda) f dp + \int_{p_\lambda^Z}^{p^Y} 1 f dp + \int_{p^Y}^1 (1-\lambda) f dp} - I$$

The lender's payout on the secured market is

$$\Pi_L^s = R^s I - I.$$

Setting the two equal, we obtain

$$\begin{aligned} R^s I - I &= \frac{R^u I(\int_0^{p_\lambda^Z}(1-\lambda)fdp + \int_{p_\lambda^Z}^{p^Y} 1pfdp + \int_{p^Y}^1 (1-\lambda)pfdp)}{\int_0^{p_\lambda^Z}(1-\lambda)fdp + \int_{p_\lambda^Z}^{p^Y} 1fdp + \int_{p^Y}^1 (1-\lambda)fdp} - I \\ R^s &= R^u \frac{(\int_0^{p_\lambda^Z}(1-\lambda)fdp + \int_{p_\lambda^Z}^{p^Y} 1pfdp + \int_{p^Y}^1 (1-\lambda)pfdp)}{\int_0^{p_\lambda^Z}(1-\lambda)fdp + \int_{p_\lambda^Z}^{p^Y} 1fdp + \int_{p^Y}^1 (1-\lambda)fdp} \\ &= R^u \left(1 - \frac{(\int_{p_\lambda^Z}^{p^Y} 1(1-p)fdp + \int_{p^Y}^1 (1-\lambda)(1-p)fdp)}{\int_0^{p_\lambda^Z}(1-\lambda)fdp + \int_{p_\lambda^Z}^{p^Y} 1fdp + \int_{p^Y}^1 (1-\lambda)fdp}\right) \end{aligned}$$

as equilibrium condition for the secured/unsecured rate.

We note that the borrower's choices, and thus the threshold values p_λ^Z and p_λ^Y, depend on the interest rates that will be applied by the lender. Thus, the resulting equations would have to be solved recursively, until an equilibrium is found.

In the *collateral-constrained* case, we have a *pooling equilibrium* in one case (where $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$, i.e. $p_\lambda^T > p^Y$) and a *partial pooling equilibrium* in the other case. In the first case, all borrowers borrow on the secured market as far as possible and are not distinguishable. In the second case, borrowers adjust their market behaviour according to type (borrowing either on the secured market as far as possible, or fully on the unsecured market), but not sufficiently for lenders to clearly distinguish the borrower's type.

In the *externality* case, the borrowers and the lenders do not take the externality into account in the derivation of their decisions. Thus, the analysis is the same as in the basic model.

The lender will set $R^u \geq \frac{R^s\theta}{A}$, and all borrowers will borrow on the secured market, with borrowers with $p_i < p^T$ investing in the safe asset and borrowers with $p_i > p^T$ investing in the risky asset.

Again, we have a *pooling equilibrium*, with all borrowers borrowing on the secured market.

3 Normative analysis and comparison with the positive case

We first consider the social planner's choice, in order to derive a benchmark allocation to which we can compare the market outcome. The social planner would choose the borrowers that should invest in the risky asset to maximise total welfare. We define total welfare as the sum of lenders' and borrowers' expected payoffs. The social planner is risk-neutral.

We assume that investments are worth undertaking, i.e. that both θ and A are greater than I. In this case, it is the interest of the social planner to ensure that investments are always undertaken. The only question is whether a borrower should invest in the safe or the risky asset.

From the perspective of the social planner, i.e. from an economy-wide perspective, the distribution of losses if an investment is not successful does not play a role. Moreover, collateral is not lost for the overall economy, and the distribution of collateral between market participants is not relevant for total welfare, so the choice of market (secured or unsecured) does not play a role from a perspective of the social planner in assigning borrowers to the risky or safe asset. Finally, interest payments are not relevant from the perspective of the social planner, as they do not change the aggregate situation.

(This assumption can be relaxed easily - if A is less than 1, then it is not in the interest of the social planner that investments are always undertaken, but only if θp_i is greater than 1. Replacing 'safe investment' by 'no investment', the discussion below can easily be generalised to this case.)

3.1 Baseline model

In this case, it is the interest of the social planner to ensure that the risky investment is chosen whenever $\theta p_i > A$ and that the safe investment is chosen otherwise. Thus, there is a clear cut-off value $p^T = A/\theta$. Borrowers with $p_i < p^T$ should invest in the safe asset, borrowers with $p_i > p^T$ should invest in the risky asset. (Borrowers with $p_i = p^T$ are indifferent, as the expected payout from the safe and the risky asset is the same.)

This can be seen as follows:

For a borrower of type i, the sum of the lenders' and the borrowers' payoff does not depend on the market chosen. It is independent of the interest rates that are set on the markets and of the collateral situation. The expected payoff is $A - I$ for the safe asset and $(\theta - I)p_i + (-I)(1 - p_i)$ for the risky asset. The social planner will thus wish all borrowers with $(\theta - I)p_i + (-I)(1 - p_i) \leq A - I$, which is equivalent to $p_i \leq A/\theta$ to invest in the safe asset.

In order for a borrower to invest in the risky asset, the expected return from the risky asset (the return in case of success times the probability of success) must be equal to or greater than the return from the safe asset. If F is the cumulative distribution function associated with f, then $F(p^T)$ borrowers invest in the safe asset and $1 - F(p^T)$ borrowers invest in the risky asset.

The total welfare, according to the social planner, is then

$$\begin{aligned} W_{SP} &= \int_0^{p^T} (A - I) f(p) dp + \int_{p^T}^1 (\theta p - I) f(p) dp \\ &= \left[F(p^T) A + \int_{p^T}^1 \theta p f(p) dp \right] - I. \end{aligned}$$

In the basic case, the market outcome is the same as the one that the social planner would choose, and no friction arises. All borrowers will borrow on the secured market, with borrowers with $p_i < p^T$ investing in the safe asset and borrowers with $p_i > p^T$ investing in the risky asset.

3.2 Collateral constraints

As argued before, the social planner does not care about the use of collateral or about interest payments, as these are between economic agents only and do not change the aggregate situation. Thus, the benchmark allocation that would be chosen by the social planner is the same as in the non-constrained case (see above).

In the collateral-constrained case, we recall that we have to distinguish two cases: $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$, i.e. i.e. $p_\lambda^T > p^Y$, and $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} < \frac{R^s}{R^u}$, i.e. $p_\lambda^T < p^Y$. We also recall that as long as $\theta > A > R^u I(1-\lambda)$ and $\lambda < 1$, we have $p_\lambda^T < p^T$.

In the first case, the borrower will invest in the safe asset whenever $p_i \leq p_\lambda^T$. In the second case, the borrower will invest in the safe asset for $p_i < p_\lambda^Z$.

In both cases, the market solution differs from the socially optimal one that would be chosen by the social planner (where the borrower would invest in the safe asset if and only if $p_i \leq p^T$). Thus collateral shortage will always lead to a sub-optimal market outcome.

The reason is "moral hazard" behaviour: When borrowing on the unsecured market, the borrower can pass losses on to the lender. Thus, a risky investment can become profitable for the borrower even when the expected profit lies below the profit from the safe investment. The lender compensates for his expected losses by charging higher interest rates on the unsecured market on average. But he cannot distinguish between borrowers that will invest in the safe asset and those that will invest in the risky asset. Thus, borrowers that invest in the safe asset (or which have a very high probability of success) cross-subsidise borrowers which have a medium-high probability of success for the risky asset and invest in this anyway, even though this is not socially optimal.

This is a problem for the social planner (as unproductive risky investments are being realised instead of the safe investment that yields a higher expected return). The borrowers with $p_i \in [p_\lambda^T, p^T]$ will invest in the risky asset, while the social planner would like them to invest in the safe asset.

3.3 Externality

In the presence of external effects, the existence of a regulator would be welfare-improving. We introduce an externality by assuming that each borrower's success probability decreases with an increasing aggregate investment in the risky asset. Define a cutoff probability p^E such that all borrowers with $p_i \geq p^E$ invest in the risky asset. The success probability of each borrower p_i is reduced by multiplication with a factor $\delta \leq 1$, where δ is an increasing function of the share of borrowers that invest in the risky asset $1 - F(p^E)$. We set $\delta := F(p^E) \leq 1$ and define $\tilde{p}_i := p_i\delta$ as the new success probability of the borrower of type i. The more borrowers invest in the risky asset, the more the probability distribution of p becomes skewed to the left, i.e. the lower the aggregate probability of success.

An individual borrower will typically not consider their effect on the aggregate probability of success. The straightforward consequence is a share of investment in the risky asset that is too high from an economy-wide perspective. The social planner would choose a lower share. In this sense, the interpretation of the externality in our model is similar to the externality in Perotti and Suarez (2011). However, we combine this with a full model of the money market.

For simplicity, we assume that the lenders do not consider the overall reduction in the success probability either, i.e. the effect of the externality is not considered by borrowers and lenders and only known to the social planner. As a consequence, the payout structure tables are the same as for the basic case. As the success probability changes, the expected value of the risky investment changes. The effect of the externality is not taken into account by borrowers and lenders. Thus, the payout structure given for the basic case, with the undistorted success probability, is assumed by market participants and forms the basis for their decisions.

The actual payout structure is lower, given that the success probabilities are reduced. However, this will only be known to borrowers and lenders ex post. The borrower's true return when investing on the risky asset on the secured market is $\Pi_B^s(risky) = (\theta - R^s I)\tilde{p}_i\delta + (-R^s I)(1 - \tilde{p}_i\delta)$. The borrower's true return when investing on the risky asset on the unsecured market is $\Pi_B^u(risky) = (\theta - R^u I)\tilde{p}_i\delta + (0)(1 - \tilde{p}_i\delta)$. Accordingly, the lender's true return is reduced. The trade-off for the borrower is identical to the case without an externality, but overall welfare is lower due to the externality.

The externality represented by the factor δ lowers the probability of success if the aggregate risk taken in the economy is too high. We recall that the externality materialises by changing p_i into $\tilde{p}_i = p_i\delta$, with $\delta = F(p^E)$, where p^E is the cut-off value between borrowers choosing the safe and the risky investment.

Based on the same argumentation as for the basic case, one sees that it is the interest of the social planner to ensure that the risky investment is chosen whenever $\theta\tilde{p}_i = \theta p_i\delta > A$ and that the safe investment is chosen otherwise. Thus, the cut-off value $p^T = \frac{A}{\theta}$ still holds, but now it has to be applied to the distorted values $\tilde{p}$. Borrowers with $p_i\delta = \tilde{p}_i < p^T$ should invest in the safe asset, borrowers with $\tilde{p}_i > p^T$ should invest in the risky asset. (Borrowers with $\tilde{p}_i = p^T$ are indifferent.)

Put differently, borrowers with $p_i < p^T/\delta$ should invest in the safe asset and borrowers with $p_i > p^T/\delta$ should invest in the risky asset.

The total welfare, according to the social planner, is then

$$W_{SP}^E = \left[F(p^T/\delta)A + \int_{p^T/\delta}^{1} \theta p f(p) dp \right] - I.$$

For the positive case, there would again be a deviation of the market solution from the optimal outcome that would be chosen by the social planner. Since borrowers and lenders do not take the externality into account, borrowers with $p_i < p^T$ invest in the safe asset and borrowers with $p_i > p^T$ invest in the risky asset.

The externality materialises as multiplication of p_i with δ, where δ was defined as $F(p^E)$, with p^E the threshold value for choosing between a safe and a risky investment. Note that we have $p^E = p^T$.

According to the social planner, borrowers with $p_i < p^T/\delta = p^T/F(p^T)$ should invest in the safe asset and borrowers with $p_i > p^T/\delta = p^T/F(p^T)$ should invest in the risky asset.

The borrowers with $p_i \in [p^T, p^T/F(p^T)]$ will thus invest in the risky asset, while the social planner would like them to invest in the safe asset.

3.4 Discussion

In the basic case, despite the existence of asymmetric information, the market outcome is socially optimal.

Collateral shortage always leads to a suboptimal market outcome. However, two cases can be distinguished: The case where all borrowers borrow in the secured market as much as possible (*pooling equilibrium*), and the case where some borrowers borrow all their funding needs on the unsecured market (*partial pooling equilibrium*). In both cases, the outcome is suboptimal, because a moral hazard area arises where losses are passed on from the borrower to the lender. This area is larger in the second case than in the first case.

An externality also leads to a suboptimal market outcome, as there is a range of borrowers which will invest in the risky asset while the social planner would like them to invest in the safe asset.

The suboptimal market outcomes show the need for intervention by a public authority, i.e. a central bank or a regulator.

4 The model with a central bank

The central bank in our model can act as a mediator on the interbank market via a corridor system. We establish the cases for which such a mediation function is welfare-improving. The central bank provides a deposit facility with an interest rate R^{df}. For the lender, the option to hold deposits with the central bank is an alternative to lending on the secured market, as both actions are risk-free. By setting the interest rate on the deposit facility, the central bank can thus give a lower bound for the interest rate on the secured market. The central bank can also provide a lending facility with an interest rate R^{CB}, where it lends funds against central bank eligible collateral. Obviously, the central bank will set its interest rates such that $R^{df} < R^{CB}$. The width of the corridor is then $R^{CB} - R^{df}$.

To model the central bank as a lender of last resort for banks, we assume that the collateral range accepted by the central bank is wider than that assumed by markets. We also assume that market participants have enough central bank eligible collateral available, even if they have used up all collateral eligible on the secured market. Thus, even in the collateral-constrained case, market participants can satisfy all their funding needs by borrowing from the central bank.

We assume that a lender always has the option to deposit his funds at the central bank's deposit facility.

Corollary 4 *If there is a central bank that offers a deposit facility and if the interest rate at the deposit facility is R^{df}, then $R^s \geq R^{df}$ holds for the equilibrium market interest rate on the secured market.*

With a well-functioning capital market, the secured interest rate will therefore be equal to the deposit facility rate, R^{df}, or $max(1, R^{df})$ if there is a safe store of assets as an alternative. In principle, R^{df} could also be negative. If there is no safe store of assets, then R^s could become negative as well.

Note that we do not assume any further liquidity-providing operations as this does not unduly restrict our model. Some central banks operate with a corridor system only, so that the model would perfectly describe their behaviour. For other central banks, such as the ECB, the model describes the key features of the framework that is currently in place. In the current situation of a liquidity surplus, the market rate is effectively steered with the rate at the deposit facility.[6] With the fixed rate full allotment procedure, the interest rate at the main refinancing operations takes the role of R^{CB} in our model.

Given our model setup there is no need for a central bank when there is enough collateral available, but the existence of the central bank can be welfare-improving when collateral is scarce. Our model illustrates that in this case the unsecured rate can be higher than the central bank rate as a result of the combination of insufficient collateral and asymmetric information. Without the central bank, some borrowers would then refrain from investing at all. In such a case, the existence of the central bank can move the market outcome closer to the first best outcome. Thus, the crucial assumption is that the central bank collateral framework is wider than only the collateral accepted in the secured market. As $R^s \geq R^{df}$, borrowers will in principle prefer borrowing on the secured market to borrowing at the central bank if they have collateral available that can be used for both.[7]

As outlined above, we model the central bank as a secured lender with interest rate R^{CB} and a deposit facility with interest rate $R^{df} < R^{CB}$. For example, one can consider a situation where there is a marginal lending facility where banks can receive unlimited amounts of liquidity.

We have seen that, with a well-functioning capital market, $R^{df} = R^S$ (if $R^{df} \geq 1$, which we assume in the following).

We also assume that the central bank collateral framework is wider than the range of collateral accepted by markets, such that collateral constraints would not hold for borrowing at the central bank - even if a borrower is collateral

[6]The liquidity surplus has been created by central bank action, e.g. by generous liquidity provision in liquidity-providing operations to attenuate stress in the interbank market after the onset of the financial crisis. For example, the ECB's 3-year LTROs are outstanding and have provided substantial liquidity to the banking system.

[7]Note that of course this simplification does not explicitly model the need for adequate risk-control measures for different types of collateral. While this is important, not explicitly modelling such risk-control measures does not change the implications of the model.

constrained, he could obtain the remaining funding at the central bank via other types of collateral.

[This assumption is motivated by the concrete situation in the case of the ECB, by the fact that the central bank in general plays the role of a lender of last resort, and by the fact that the central bank is not liquidity constrained and can thus take illiquid, but otherwise valuable, collateral.]

4.1 Baseline model

As $R^{CB} > R^{df}$ and $R^{df} = R^s$, we have $R^{CB} > R^s$. Thus, no borrower will borrow at the central bank, as borrowing on the secured market is always cheaper. The central bank cannot exert an influence on market conditions in this case, which is also not necessary, as they are socially optimal.

4.2 Collateral constraints

The existence of the central bank creates an additional funding opportunity for borrowers. Borrowing from the central bank replaces borrowing on the unsecured market when the unsecured market rate rises above the central bank rate. The central bank thus effectively provides a corridor for market interest rates.

Recall that we had two cases in the *collateral constrained* case: First the *pooling equilibrium*, arising if $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$, i.e. $p^T_\lambda > p^Y$, and second the *partial pooling equilibrium*, arising if $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} < \frac{R^s}{R^u}$, i.e. $p^T_\lambda < p^Y$. The analysis is similar in the two cases.

If $R^{CB} > R^u$, then the existence of the central bank has no effect. If $R^{CB} < R^u$, then some borrowers will move from the unsecured market to the central bank. In particular, borrowers which are planning to invest safe anyway or borrowers with very high success probabilities will move towards central bank funding. This will induce lenders to increase the unsecured rate (as an increased share of "moral hazard" borrowers would participate in the unsecured market), again pushing more borrowers to borrow at the central bank. An equilibrium arises when central bank lending has completely crowded out the unsecured market: This can happen with a central bank rate above the secured market rate (as long as the unsecured rate has become so high that even moral hazard borrowers could not make a profit from passing on their losses). In this case, borrowers will invest in the safe asset exactly if $p_i < p^T$, and social welfare is optimal.

However, as the central bank may have an intrinsic interest in keeping up market activity, this may not be an overall desirable setup. Rather, the central bank would most likely set the interest rate R^{CB} somewhere above a normal market rate R^u, in order to only step in when the deviation of R^u from R^S is too large.

Thus, the central bank would tolerate a limited amount of suboptimality in the market outcome in order to sustain a functioning unsecured interbank market, but would step in as a lender of last resort in case of disruptions that drive up the unsecured interest rate above the level tolerated by the central bank.

Alternatively, the central bank could consider to restrict the amounts of liquidity that can be borrowed at the central bank, whereby some unsecured market activity would be preserved, but social welfare could still be improved overall. However, it would need to be explicitly modelled how the central bank would allocate its liquidity to borrowers, which in turn would determine the equilibrium outcome. We do not explore this route further here.

Finally, we also see that it is important for the central bank to take a wide range of collateral (as otherwise investment opportunities could not be realised), but that it is likewise important that this collateral is valued appropriately (as otherwise a moral hazard region could arise, similar to the case of the unsecured market that we have analysed above, when the borrower expects that he could pass some share of the costs of a failure to the central bank).

4.3 Externality

Consider the inefficient pooling equilibrium given in the *externality* case. As we assume that $R^{CB} > R^{df}$ and $R^{df} = R^s$, we have $R^{CB} > R^s$. Thus, no borrower will borrow at the central bank. The central bank cannot exert an influence on market conditions in this case, even though they are socially suboptimal.

4.4 Discussion

The existence of the central bank can be welfare-improving in certain cases (e.g. collateral constraints), but it cannot improve welfare in other cases (e.g. externality). In this case, other authorities (e.g. regulatory authorities) may be needed.

5 The model with a regulator

We model the case of an externality to establish the need for regulation and compare the effect of central bank intervention with the intervention of a regulator.

If the share of borrowers investing in the risky asset is above the social optimum, there is a case for the regulator to intervene. The regulator can intervene either on prices or on quantities. As regards price action, the regulator can intervene via a tax on θ or a subsidy of A; the regulator could also intervene via a tax on R^u or a subsidy of R^s. As regards quantity action, the regulator could limit investment in θ or activity on the unsecured market, or set a minimum level for investment in A.

In the literature, the LCR has been interpreted as a quantity restriction rather than a as a tax on certain assets (e.g. Perotti and Suarez 2011). This interpretation results from the fact that the LCR restricts the amount of what would correspond to risky assets in our model. As taxing an activity that creates an externality (Pigou 1920) has a different effect on welfare than imposing a quantity restriction, we also discuss two possible ways of introducing the LCR into the model.

Taking our model in an abstract setting, a regulator that wants to achieve the optimal outcome from the perspective of the social planner could act via regulating either quantities or prices, i.e. in terms of a quantity restriction or in terms of a Pigovian tax. In Perotti and Suarez (2011), a combination of both measures is most efficient. The effect of the regulation depends on the source of the inefficiency. We consider both cases separately. Our framework is designed to model Basel III liquidity regulation as described in further detail in Appendix A.1.

5.1 Baseline model

In the baseline model, there is no need for the regulator to intervene, as the market outcome is socially optimal.

5.2 Collateral constraints

In the case of collateral constraints, the regulator could use price or quantity constraints. However, as the secured market cannot supply all liquidity needed for an adequate overall level of investment, the regulator must not remove the unsecured market completely.

If the regulator wants to act on the interbank markets, he could either limit the volume on the unsecured market by increasing (taxing) the interest rate on the unsecured market, or lowering (subsidising) the interest rate on the secured market. The regulator could, for example, raise the unsecured interest rate R^u. The regulator cannot raise it too high, since investment still has to be profitable for safe investors, i.e. $R^u(1-\lambda)I + R^s\lambda I \leq A$ still has to hold. Thus, $R^u = \frac{A-R^s\lambda I}{(1-\lambda)I}$ is the upper limit for R^u.

Could the regulator achieve an optimal outcome by raising R^u as far as possible, i.e. to $R^u = \frac{A-R^s\lambda I}{(1-\lambda)I}$? The answer of course depends on the values of the parameters, but in most cases the regulator cannot achieve the first-best outcome by raising R^u to $R^u = \frac{A-R^s\lambda I}{(1-\lambda)I}$. As seen in Proposition 1, the suboptimal case arises when $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} < \frac{R^s}{R^u}$, i.e. $p_\lambda^T > p^Y$. Inserting the value $R^u = \frac{A-R^s\lambda I}{(1-\lambda)I}$ in this formula, we see that this is equivalent to $\theta(1-\lambda) + R^s\lambda I - A > 0$. Inspection shows that there are plausible numerical values for θ, A, R^s and λ that fulfil this equation.[8] Thus, the 'moral hazard area' arises in this case, and the optimal outcome cannot be achieved.

If the regulator prefers a quantity restriction, the regulator could limit the volume on the interbank market to exactly the share that borrowers need to obtain on the unsecured market (due to collateral constraints). However, unless

[8]For example, one can choose $I = 100$, $\theta = 110$, $A = 101$, $R^s = 1$ and $\lambda = 0.7$.

the regulator could perfectly control the allocation on the interbank markets there will always be some borrowers that have the incentive to borrow the full amount on the unsecured market, so some borrowers would not obtain any liquidity on the unsecured market and could not realise their investment opportunities. The result is thus clearly suboptimal.

If the regulator wants to act on investment opportunities, the regulator could act with taxes or subsidies, not on interest rates, but by charging a tax on risky investment or subsidising the safe investment. Subsidising safe investments means raising A and taxing risky investments means lowering θ. As the value of A increases or the value of θ decreases, more borrowers will invest in the safe asset.

With given values for R^s, R^u, λ and I, it is always possible to change A and θ such that $\frac{A-R^u(1-\lambda)I}{\theta-R^u(1-\lambda)I} > \frac{R^s}{R^u}$, i.e. $p_\lambda^T < p^Y$, and the moral hazard area does not arise. However, the changed incentive for the borrowers would also change the equilibrium interest rates that would be charged by the lender.

[Details to be added.]

5.3 Externality

In the externality case, acting on markets does not make sense for the regulator, as all borrowers borrow on the secured market and there is no activity on the unsecured market. Thus, the regulator could act only via directly influencing the investment opportunities of borrowers.

With respect to the regulation on quantities, the regulator could limit the volume invested in the risky asset θ, or set a minimum level for the volume invested in the safe asset A. Note that every borrower needs to invest the full amount I in a project, so the regulator cannot impose restrictions at an individual level, otherwise no investment would take place at all. The regulator would thus have to impose this restriction on an aggregate level.

The effect of this regulatory activity depends on how the remaining investment possibilities would be distributed among borrowers. With a market/price-driven mechanism, it could be possible that borrowers sort according to their type, whereby the borrowers with the highest success probabilities invest in the risky asset and the borrowers with lower success probabilities invest in the safe asset. In this case, if the regulator sets the quantity thresholds at the socially optimal amounts corresponding to a share of $F(p^T)$ (to be set as a minimum share) for the safe asset or a share of $1 - F(p^T)$ (to be set as a maximum share) for the risky asset, a socially optimal outcome can be achieved.

With a mechanism whereby the restriction is allocated to individual borrowers without taking their type into account, i.e. by chance or via a process driven by another characteristic of the borrower, independent of their success probability, the outcome will be suboptimal.

With respect to regulating prices, the regulator could act with taxes or subsidies, i.e. by charging a tax on risky investment or subsidising the safe investment. For the borrower, this would correspond to changing the values of A and θ. Subsidising safe investments means raising A and taxing risky investments means lowering θ. As the value of A increases or the value of θ decreases, more borrowers will invest in the safe asset. The threshold value between borrowers that invest safe and those that invest risky will increase. In the *externality* case, this can be used to shift this threshold value to the desired, socially optimal level.

The regulator could set this tax/subsidy such that the risky investment is profitable for a borrower if and only if $p_i > p^T/\delta$, by either multiplying A with $1/\delta$ or multiplying θ with δ. The new threshold value would then be $\frac{A}{\theta\delta} = p^T/\delta$. (A combination of subsidising A and taxing θ is of course also possible. There is always a combination of subsidy and tax that renders the outcome cost-neutral for the regulator: Multiply A with $c = \frac{A+\theta}{a+\theta\delta}$ as subsidy and θ with δc as tax.)

The interest rate on the secured market does not change, as it is driven by the central bank (deposit) rate (or zero, in the absence of a central bank but in the presence of a store of value). The unsecured market will disappear completely, so no statement about unsecured interest rates can be made.

If the value of A is raised above θ, the market equilibrium differs, because no borrower would want to invest in the risky asset. As raising the value of A above what is necessary to compensate the externality is inefficient, this would not constitute an efficient equilibrium. Thus, regulation can achieve a socially optimal outcome in case of an externality of the type considered in the model, either by price or quantity restrictions.

Proposition 2 *Liquidity regulation can lead to a socially optimal outcome in certain cases of inefficiencies (e.g. externalities of the type considered in our model).*

This can be achieved via a quantity restriction (limiting the volume of risky investments or setting a minimum volume for safe investments), a price restriction (taxing risky investments and/or subsidising safe investments), or both. The quantity restriction would lead to a socially optimal outcome if adequate market mechanisms for the allocation of the restriction can be assumed. The price restriction, if calibrated correctly, will always lead to a socially optimal outcome.

6 The model with a central bank and a regulator

As seen in previous sections, the central bank intervenes by setting an interest rate R^{CB}. The regulator can intervene either by influencing the interbank markets or investment opportunities, via prices or quantities.

Intervening on the interbank markets can create a conflict with the central bank:

- Intervention on R^s is a direct conflict with the central bank deposit facility and leaves room for infinite arbitrage.

- Intervention on R^u can lead to a conflict with central bank, as in effect the corridor will not function in the way designed by the central bank - banks could be pushed into central bank operations because R^u (plus the tax) rises above R^{CB} in a situation where the central bank did not want to replace the unsecured market yet. So the regulator could achieve an efficient outcome, but at the expense of the central bank.
- Quantity restriction on the unsecured market may work in the same way, as it could drive up the price to a level $R^u > R^{CB}$ where the central bank intermediation takes over.

Intervening on A or θ does not have the same potential for direct conflicts with the central bank. However, in case an unsecured market exists, these interventions would always be designed with the ultimate aim of pushing borrowers onto the secured market, as any activity on the unsecured market creates inefficiencies from the viewpoint of the social planner. Thus, such regulatory intervention would ultimately also lead to a situation where the activity on the unsecured market is reduced, which may not be the aim of the central bank.

This leads to a conflict of interest between the central bank and the regulator: The central bank has two aims: (1) to steer funding conditions for the economy by setting the lower bound for the corridor, R^{df}, which becomes the anchor for the secured rate R^s; and (2) allowing for activity on the unsecured market while addressing tails risks, i.e. a situation where the unsecured rate would rise too much above the secured rate, by setting an upper bound of the corridor R^{CB}. When the upper bound is set such that safe investments are always profitable, this allows the central bank to pursue the aim of getting investment to take place (i.e. pursuing the secondary aim of supporting growth, as long it is in line with the primary aim of ensuring price stability). Thus, the central bank addresses inefficiencies that would come from collateral constraints to some extent, but not fully, while not touching inefficiencies coming from negative externalities.

The regulator has the aim of reducing inefficiencies in the market, arising from the negative externalities as well as from collateral constraints. Given that inefficiencies exist as soon as there is an unsecured market, the regulator would in principle design regulation such that (almost) all activity is pushed onto the secured market - just leaving enough activity on the unsecured market so that the collateral constraints do not keep investors from investing at all. Furthermore, the regulator would steer A and θ to influence investment decisions directly.

[Details to be added.]

7 Conclusion

The introduction of liquidity regulation in the aftermath of the financial crisis has led to the question how the new equilibrium inthe money market would look like. Models of the money market usually focus on asymmetric information as the key driver of market activity and other constrains that justify the intervention of a central bank in the money market. Our model allows for such a 'classic' case. However, we combine such a traditional model of the money market with an endogenous external effect to establish the case for the intervention of a regulator in this setting. Externalities on the interbank market have been analysed before, but not while at the same time modelling the interbank market explicitly.

Our model is thus the first to analyse the interaction of a central bank and a regulator when both have a reason to be present in the money market. Our most important finding is that whereas a regulatory intervention can be beneficial in the presence of externalities, it might not be so in the presence of a central bank.

The model offers several important insights. First, even in the presence of asymmetric information, the market can lead to an efficient outcome if there are no collateral constraints and no external effects.

Second, the existence of a central bank may be welfare-improving if the initial market outcome is not efficient due to asymmetric information and collateral constraints. Then central bank lending under certain conditions can supplement the market by offering contracts that the market will not provide. However, it is advisable that the central bank lending rate and collateral requirements are not chosen such that they partially or completely crowd out the secured market. Moreover, the central bank cannot achieve the first-best outcome in the presence of an externality.

Third, the regulator can achieve an efficient outcome in the presence of the externality. When there are collateral constraints and asymmetric information in the market, the regulator may improve upon the market outcome, but can never achieve the first-best outcome.

Fourth, depending on the initial market outcome and the regulator's reaction to inefficiencies, the regulator may interfere with the central bank policy in a way that makes the final outcome clearly inferior to either central bank intervention or regulatory intervention. If the regulator intervenes on the unsecured market, the outcome may be inferior as the intermediation function is then completely shifted to the central bank, depending on how the regulator intervenes in the market.

Our results suggest that it is of utmost importance to acknowledge that central bank lending can improve on the market outcome in a crisis situation and that an additional regulatory response would not necessarily increase efficiency. It depends on the design of the regulatory response. This has direct implications for the implementation of global liquidity regulation. An analysis and identification of the market equilibrium, in which the regulator intervenes, is crucial for the regulatory intervention to be actually welfare-improving.

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Appendix A

A.1 Basel III liquidity regulation and the Liquidity Coverage Ratio (LCR)

In this section, we discuss the effect of Basel III liquidity regulation and the Liquidity Coverage Ratio (LCR) in the context of our model.

Regulation prescribes that the LCR has to be above 100% at all times, where

$$LCR = \frac{\text{High-quality liquid assets}}{\text{net outflows over 30-day horizon}}.$$

In our model, regulation can be used to address negative externalities and inefficiencies stemming from collateral constraints, as outlined above. The LCR can be seen as a form of regulation in the model, via a price effect (indirectly, by making certain investments more attractive than others, as they will improve the LCR) and a quantity effect (by limiting the activity of certain economic agents on certain markets, given the restriction that they have to comply with the LCR requirements). How exactly these effects play out depends on the original situation of the market player, but we take a stylized approach in the model.

We model short-term interbank markets (maturity below 30 days), so that the borrowing/lending on the interbank market affects the *net outflows over a*

30-day horizon (NetOut). Furthermore, we assume that collateral consists of *High-quality liquid assets (HQLA)*.

Thus, borrowing/lending on the secured market has no effect on the LCR (as the outflow of funds is matched with an inflow of collateral of the same magnitude). By contrast, borrowing/lending on the unsecured market will have an effect. Namely, if an amount I is borrowed on the unsecured market, then a bank which had $LCR = HQLA/NetOut$ before will have LCR' with

$$LCR' = \frac{HQLA + I}{NetOut + I}.$$

This will move the LCR closer to 1. Thus, if a bank was LCR-constrained (i.e. had an $LCR < 1$), then it has an incentive to borrow unsecured instead of secured.

Furthermore, the regulatory treatment favours borrowing from the central bank (by applying lower outflow rates). Thus, LCR-constrained banks have a strong incentive to borrow at the central bank. The above discussion only touches on the interbank market, without considering the investment opportunities discussed in our model.

Taking the investment into account, we can make the following distinction: We assume that the safe asset can be easily liquidated and is included in *high-quality liquid assets*, while the risky asset is not included therein. Thus, LCR-constrained banks will have an incentive to invest in the safe asset.

To summarise, we can identify three effects of the LCR in our model:

- LCR-constrained banks have an incentive to borrow unsecured instead of secured.
- LCR-constrained banks have a strong incentive to borrow from the central bank.
- LCR-constrained banks have an incentive to invest in the safe asset.

The impact of the LCR depends on the number of banks that are LCR-constrained. In case this number of LCR-constrained banks is not too high and the LCR can be satisfied by the banking system as a whole, the unsecured market can be used to shift LCR-leeway from one bank to the other. This shift can be seen as an additional reason for the existence of an unsecured market.

The shift to the unsecured market may blur the price signal on the unsecured market. It may also lead to higher risk-taking behaviour that could be suboptimal. To the extent that this path cannot be pursued to satisfy the LCR-requirements of all banks, or banks choose not to fully exploit this "socially neutral" way of fulfilling the LCR, LCR-constrained banks remain.

In this case, the LCR requirements may lead to further reasons for a suboptimal outcome in our model. First, the fact that LCR-constrained banks have a strong incentive to borrow from the central bank will lead to the use of central bank funding instead of market funding. This has a negative impact on market functioning. (However, there is no effect on the allocation of investment, as seen by the social planner. - Only to the extent that lenders do not have a safe store of value, while the central bank does, or that moral hazard is attached to central bank borrowing; one could explore that avenue further.)

Second, depending on the relationship between p_i and LCR_i at the individual bank level, the fact that LCR-constrained banks have an incentive to invest in the safe asset may lead banks with good risky investment opportunities (i.e. where $\theta p_i > A$) to nevertheless invest in the safe asset. This would happen if costs of not fulfilling the LCR, e.g. fines charged by the regulator or the financial effect of credibility losses (that could for example lead to higher bank funding costs in general), would be greater than the additional expected gain from investing in the risky opportunity.

In crisis times, interbank markets do not work properly, and the central bank has assumed an intermediation role. LCR-constraints cannot be shifted around any more via the unsecured market, so this exacerbates the central bank role and leads to more central bank funding for LCR-constrained banks.

Furthermore, there is a higher incentive to invest in safe, liquid assets if banks are LCR-constrained (independent of the value of p_i), which means that less risky projects are realised even if they would be profitable. This in turn leads to a less optimal social outcome. (For example, this could be seen as exacerbating a credit crunch that may be one of the big risk factors in crisis times anyway.) The interplay between the central bank and liquidity regulation could be beneficial if the incentive to invest in risky assets would be countered by the need to invest in safe assets because of the HQLA.

Appendix B

Basic Model without Collateral Constraints

These are illustrated in Table B.1.[9]

Table B.1: Payout structure for the borrower

market	safe investment	risky investment
secured	$A - R^s I$	if project successful: $\theta - R^s I$
		if project unsuccessful: $-R^s I$
unsecured	$A - R^u I$	if project successful: $\theta - R^u I$
		if project unsuccessful : 0

A borrower's return is the difference between the return from the investment, which is A from the safe asset or θ with probability p_i from the risky asset, and the interest payment to the lender, which is $R^s I$ if borrowing on the secured market and $R^u I$ when borrowing on the unsecured market. A borrower's return when investing in the safe asset on the secured market is thus

[9]As we will see that the mixed strategy is never optimal, we exclude this option in the table and the detailed discussion.

$\Pi_B^s(safe) = A - R^s I$. A borrowers return when investing in the safe asset on the unsecured market is $\Pi_B^u(safe) = A - R^u I$. The borrower's expected return when investing in the risky asset on the secured market is $\Pi_B^s(risky) = (\theta - R^s I)p_i + (-R^s I)(1-p_i)$. The borrower's expected return when investing in the risky asset on the unsecured market is $\Pi_B^u(risky) = (\theta - R^u I)p_i + (0)(1-p_i)$. The borrower's expected return with a mixed strategy is a linear combination of the two pure strategies, weighed by the share of funding that is obtained on the secured/unsecured market.

If there is abundant collateral, a borrower will always either choose the secured market or choose the unsecured market. For any individual borrower, the choice of the (expected) higher-paying strategy is superior to a mixed strategy, thus the mixed strategy is never optimal.

We assume that investing is profitable, i.e. $A > I$ and $\theta > I$. We first analyse the situation of the borrower. When analysing the behaviour of the borrower, we take the interest rates as given. (These will then be set by looking at the optimisation behaviour of the lender - in equilibrium, the interest rates would be such that the lender is indifferent with respect to lending secured or unsecured.)

In order to determine the funding strategy when the investment strategy is clear, we have to compare the expected payout in the four basic cases:

$$\begin{aligned}
\Pi_B^s(safe) &= A - R^s I \\
\Pi_B^u(safe) &= A - R^u I \\
\Pi_B^s(risky) &= (\theta - R^s I)p_i + (-R^s I)(1-p_i) = \theta p_i - R^s I \\
\Pi_B^u(risky) &= (\theta - R^u I)p_i + (0)(1-p_i) = (\theta - R^u I)p_i.
\end{aligned}$$

The mixed funding strategy is a linear combination of the respective payout functions using only the secured or the unsecured market.

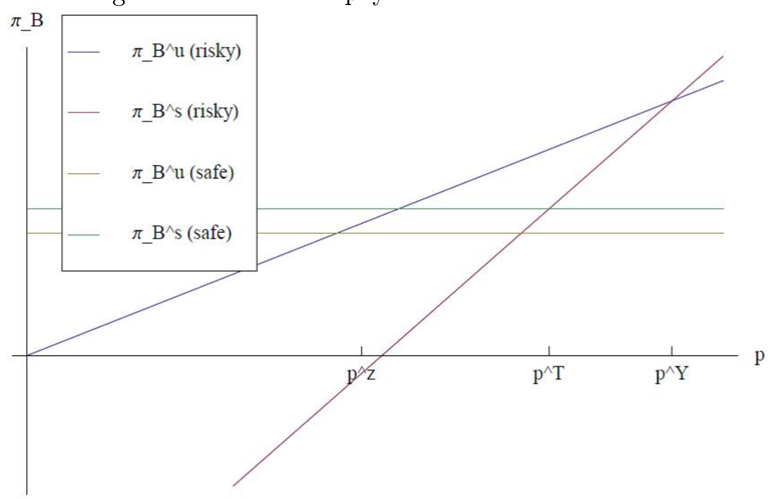
We assume that a borrower always wants to invest, i.e. $\Pi_B^s(safe) = A - R^s I > 0$. We see that in all four cases the expected payout is a linear function of p_i.

Both $\Pi_B^s(safe)$ and $\Pi_B^u(safe)$ do not depend on p_i, which can be interpreted as a linear relationship with coefficient 0 and with intercept $A - R^s I$ and $A - R^u I$ respectively. We have $\Pi_B^s(safe) \geq \Pi_B^u(safe)$ since $R^s \leq R^u$. (If $A > R^s I$, then the intercept for $\Pi_B^s(safe)$ is positive.) Thus, it will always be preferred by the borrower to borrow on the secured market if he invests in the safe asset.

Both $\Pi_B^s(risky)$ and $\Pi_B^u(risky)$ depend on p_i in a linear way, but with different coefficients: the coefficient θ arising in $\Pi_B^s(risky)$ is higher than the coefficient $\theta - R^u I$ arising in $\Pi_B^u(risky)$. Conversely, the intercept is negative $(-R^s I)$ for $\Pi_B^s(risky)$ and 0 for $\Pi_B^u(risky)$. (Both coefficients are positive if $\theta > R^u I$. Note that we can in principle safely assume this, as otherwise no activity would take place on the unsecured market and R^u would therefore become irrelevant.)

Figure A.1 below illustrates the situation.

Figure B.1: Borrower payoff structure for the basic case



The borrowers want to choose the highest expected payout by deciding on the investment strategy and the funding market. The borrowers' optimal choices vary depending on p_i (while the other parameters are the same for all borrowers). We see that it never makes sense for a borrower to choose a mixed strategy, since this would be a linear combination of the respective two lines, and it always makes sense to choose the higher of the two lines, i.e. one of the corner options.

Given the relationship of the four lines defined by the profit functions, it is clear that a borrower with a very low p_i will choose to invest in the safe asset and borrow on the secured market, and a borrower with a very high p_i will choose to invest in the risky asset and borrow on the secured market (this can also be seen by looking at the extreme values $p_i = 0$ and $p_i = 1$).

As regards intermediate values of p_i, two possibilities exist:

The first case is where the line determining $\Pi^u_B(risky)$ always lies below $max(\Pi^s_B(safe), \Pi^s_B(risky))$. In this case, all market participants will borrow on the secured market and invest either safe or risky, depending on whether p_i lies above or below the value determining the intercept of $\Pi^s_B(safe)$ and $\Pi^s_B(risky)$, which we denote by p^T. The unsecured market will not be used. From setting the equations for the two lines given by $\Pi^s_B(safe)$ and $\Pi^s_B(risky)$ equal, we see that

$$p^T := A/\theta.$$

The second, more interesting case is where there is a range where the line determining $\Pi^u_B(risky)$ lies above all others. This is the case if $(\theta - R^u I)p^T > A - R^s I$, i.e. if $\frac{A}{\theta} < \frac{R^s}{R^u}$.

In this case, two intercept values are relevant, the intercept of $\Pi^s_B(safe)$ and $\Pi^u_B(risky)$ and the intercept of $\Pi^u_B(risky)$ and $\Pi^s_B(risky)$. We denote the relevant values of p_i by p^Z and p^Y respectively. It is evident that $p^T \in [p^Z, p^Y]$.

(Note that $p^T \in (p^Z, p^Y)$ if the linear coefficients of all three lines are clearly different, i.e. if $\theta > R^u I$.)

By definition as intercept of the respective lines, we have $\theta p^Y - R^s I = (\theta - R^u I)p^Y$, i.e. $p^Y = R^s / R^u$. This is the threshold value between borrowing secured and unsecured for risky investors. Similarly, we have $A - R^s I = (\theta - R^u I)p^Z$, i.e. $p^Z = \frac{A - R^s I}{\theta - R^u I}$. This is the threshold value between borrowing secured and investing safe, and borrowing unsecured and investing risky.

All of these considerations are summarised in the following proposition.

Proposition 3 *We assume that investing is profitable, i.e.* $A > I$ *and* $\theta > I$. *We assume that a borrower always wants to invest, i.e.* $\Pi_B^s(safe) = A - R^s I > 0$.

A borrower will never choose a mixed strategy. He will choose to borrow either fully on the secured or fully on the unsecured market.

The behaviour of the borrower depends on the value of p_i *and on the relationship of* A, θ, R^s *and* R^u. *We define* $p^Z := \frac{A - R^s I}{\theta - R^u I}$, $p^T := A/\theta$ *and* $p^Y := R^s / R^u$.

We have to distinguish two cases: $\frac{A}{\theta} > \frac{R^s}{R^u}$, *i.e.* $p^T > p^Y$, *and* $\frac{A}{\theta} < \frac{R^s}{R^u}$, *i.e.* $p^T < p^Y$.

In the first case, the borrower will always borrow on the secured market. He will invest in the safe asset whenever $p_i \leq p^T$ *and in the risky asset whenever* $p_i > p^T$.

In the second case, the borrower will borrow on the secured market and invest in the safe asset for $p_i \in [0, p^Z]$, *he will borrow on the unsecured market and invest in the risky asset for* $p_i \in [p^Z, p^Y]$, *and he will borrow on the secured market and invest in the risky asset for* $p_i \in [p^Y, 1]$. *(In this case,* $p^Z < p^T < p^Y$.*)*

This situation is intuitively plausible: If the chances of success of the risky asset are so low that the expected gains are greater from the safe asset even if losses could be passed on to the lender, then the borrower will invest in the safe asset and borrow on the secured market. If the risky asset is clearly preferable, even if the borrower has to take the losses himself, because the probability of success is very high, then he will invest in the risky asset and borrow on the secured market. But in between, a moral hazard problem arises: If the borrower would have to take the losses, he would opt for the safe asset, but if he can pass on the losses to the lender, he will invest in the risky asset.