A Dynamic Network Model of the Unsecured Interbank Lending Market*

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November 1, 2014

Job Market Paper
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Abstract

We introduce a structural dynamic network model of the formation of lending relationships in the unsecured interbank market. Banks are subject to random liquidity shocks and can form links with potential trading partners to bilaterally Nash bargain about loan conditions. To reduce credit risk uncertainty, banks can engage in costly peer monitoring of counterparties. We estimate the structural model parameters by indirect inference using network statistics of the Dutch interbank market from 2008 to 2011. The estimated model accurately explains the high sparsity and stability of the lending network. In particular, peer monitoring and credit risk uncertainty are key factors in the formation of stable interbank lending relationships that are associated with improved credit conditions. Moreover, the estimated degree distribution of the lending network is highly skewed with a few very interconnected core banks and many peripheral banks that trade mainly with core banks. Shocks to credit risk uncertainty can lead to extended periods of low market activity, amplified by a reduction in peer monitoring. Finally, our monetary policy analysis shows that a wider interest rate corridor leads to a more active market through a direct effect on the outside options and an indirect multiplier effect by increasing banks' monitoring and search efforts.

Keywords: Interbank liquidity, financial networks, credit risk uncertainty, peer monitoring, monetary policy, trading relationships, indirect parameter estimation

JEL classification codes: C33, C51, E52, G01, G21

*Corresponding author: F.Brauning@vu.nl. We thank Henrique Basso, Darrell Duffie, Lucy Gornicka, Siem Jan Koopman, Marco van der Leij, Andre Lucas, Albert Menkveld, Gabriel Perez Quiros, Gerhard Rünstler and Hyun Song Shin for comments. Participants at seminars at De Nederlandsche Bank, Deutsche Bundesbank, the Bank of England, the Bank for International Settlements, the Duisenberg School of Finance, the ECB Money Market Workshop, the Cambridge Workshop on Financial Risk and Network Theory, the 7th Annual SoFiE Conference, the 1st Annual IAAE Conference, the Banque de France - SoFiE Conference on Systemic Risk and Financial Regulation, and the GENED Workshop on Networks in Macroeconomics and Finance also provided useful feedback. Blasques and Bräuning gratefully acknowledge financial support from the Netherlands Organisation for Scientific Research (NWO) and the SWIFT institute. The opinions expressed in this paper do not necessarily reflect those of De Nederlandsche Bank, the Eurosystem or the Bank for International Settlements.

1 Introduction

The global financial crisis of 2007-2008 highlighted again the crucial role of interbank lending markets in the financial system and the entire economy. Particularly, after the September 2008 fall of Lehman Brothers , increased uncertainty in the banking system led to severely distressed unsecured interbank lending markets. As a result, monetary policy implementation was hampered and credit supply to the non-financial sector dropped substantially, with adverse consequences for both the financial sector and the real economy. Moreover, the freeze of the euro area interbank markets within some member countries amplified the sovereign debt crisis in Europe. In order to mitigate these adverse effects, central banks intervened not only by injecting additional liquidity into the banking sector but also by adjusting their monetary policy instruments. As a consequence, central banks became the intermediary for large parts of the money market during the crisis.[1]

But why should central banks not resume the role of a central counterparty for money markets also during normal times? Generally having a central counterparty for the unsecured interbank market reduces contagion effects as credit exposures between banks can no longer give rise to a chain reaction that might bring down large parts of the banking sector, see Allen and Gale (2000). Likewise search frictions that result from asymmetric information about liquidity conditions of other banks are mitigated. On the other hand, with a central counterparty, private information that banks have about the credit risk of other banks is no longer reflected in the price at which banks can obtain liquidity. Thus market discipline is impaired. Moreover, the incentives for banks to acquire and process such information are largely eliminated. However, as Rochet and Tirole (1996) argue, if banks can assess the creditworthiness of other institutions more efficiently than a regulatory authority, a decentralized organization of the interbank market can be optimal.[2] Consequently, in order to assess whether a central counterparty in the unsecured money market is welfare enhancing, one has to gauge the extent to which credit risk uncertainty and peer monitoring affect liquidity allocation in money markets.

Our paper contributes to this debate by introducing and estimating a structural dynamic network model of unsecured interbank lending. The two key economic drivers of the model outcomes are asymmetric information about counterparty risk and liquidity conditions elsewhere in the market. In particular, our focus is on the role of peer monitoring in reducing bank-to-bank credit risk uncertainty and endogenous counterparty selection (directed counterparty search) in mitigate search frictions resulting from the over-the-counter market structure. Therefore we model the interbank market as a

[1] See Cœuré (2013) and Heijmans et al. (2014), Video 3 for evidence of the Dutch central bank's role.

[2] Indeed, the ECB highlights the role of monitoring and private information as well: "Specifically, in the unsecured money markets, where loans are uncollateralised, interbank lenders are directly exposed to losses if the interbank loan is not repaid. This gives lenders incentives to collect information about borrowers and to monitor them over the lifetime of the interbank loan [...]. Therefore, unsecured money markets play a key peer monitoring role.", see the speech by Benoît Cœuré, Member of the Executive Board of the ECB, at the Morgan Stanley 16th Annual Global Investment seminar, Tourrettes, Provence, 16 June 2012. `http://www.ecb.europa.eu/press/key/date/2012/html/sp120616.en.html`, retrieved 10/10/2013.

network of potential trading relationships that banks may use to smooth random liquidity shocks drawn from a latent bank-specific distribution. Each period, banks choose bilateral monitoring efforts and can establish costly links to maximize profits from interbank trading. Upon linking to a potential lending partner, banks engage in bilateral Nash bargaining about the loan conditions. We characterize the bilateral equilibrium interest rate as an increasing function of the outside option for borrowing and lending (given by the central bank's standing facilities), counterparty risk uncertainty and the lender's market power relative to the borrower. Further, we show that banks' optimal dynamic monitoring efforts towards potential borrowing banks are functions of the expected profitability of trade with each counterparty, in particular the expected probability of being contacted by distinct borrowers and the expected loan volumes to be traded with these borrowers. Similarly, banks' search efforts depend on expected bilateral interest rates and volumes. To derive the reduced form of the model, we specify an expectations mechanism based on an exponentially weighted moving average.

The second novelty of this paper lies in the econometric analysis of the dynamic network model, in particular the estimation of the structural parameters. To this end we use a unique transaction-level dataset of unsecured overnight loans between the largest 50 Dutch banks derived from the TARGET2 payment system records from 2008 to 2011. This data contains information on the bilateral volumes and interest rates of granted loans as well as the identity of the borrowing and lending bank of each transaction. Given the dynamic complexity of the model and the presence of latent variables, likelihood-based estimation is infeasible and we turn to the unifying approach of indirect inference for simulation-based parameter estimation, introduced in Gourieroux et al. (1993) and Smith (1993). Specifically, our indirect inference estimator of the structural parameters is based on an auxiliary vector that contains network statistics (e.g. density, reciprocity and centrality) that have become popular in characterizing the topological structure of interbank markets, see for instance Bech and Atalay (2010). We further complement these network statistics by moment statistics of bilateral interest rates and volumes, and measures of bilateral lending relationships as in Furfine (1999) and Cocco et al. (2009). Our estimator is then obtained as the parameter that minimized the distance between the observed and simulated values of the auxiliary vector.

Our estimated model parameters indicate that banks' monitoring efforts significantly reduce bank-to-bank credit risk uncertainty that prevails in the market. In particular, we find that peer monitoring in interrelation with endogenous counterparty selection generates an amplification mechanism that is a crucial driver of interbank lending: Lending banks invest in monitoring those borrowers from whom they expect high profitability; either because of large volumes or higher expected return of granted loans, or because of a high frequency of borrowing contacts. Borrowing banks obtain part of the surplus generated by monitoring; this foster their borrowing relation with the lending bank. As a consequence, more loans are granted and lending banks further increase monitoring efforts in expectation of higher profitability. Our analysis of the estimated model shows that this monitoring multiplier effect has important implications for the endogenously arising network structure as well as for the amplification of shocks to credit risk uncertainty and changes in monetary policy.

First, we find that monitoring, search frictions and counterparty risk uncertainty are significant factors in matching the topological structure of the trading network, such as the high sparsity, low reciprocity and skewed degree distribution. In particular, the model estimates imply a tiered network structure with a few highly interconnected core banks that intermediate in the market and many sparsely connected peripheral banks that almost exclusively trade with core banks. Banks in the core typically have a structural liquidity deficit (investment opportunity) but feature a large variance in liquidity shocks, on the other hand peripheral banks typically have a structural funding surplus and small scaled shocks. Part of the tiered structure can be explained by banks' heterogeneous liquidity shocks. However, comparison of the estimated model with a model calibration without monitoring (but everything else equal) shows that credit risk uncertainty and peer monitoring are crucial in reinforcing the core-periphery structure: large money center banks are more intensively monitored by their lenders and they in turn monitor closely their borrowers, leading to both lower bid and offer rates and that fuels their role as market intermediaries.

Second, the core-periphery structure of both the estimated and the observed lending network is very stable across time. In particular, we find that bank pairs form long-term trading relationships that are associated with lower interest rates and improved credit availability. Bank-to-bank uncertainty problems are small as these relationship pairs engage in repeated peer monitoring and counterparty search which crucially depend on banks' persistent expectations about bilateral credit availability and conditions. In this respect, our findings also highlight the role of banks' heterogeneous liquidity shock distributions (e.g. differences in funding and investment opportunities) in determining bilateral trading opportunities as the fundamental source of interbank lending relationships. Specifically, banks with on average complementary shocks or large variance of liquidity shocks profit from forming a bilateral lending relationship. However, our analysis shows that the multiplier effect of monitoring is necessary to generate a bilateral stability and impact of relationship lending on interest rates of a similar magnitude as observed in the data.

Third, our dynamic analysis reveals that shocks to credit risk uncertainty can bring down market activity for extended periods of time. The lending network shrinks because bilateral interest rates increase as a response to the higher perceived riskiness of counterparties. Hence, interbank lending becomes less profitable relative to the outside options (given by the central bank's standing facilities) and a number of trades are simply substituted by recourse to the standing facilities. Moreover, as a response to the shock, banks invest less in peer monitoring in expectation of future higher uncertainty and associated lower profitability. Feedback loops between monitoring and search amplify this reduction thereby preventing a faster recovery of the market. We also find that after the shock, the lending network becomes less connected but more concentrated (larger reciprocity and higher skewness of degree distribution) as banks with extensive relationships stay in the market. In particular, bank pairs that face low bank-to-bank credit risk uncertainty (due to private information acquired by previous monitoring) continue to lend to each other and, as a consequence, the average interest rate spread of granted loans decreases during the crisis period.

Fourth, the analysis of the estimated model shows that the central bank's interest rate corridor is a crucial determinant of interbank lending activity. In particular, we find that by increasing the discount window width the central bank fosters interbank lending as it directly reduces the attractiveness of the outside options thereby increasing the potential surplus from (bilateral) interbank lending. However, we document a further indirect multiplier effect as increased expected surplus from interbank trading intensifies banks' monitoring and search effort that in turn further improve credit conditions and availability in the market leading to more liquidity and a more efficient market usage. Moreover, we find that as a response to an increase in the corridor width, the lending network becomes less stable as more loans are settled outside of established relationships (spot lending increases). Finally, under the new policy regime loans associated with higher bank-to-bank uncertainty are settled and, as a consequence, both the interest spread (relative to the corridor center) and the cross-sectional variation of spreads increases.

The paper is structured as follows. Section 2 discusses the related literature. Section 3 introduces the structural model. Section 4 provides details on the estimation procedure, discusses the parameter estimates of the model and analyzes the relative fit in terms of various criteria. Finally, Section 5 analyzes the estimated model and studies policy implications. Section 6 concludes.

2 Stylized Facts and Related Literature

Interbank lending networks exhibit two stylized facts. First, interbank markets exhibit a sparse core-periphery structure whereby a few highly interconnected core banks account for most of the observed trade. Peripheral banks have a low number of counterparties and almost exclusively trade with core banks.[3] Second, interbank lending is based on stable bilateral trading relationships that facilitate access to credit and offer better loan conditions.[4] By introducing a model that explains these two stylized facts, our paper relates to several strands of the literature.

First, the basic economic driving forces of the proposed interbank lending model are credit risk uncertainty, peer monitoring and search frictions. Thereby, our paper is related to recent work by Afonso and Lagos (2012) who propose a search model to explain intraday trade dynamics in the spirit of over-the-counter models as in Duffie et al. (2005). Similar to these authors, we also build our dynamic model on bilateral bargaining and search frictions. However, Afonso and Lagos (2012) abstract from the role of bank default which is introduced by Bech and Monnet (2013). Neither model accounts for credit risk uncertainty nor do they focus on explaining the network

[3] For empirical evidence on the topological structure of interbank markets see, for instance, Soramäki et al. (2007), May et al. (2008) and Bech and Atalay (2010) for the US, Boss et al. (2004) for Austria, Iori et al. (2008) and Lux and Fricke (2012) for Italy, Becher et al. (2008) for the UK, Craig and von Peter (2014) for Germany, and van Lelyveld and in 't Veld (2012) for the Netherlands.

[4] The existence of interbank relationship lending has been documented by, for instance, Furfine (1999), Furfine (2001), Ashcraft and Duffie (2007) and Afonso et al. (2013) for the US, Iori et al. (2008) and Affinito (2011) for Italy, Cocco et al. (2009) for Portugal, Bräuning and Fecht (2012) for Germany.

structure of interbank markets. Moreover, since they assume a continuum of atomistic agents where the probability of two banks are matched repeatedly is zero, there is no role for the emergence of long-term trading relationships.

On the other hand, building on the classical banking model by Diamond and Dybvig (1983), Freixas and Holthausen (2005), Freixas and Jorge (2008) and Heider et al. (2009) have focused on the role of asymmetric information about counterparty risk in the liquidity allocation. In particular, Heider et al. (2009) show that informational frictions can lead to adverse selection and a market freeze with liquidity hoarding. In these models, however, interbank markets are anonymous and competitive and they hence abstract from the OTC structure where deals are negotiated on a bilateral basis and credit conditions depend on heterogeneous expectations about counterparty risk and credit conditions. The role of peer monitoring and private information in interbank markets that we consider a key driver has been highlighted by Broecker (1990), Rochet and Tirole (1996) and Furfine (2001). The literature however lacks a model of peer monitoring at the bank-to-bank level in an OTC market.[5]

Second, our paper relates to the growing literature on the formation of financial networks (see Gale and Kariv (2007), Babus (2013), in 't Veld et al. (2014), Vuillemey and Breton (2014) and Farboodi (2014)).[6] In particular, Babus (2013) shows that when agents trade risky assets over-the-counter, asymmetric information and costly link formation can endogenously lead to a (undirected) star network with one intermediary. Farboodi (2014) develops a model that generates a core-periphery structure where banks try to capture intermediation rents. Crucially, her model relies on the assumption of differences in investment opportunities (see also in 't Veld et al. (2014)). Our model confirms the importance of this type of bank heterogeneity for the emergence of a core-periphery structure, but credit risk uncertainty and peer monitoring are the key drivers of persistent bilateral lending relations that reinforce the core-periphery structure. As opposed to these studies that are concerned with the emergence of a static network, our paper also analyzes the dynamic behavior of the lending network and contributes to parameter estimation of structural network models.[7]

Third, our findings relate to empirical studies analyzing the functioning of interbank markets during the financial crisis. For the US overnight interbank market, Afonso et al. (2011) provide evidence that counterparty risk concerns plays a larger role than liquidity hoarding (Acharya and Merrouche (2013)) in explaining the disruption of interbank lending around the days of Lehman Brothers' bankruptcy. Our estimated model dynamics confirm that shocks to counterparty risk

[5]Babus and Kondor (2013) consider information aggregation in OTC markets for a given network structure, where agents infer the asset's value based on other observed bilateral prices and quantities from other transactions. In contrast, in our model of bank-to-bank uncertainty banks engage in bilateral monitoring and do not learn about a counterparties riskiness from other bilateral prices.

[6]The effects of the network structure on financial contagion has been studied, for instance, by Georg (2013), Gai et al. (2011), Acemoglu et al. (2013) and Gofman (2014). We do not focus on contagion effects in this paper.

[7]Most of the literature on estimation of network models discuss estimation of statistical, reduced form models. A recent attempt to calibrate a structural network model is presented by Gofman (2014) who matches the density, maximum degree and the number of intermediaries) with those of the federal funds market as reported by Bech and Atalay (2010).

uncertainty can bring down lending activity for extended periods of time accompanied with a more concentrated lending network. In the latter respect, Gabrieli and Georg (2014) provide empirical evidence on the network shrinkage for the Euro money market during the financial crisis 2007/8.

Fourth, our paper relates to the literature on monetary policy. Theoretical contributions on the implementation of monetary policy in a corridor system with standing facilities in the context of competitive markets include Poole (1968), Whitesell (2006) and Berentsen and Monnet (2008). See also Kahn (2010) for a non-technical overview and evidence for monetary policy regimes in several countries. In this paper, we extend this literature by analyzing effects of changes in the interest rate corridor on the structure of the lending network and the (cross-sectional) distribution of interest rates. In particular, our model suggests that increasing the corridor incentivizes peer monitoring and private interbank lending. However, absent a view on the central bank's preferences, we cannot make statements about the optimal corridor width (cf. Bindseil and Jabłecki (2011), Berentsen and Monnet (2008)).

3 The Interbank Network Model

We model the interbank lending market as a network consisting of N nodes with a time-varying number of directed links between them. Each node represents a bank and each link represents an unsecured interbank loan that is characterized by a loan amount and an interest rate. Time periods are indexed by $t \in \mathbb{N}$. Banks are indexed by $i \in \{1, ..., N\}$.

Each period, banks are subject to liquidity shocks that hit banks' payment accounts in their daily business operations (e.g. clients that want to make payments). Banks wish to smooth these shocks by borrowing and lending unsecured funds from each other in an over-the-counter market. As an outside option, banks have unlimited recourse to the central bank's standing facilities (discount window) with deposit rate $\underline{r}$ and lending rate $\overline{r}$ with $\overline{r} \geq \underline{r}$.

Banks enter the market with the objective to maximize expected discounted interbank market profits from lending and borrowing funds by: *(i)* choosing which banks to approach for bilateral Nash bargaining about interest rates with other banks; and *(ii)* setting bilateral monitoring expenditures to mitigate uncertainty about counterparty credit risk.

In the following, we discuss the model structure, solve for banks' optimal dynamic monitoring and search decisions and specify an adaptive expectation mechanism to derive the reduced form of the model.

3.1 Liquidity Shocks

At each period t, every bank enters the market as a potential borrower or lender according to the random vector $\zeta_t = (\zeta_{1,t}, ..., \zeta_{N,t})'$. Each element $\zeta_{i,t} \in \mathbb{R}$, denotes the period t liquidity shock of bank i with $\zeta_{i,t} < 0$ ($\zeta_{i,t} > 0$) being a deficit (surplus). For instance, a bank might demand funds because it has an investment opportunity, or it might have funds available as customers deposit money at their accounts.

The distribution of liquidity shocks in the banking system has been identified as crucial for the interbank lending structure by Allen and Gale (2000) amongst others. We assume that liquidity shocks $\zeta_{i,t}$ are independently and normally distributed with bank specific mean μ_{ζ_i} and variance $\sigma^2_{\zeta_i}$ parameters

$$\zeta_{i,t} \overset{i.i.d.}{\sim} \mathcal{N}(\mu_{\zeta_i}, \sigma^2_{\zeta_i}) \quad \text{where} \quad \mu_{\zeta_i} \sim \mathcal{N}(\mu_\mu, \sigma^2_\mu) \quad \text{and} \quad \log \sigma_{\zeta_i} \sim \mathcal{N}(\mu_\sigma, \sigma^2_\sigma),$$

and we allow for correlation between μ_{ζ_i} and $\sigma^2_{\zeta_i}$ through the parameter $\rho_\zeta := \mathrm{Corr}(\mu_{\zeta_i}, \sigma_{\zeta_i})$. Note that we assume (conditional) independence and normality of liquidity shocks for convenience as it allows us to analytically compute part of the model's solution. This simple type of heterogeneity in the distribution of banks' liquidity shocks allows us to model size effects related to the scale of banks' businesses through larger variances that are drawn from a log-normal distribution. Moreover, it allows us to account for structural liquidity provision or demand by some banks through a nonzero mean $\mu_{\zeta_{i,t}}$. The parameter ρ_ζ allows both effects to be correlated, for instance some banks might on average supply small amounts of liquidity to the market (e.g. deposit collecting institutions).[8]

3.2 Counterparty Risk Uncertainty

Borrowers may default on interbank loans and – due to the unsecured nature of interbank lending – impose losses on lenders. The *true probability of default* of bank j at time t is derived as the tail probability of a random variable $z_{j,t}$ that measures the *true financial distress* of bank j,

$$\mathbb{P}(z_{j,t} > \epsilon).$$

In particular, $z_{j,t}$ is constructed so that bank j is forced into default whenever $z_{j,t}$ takes values above some common time-invariant threshold $\epsilon > 0$. This threshold can be interpreted as either a minimum regulatory requirement or a level that is (seen to be) sufficient to operate in the market. We focus on the case that $z_{j,t}$ is identically and independently distributed for each bank j with $\mathbb{E}(z_{j,t}) = 0$ and $\sigma^2 = \mathrm{Var}(z_{j,t})$, such that there is no heterogeneity in banks' true probability of default.

While counterparty credit risk relates to the riskiness and liquidity of a borrower's assets, asymmetric information about counterparty risk is seen as a major characteristic of financial crises

[8] We model bank heterogeneity by latent random variables (random effects) that are characterized by five parameters, instead of estimating the $\mu_{\zeta_i}, \sigma^2_{\zeta_i}$ as fix parameters (fixed effects) for each i. In our model estimation we have data for $N = 50$ banks rendering the latter approach computationally infeasible.

that led to inefficient allocations in money markets, see Heider et al. (2009).[9] Asymmetric information problems arise because counterparty risk assessment is not based on the true but on the *perceived probability of default* that bank i attributes to bank j at time t. This probability is denoted $P_{i,j,t}$ and obtained as the tail probability of a random variable $z_{i,j,t}$ that measures bank i's *perceived financial distress* of bank j. The perceived financial distress $z_{i,j,t}$ contains an added bank-to-bank uncertainty that is modeled by the addition of an independent *perception error* $e_{i,j,t}$ so that,

$$z_{i,j,t} = z_{j,t} + e_{i,j,t}.$$

Note that if the sequence of perception errors $\{e_{i,j,t}\}_{t\in\mathbb{N}}$ is *iid* with $\mathbb{E}(e_{i,j,t}) = 0$ and small variance $\text{Var}(e_{i,j,t}) \approx 0$, then the perceived financial distress is correct on average $\mathbb{E}(z_{i,j,t}) = \mathbb{E}(z_{j,t})$ and added uncertainty is small $\text{Var}(z_{i,j,t}) = \text{Var}(z_{j,t}) + \text{Var}(e_{i,j,t}) \approx \text{Var}(z_{j,t})$. As a result the perceived probability of default $P_{i,j,t}$ is likely to follow closely the true probability of default $P_{j,t}$.

The evolution of the distribution of the perception errors is determined by the knowledge that bank i has about the default risk of bank j. This knowledge depends on factors such as the past trading history and, in particular, the monitoring expenditure that bank i allocates to learn about bank j's financial situation, as discussed in more detail in the following section. Specifically, we assume that banks' perception errors have mean zero and variance $\tilde{\sigma}^2_{i,j,t} = \text{Var}(e_{i,j,t})$ that evolves over time according to autoregressive dynamics given by

$$\log \tilde{\sigma}^2_{i,j,t+1} = \alpha_\sigma + \gamma_\sigma \log \tilde{\sigma}^2_{i,j,t} + \beta_\sigma \phi_{i,j,t} + \delta_\sigma u_{i,j,t}. \tag{1}$$

Here $\phi_{i,j,t-1}$ is a function of past bilateral trading intensity and monitoring expenditure that measures the amount of *new information* that bank i collects about the financial situation of bank j in period t, and $u_{i,j,t} \sim \mathcal{N}(0,1)$ is a shock to the counterparty risk uncertainty scaled by $\delta_\sigma > 0$. Moreover we impose that $\beta_\sigma \leq 0$ and hence the added information (weakly) reduces the perception error variance. Note that due to the log specification, $\tilde{\sigma}^2_{i,j,t}$ follows a nonlinear process $\tilde{\sigma}^2_{i,j,t+1} = \xi_{i,j,t}(\phi_{i,j,t}, \tilde{\sigma}^2_{i,j,t}) = \xi(\phi_{i,j,t}, \tilde{\sigma}^2_{i,j,t}, u_{i,j,t})$. Further, $\frac{\partial \xi_{i,j,t}}{\partial \phi_{i,j,t}} < 0$ and $\frac{\partial^2 \xi_{i,j,t}}{\partial \phi^2_{i,j,t}} > 0$ and hence the effect of additional information on the perception error variance is weaker for higher levels of $\phi_{i,j,t-1}$. Thus our model dictates decreasing returns to scale in information gathering.

Since the exact distribution of the perception error $e_{i,j,t}$ is unknown to bank i, every bank is assumed to approximate the tail probability of the extreme event of default by the conservative

[9] In this respect William Dudley, President and CEO of the Federal Reserve Bank of New York, remarked: «So what happens in a financial crisis? First, the probability distribution representing a creditor's assessment of the value of a financial firm shifts to the left as the financial environment deteriorates [...]. Second, and even more importantly, the dispersion of the probability distribution widens - lenders become more uncertain about the value of the firm. [...] A lack of transparency in the underlying assets will exacerbate this increase in dispersion. ("More Lessons from the Crisis", Nov. 13, 2009)», see `http://www.bis.org/review/r091117a.pdf`

bound provided by Chebyshev's one-tailed inequality,[10]

$$\mathbb{P}(z_{i,j,t} > \epsilon) \leq \frac{\sigma^2_{i,j,t}}{\sigma^2_{i,j,t} + \epsilon^2} = \frac{\sigma^2 + \tilde{\sigma}^2_{i,j,t}}{\sigma^2 + \tilde{\sigma}^2_{i,j,t} + \epsilon^2} = P_{i,j,t}.$$

Hence, both the true riskiness of a bank as well as the additional uncertainty resulting from the perception error increase the perceived probability of default on which lender banks base their credit risk assessment. Conditional on these bank-specific perceived probabilities of defaults banks bargain about the loan conditions.

3.3 Bargaining and Equilibrium Interest Rates

In the over-the-counter interbank market, banks bilaterally negotiate about the loan conditions. Without loss of generality, let bank i be the lender bank ($\zeta_{i,t} > 0$) in the bargaining process and bank j be the borrower bank ($\zeta_{j,t} < 0$). From the point of view of bank i, lending funds to bank j at time t is a risky investment with stochastic return

$$R_{i,j,t} = \begin{cases} r_{ijt} & \text{w.p. } 1 - P_{i,j,t} \\ -1 & \text{w.p. } P_{i,j,t}. \end{cases}$$

where we assume that loss given default is 100 percent. We further assume that bank i is risk neutral and maximizes expected lending profit conditional on the perceived probability of default $P_{i,j,t}$. The expected profit per euro at an equilibrium interest rate $r_{i,j,t}$ is then given by

$$\bar{R}_{i,j,t} = \mathbb{E}_t R_{i,j,t} = (1 - P_{i,j,t}) r_{i,j,t} - P_{i,j,t}.$$

For the borrower bank j, the borrowing cost per euro is simply given by the equilibrium interest rate $r_{i,j,t}$ when borrowing from lender bank i.

We follow the standard approach in search models and assume that banks negotiate interest rates bilaterally according to a generalized Nash bargaining (see Afonso and Lagos (2012) for a similar application to interbank markets). Written in terms of surplus relative to the outside option, the bilateral equilibrium interest rate then satisfies

$$r_{i,j,t} \in \arg\max_r \left((1 - P_{i,j,t})r - P_{i,j,t} - \underline{r}\right)^\theta \left(\overline{r} - r\right)^{1-\theta},$$

where $\underline{r}$ is the outside option for lenders (i.e., the interest rate for depositing funds at the central bank), $\overline{r}$ is the the outside option for borrowers (i.e., interest rate for borrowing from the central bank) with $\overline{r} \geq \underline{r}$. The parameter $\theta \in [0, 1]$ denotes the bargaining power of lender i relative to borrower j. As the exchange of funds is voluntary, the bilateral Nash bargaining problem is subject

[10]Instead of the Chebyshev bound, one can assume that the banks use a certain distribution to compute this probability. Then we just have to use the respective cumulative distribution function here.

to the participation constraints $r_{i,j,t} \leq \overline{r}$ and $\bar{R}_{i,j,t} \geq \underline{r}$, and hence the interest rate corridor sets upper and lower bounds for the interbank lending rates.[11]

Normalizing $\underline{r} = 0$ and denoting $\overline{r} = r$, the corresponding bilateral equilibrium interest rate between lender i and borrower j is given by

$$r_{i,j,t} = \theta r + (1-\theta)\frac{P_{i,j,t}}{1-P_{i,j,t}} \tag{2}$$

where the last term is a risk premium depending on the perceived probability of default, $P_{i,j,t}$. The minimum interest rate lender i is willing to accept is $r^{min}_{i,j,t} = \frac{P_{i,j,t}}{1-P_{i,j,t}}$ which is obtained from setting $\mathbb{E}_t R_{i,j,t}$ equal to the return of the outside option. Similarly, the borrower will not accept rates higher than $r^{max}_{i,j,t} = r_t$. Note that $\frac{\partial r_{i,j,t}}{\partial P_{i,j,t}} = (1-\theta)/(1-P_{i,j,t})^2 > 0$ and $\frac{\partial^2 r_{i,j,t}}{\partial P^2_{i,j,t}} = 2(1-\theta)/(1-P_{i,j,t})^3 > 0$. It is also easy to see that $\frac{\partial r_{i,j,t}}{\partial \theta} > 0$ for $r_{i,j,t} > r^{min}_{i,j,t}$ such that lenders with more market power are able to obtain higher rates from their borrowers. Further, $\frac{\partial r_{i,j,t}}{\partial r} = \theta > 0$ and hence the bilateral interest rate increases with the outside option for borrowing.

Using the definition of $P_{i,j,t}$ we can rewrite the equilibrium interest rate as a function of the default threshold, the true financial distress variance, and the variance of the perception error as

$$r_{i,j,t} = \theta r + (1-\theta)\frac{\sigma^2 + \tilde{\sigma}^2_{i,j,t}}{\epsilon^2}.$$

Taking the partial derivatives of this function gives $\frac{\partial r_{i,j,t}}{\partial \sigma} = \frac{(1-\theta)2\sigma}{\epsilon^2} > 0$ and $\frac{\partial^2 r_{i,j,t}}{\partial \sigma^2} = \frac{2(1-\theta)}{\epsilon^2} > 0$ and similarly $\frac{\partial r_{i,j,t}}{\partial \tilde{\sigma}_{i,j,t}} = \frac{(1-\theta)2\tilde{\sigma}_{i,j,t}}{\epsilon^2} > 0$ and $\frac{\partial^2 r_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} = \frac{2(1-\theta)}{\epsilon^2} > 0$. Thus the equilibrium interest rate increases with the uncertainty about counterparty risk. Furthermore, since the first and second derivatives with respect to the threshold parameter are given by $\frac{\partial r_{i,j,t}}{\partial \epsilon} = -2(1-\theta)\frac{\sigma^2+\tilde{\sigma}^2_{i,j,t}}{\epsilon^3} < 0$ and $\frac{\partial^2 r_{i,j,t}}{\partial \epsilon^2} = 6(1-\theta)\frac{\sigma^2+\tilde{\sigma}^2_{i,j,t}}{\epsilon^4} > 0$ it follow that, ceteris paribus, the interest rate decreases for a larger threshold when default becomes a less likely event.

The partial derivative of the expected return with respect to the perception error variance is $\frac{\partial \bar{R}_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} = -\frac{\partial P_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} + \frac{\partial 1-P_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} r_{i,j,t} + (1-P_{i,j,t})\frac{\partial r_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} = -\frac{\epsilon(1+r)\theta}{(\epsilon^2+\sigma^2+\tilde{\sigma}^2_{i,j,t})^2} < 0$. These terms show the channels through which increasing uncertainty about counterparty risk affects the expected return. First, it decreases $\bar{R}_{i,j,t}$ as $\frac{\partial P_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} > 0$ and hence loss given default becomes more likely. Second, it increases the risk premium that is obtained if the borrower survives. However, the net effect is negative and thus the expected return decreases for a larger perception error variance.

The preceding analysis reveals that the bilateral equilibrium interest rate under the asymmetric information problem, here parametrized by the perception error variance, is not Pareto efficient. Indeed, we can compute the interest rate and expected return for the perfect information case where

[11] In the model all banks have recourse to the outside options at any point in type. Thereby, we abstract for instance from collateral requirements to be eligible for recourse to the standing facilities.

$\tilde{\sigma}^2_{i,j,t} = 0$ (denoted by $r^{PI}_{i,j,t}$ and $\bar{R}^{PI}_{i,j,t}$) and compare it with the asymmetric information case

$$r_{i,j,t} - r^{PI}_{i,j,t} = \frac{(1-\theta)\tilde{\sigma}^2_{i,j,t}}{\epsilon^2} > 0 \quad \text{and} \quad \bar{R}^{PI}_{i,j,t} - \bar{R}_{i,j,t} = \frac{\epsilon^2(1+r)\theta\tilde{\sigma}^2_{i,j,t}}{(\epsilon^2+\sigma^2)(\epsilon^2+\sigma^2+\tilde{\sigma}^2_{i,j,t})} > 0$$

which gives the total reduction in (expected) surplus per euro of the loan due to the asymmetric information problem. This surplus loss depends positively on the perception error variance $\tilde{\sigma}^2_{i,j,t}$ which may be reduced by banks' peer monitoring efforts, as discussed in the next section.

3.4 Peer Monitoring, Counterparty Selection and Transaction Volumes

Banks can engage in costly peer monitoring targeted at mitigating asymmetric information problems about counterparty risk. Therefore, we introduce the $(N \times 1)$ vector $m_{i,t} = (m_{i,1,t}, ..., m_{i,N,t})'$ with each $m_{i,j,t} \in \mathbb{R}^+_0$ describing bank i's monitoring of bank j. In particular, $m_{i,j,t}$ denotes the expenditure that bank i incurres in period t in monitoring bank j. The *added information* that bank i acquires about bank j in period t is assumed to be a linear function of the monitoring expenditure in period t and the occurrence of transaction $l_{i,j,t}$ during the trading session t,

$$\phi_{i,j,t} = \phi(m_{i,j,t}, l_{i,j,t}) = \beta_\phi + \beta_{1,\phi} m_{i,j,t} + \beta_{2,\phi} l_{i,j,t}. \tag{3}$$

The added information affects the perception error variance in future periods (compare to Equation (1)). By allowing $\phi_{i,j,t}$ to be a function of both $l_{i,j,t}$ and $m_{i,j,t}$, we distinguish between (costly) active information acquisition, such as creditworthiness checks, and freely obtained information, such as trust building via repeated interaction.

Bilateral Nash bargaining between any banks i and j in the market occurs only if these two banks have established a contact. This is an important consequence of the over-the-counter structure of interbank markets (see Ashcraft and Duffie (2007) and Afonso and Lagos (2012)). Therefore, we introduce a binary variable $B_{i,j,t}$ that indicates if bank i and j are connected at time t and bargaining as described in the previous subsection is possible. Specifically, $B_{i,j,t}$ is a Bernoulli random variable with success probability $\lambda_{i,j,t}$ that can be influenced by the search efforts of bank j directed towards lender i

$$B_{i,j,t} \sim \text{Bernoulli}(\lambda_{i,j,t}) \text{ with } \lambda_{i,j,t} = \lambda(s_{j,i,t}). \tag{4}$$

The variable $s_{j,i,t} \in \mathbb{R}^+_0$ captures the search cost incurred by bank j (with a liquidity deficit) when approaching lender i. Hence, we assume loans are borrower initiated in the sense that banks with a liquidity deficit approach potential lender banks for bargaining. We collect all search efforts of bank j in the $(N \times 1)$ vector $s_{j,t} = (s_{1,i,t}, ..., s_{N,i,t})'$.

For the matching technology, we assume that for increasing search efforts two banks are more likely to establish a contact and engage in interest rate negotiations. Specifically, we model the mean of the Bernoulli $B_{i,j,t}$ by a logistic function

$$\lambda_{i,j,t} = \lambda(s_{j,i,t}) = \frac{1}{1 + \exp(-\beta_\lambda(s_{j,i,t} - \alpha_\lambda))}, \tag{5}$$

with $\beta_\lambda > 0$ and $\alpha_\lambda > 0$. Hence, if $\beta_\lambda \to \infty$ this function converges to a step function that corresponds to a deterministic link formation at fixed cost α_λ. Note that for $s_{j,i,t} = 0$ we still have $\lambda_{i,j,t} > 0$, so even with zero search expenditure there is still a positive contacting probability that allows bargaining. Our choice to use a logistic transformation of a linear function is motivated by its extensive use in the discrete choice literature as in Weisbuch et al. (2000).

Once a contact between two banks is established and bilateral Nash bargaining is successful, interbank lending takes place. This event is captured by the binary link variable that indicates an interbank loan of positive amount from lending bank i to borrowing bank j,

$$l_{i,j,t} = \begin{cases} 1 & \text{if } B_{i,j,t} = 1 \wedge r_{i,j,t} \leq r \\ 0 & \text{otherwise .} \end{cases} \tag{6}$$

Each granted interbank loan is characterized by a price-volume pair $(r_{i,j,t}, y_{i,j,t})$ that represents the loan characteristics. The bilateral interest rate $r_{i,j,t}$ is the outcome of the bargaining process (see Equation 2). The granted loan amount $y_{i,j,t}$ is given by the largest feasible amount that the banks can exchange given their current liquidity shocks

$$y_{i,j,t} = \min\{\zeta^j_{i,t}, -\zeta^i_{j,t}\}\mathbb{I}(\zeta^j_{i,t} \geq 0)\mathbb{I}(\zeta^i_{j,t} \leq 0), \tag{7}$$

where the superscripts i and j indicate that liquidity shocks are counterparty specific realizations of the liquidity shocks distributed as described in subsection 3.1. Thus within each period t for each bank $2(N-1)$ liquidity shocks are realized.[12]

Because the volumes of granted loans are exogenously determined, the interest rates are for sufficiently good risk assessment directly affected by only monitoring effort of bank i and matching only by search efforts of bank j. Thus we abstract from credit rationing on the intensive margin of volumes. The model hence focuses on explaining the extensive margin and the variation in interest rates. Note also that we do not address liquidity hoarding for precautionary reasons and solely concentrate on the role of counterparty risk uncertainty and peer monitoring.

[12] In a previous version of the paper we assumed similarly to Babus (2013) and Vuillemey and Breton (2014) that at each time instance each bank is paired with at most one counterparty (for example paring two banks randomly at each time instance) to avoid solving a complicated multilateral bargaining problem at each time. For a given observed data frequency (in our analysis daily) simulated data is then aggregated to allow for nodes with multiple links.

3.5 Profit Maximization and Optimal Dynamic Monitoring and Search

Each bank $i \in \{1, ..., N\}$ faces the dynamic problem of allocating resources to monitoring counterparties and choosing which bank to transact with to maximize expected discounted payoffs. Formally the infinite-horizon dynamic optimization problem of each bank is given by

$$\max_{\{m_{i,t}, s_{i,t}\}} \mathbb{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r^d}\right)^{s-t} \pi_{i,s}(m_{i,s}, s_{i,s}), \tag{8}$$

where $\pi_{i,t}$ denotes the interbank profit of bank i at time t, r^d denotes the discount rate, and the maximization is subject to the restrictions imposed by the structure laid down in Sections 3.1-3.4.

The only unspecified function in the objective function above is the profit function $\pi_{i,s}(m_{i,s}, s_{i,s})$. Naturally, period t expected interbank profits of bank i written in terms of surplus over the outside options are given by

$$\pi_{i,t} = \sum_{j=1}^{N} \underbrace{l_{i,j,t} \bar{R}_{i,j,t} y_{i,j,t}}_{\text{lending}} + \underbrace{l_{j,i,t}(r - r_{j,i,t}) y_{j,i,t}}_{\text{borrowing}} - m_{i,j,t} - s_{i,j,t},$$

which captures the total lending and borrowing surplus from all transactions net of search and monitoring expenditures. Note that the intertemporal optimization problem is made operational by conditioning on the bilateral equilibrium interest rates $r_{i,j,t}$ characterized in Section 3.3. Hence, in this section these interest rates appear as a restriction on the optimization instead of an argument of the objective function.

Unfortunately, the variable $l_{i,j,t} = B_{i,j,t} \cdot \mathbb{I}(r_{i,j,t} \leq r)$ introduces a discontinuity in the objective function that prevents us from obtaining analytic optimality conditions.[13] We therefore consider an approximate smooth problem where we replace the step functions of the original problem ($\mathbb{I}(r_{i,j,t} \leq r)$) by a continuously differentiable logistic function $I(r_{i,j,t}) = \frac{1}{1+\exp(-\beta_I(r - r_{i,j,t}))} =: I_{i,j,t}$.[14] Without change in notation, we redefine $l_{i,j,t} = B_{i,j,t} I_{i,j,t}$. This allows us to use the calculus of variations that is well understood and the most widely applied method to solve constrained dynamic stochastic optimization problems in structural economics; see e.g. Judd (1998) and DeJong and Dave (2006).

Substituting out all definitions except for the law of motion for $\tilde{\sigma}^2_{i,j,t}$, we can write the Lagrange

[13] Although numerical solutions are in theory possible, these would make simulation and estimation prohibitively time-consuming given the high dimensional problem.

[14] Note that for a growing scale parameter, the logistic transformation approximates the step function arbitrarily well.

function with multiplier $\mu_{i,j,t}$ given by

$$\mathcal{L} = \mathbb{E}_t \sum_{s=t}^{\infty} \Big(\frac{1}{1+r^d}\Big)^{s-t} \sum_{j=1}^{N} \pi_{i,j,t}(m_{i,j,t}, s_{j,i,t}, \tilde{\sigma}^2_{i,j,t}) + \mu_{i,j,t}(\xi(m_{i,j,t}, \tilde{\sigma}^2_{i,j,t}) - \tilde{\sigma}^2_{i,j,t+1}),$$

where we made the arguments that can be influenced by bank i's decision explicit. The Euler equations that establish the first-order-conditions to the infinite-horizon nonlinear dynamic stochastic optimization problem can then be obtained by optimizing the Lagrange function w.r.t. to the control variables and the dynamic constraints (see e.g. Heer and Maußner (2005)).

Under usual regularity conditions the integration and differentiation steps can be interchanged and we obtain

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial m_{i,j,t}} = 0 &\Leftrightarrow \mathbb{E}_t\Big[\frac{\partial \pi_{i,j,t}}{\partial m_{i,j,t}} + \mu_{i,j,t}\frac{\partial \xi_{i,j,t}}{\partial m_{i,j,t}}\Big] = 0 \\
\frac{\partial \mathcal{L}}{\partial \tilde{\sigma}^2_{i,j,t+1}} = 0 &\Leftrightarrow \mathbb{E}_t\Big[- \mu_{i,j,t} + \frac{1}{1+r^d}\left(\frac{\partial \pi_{i,j,t+1}}{\partial \tilde{\sigma}^2_{i,j,t+1}} + \mu_{i,j,t+1}\frac{\partial \xi_{i,j,t+1}}{\partial \tilde{\sigma}^2_{i,j,t+1}}\right)\Big] = 0 \\
\frac{\partial \mathcal{L}}{\partial s_{i,j,t}} = 0 &\Leftrightarrow \mathbb{E}_t\Big[\frac{\partial \pi_{i,j,t}}{\partial s_{i,j,t}}\Big] = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu_{i,j,t}} = 0 &\Leftrightarrow \mathbb{E}_t\Big[\tilde{\sigma}^2_{i,j,t+1} - \xi(\phi_{i,j,t}, \tilde{\sigma}^2_{i,j,t})\Big] = 0
\end{aligned}$$

for all counterparties $j \neq i$ and all t. Substituting out the Lagrange multipliers and taking fixed values at time t out of the expectation gives the Euler equation for the optimal monitoring path that equates marginal cost and discounted expected future marginal benefits of monitoring,

$$\frac{1}{1+r^d}\frac{\partial \xi_{i,j,t}}{\partial m_{i,j,t}}\mathbb{E}_t\left(\frac{\frac{\partial \xi_{i,j,t+1}}{\partial \tilde{\sigma}^2_{i,j,t+1}}}{\frac{\partial \xi_{i,j,t+1}}{\partial m_{i,j,t+1}}} + \frac{\partial \pi_{i,j,t+1}}{\partial \tilde{\sigma}^2_{i,j,t+1}}\right) = 1. \tag{9}$$

Unlike monitoring expenditures, search becomes effective in the same period it is exerted and does not directly alter future matching probabilities via a dynamic constraint. Thus the first-order condition for the optimal search path is given by

$$\frac{\partial}{\partial s_{i,j,t}}\mathbb{E}_t\Big[(r - r_{j,i,t})y_{j,i,t}l_{j,i,t}\Big] = 1 \tag{10}$$

leading to the usual condition that expected marginal benefit equals marginal cost in each period without any discounting. Note that because the first-order conditions hold for all $j \neq i$ and the marginal cost of monitoring and search is the same across all j, the conditions also imply that (discounted) expected marginal profits of monitoring and search must be the same across different banks j.

The transversality condition for the dynamic problem is obtained as the limit to the endpoint condition from the corresponding finite horizon problem and requires that

$$\lim_{T\to\infty} \mathbb{E}_t \left[\left(\frac{1}{1+r^d}\right)^{T-2} \frac{\partial \pi_{i,j,T-1}}{\partial m_{i,j,T-1}} - \left(\frac{1}{1+r^d}\right)^{T-1} \frac{\partial \pi_{i,j,T}}{\partial \tilde{\sigma}^2_{i,j,T}} \frac{\partial \xi_{i,j,T-1}}{\partial m_{i,j,T-1}} \right] = 0.$$

Thus in the limit the expected marginal cost of investing in monitoring must be equal to the expected marginal return.

Following the the literature on stochastic dynamic modeling (see e.g. DeJong and Dave (2006) and Ruge-Murcia (2007)), we simulate from the structural model by first deriving the optimal policy rules, linearizing them and then adopting an expectation generating mechanism that delivers the reduced form of the model.

3.5.1 Optimal Monitoring and Search

We locally approximate the analytically intractable, non-linear Euler equation for monitoring in Equation (9) by a linear function. We therefore linearize Equation (9) by a first-order Taylor expansion around a stable steady state, as discussed in detail in Appendix A. We then obtain the optimal bank-to-bank monitoring choice as the affine function

$$m_{i,j,t} = a_m + b_m \tilde{\sigma}^2_{i,j,t} + c_m \mathbb{E}_t \tilde{\sigma}^2_{i,j,t+1} + d_m \mathbb{E}_t B_{i,j,t+1} + e_m \mathbb{E}_t y_{i,j,t+1}, \tag{11}$$

where the intercept and coefficients are functions of the structural parameters, cf. Appendix A. The policy rule shows that the optimal monitoring expenditures of bank i towards bank j depends on the current state of bank-to-ban credit risk uncertainty, the expected future uncertainty, the expected volume of the loan and on the expected probability of being contacted by bank j.[15] Note that because all first order conditions are linearized at the same point, the parameters of the policy function are the same for all banks and all counterparties.

We obtain an analytical solution for the optimal bank-to-bank search level. The solution depends on the borrower's expected surplus $\Delta_{i,j,t} := \mathbb{E}_t[y_{j,i,t}(r - r_{j,i,t}) I_{j,i,t}]$ from borrowing from bank i

$$s_{i,j,t} = \begin{cases} s(\Delta_{i,j,t}) & \text{for } \Delta_{i,j,t}\lambda(s(\Delta_{i,j,t})) - s(\Delta_{i,j,t}) \geq 0 \\ 0 & \text{for } \Delta_{i,j,t}\lambda(s(\Delta_{i,j,t})) - s(\Delta_{i,j,t}) < 0, \end{cases} \tag{12}$$

where the interior solution with positive search levels is obtained from the analytical solution to the first order-condition in Equation (10) as

$$s(\Delta_{i,j,t}) := 1/\beta_\lambda \log \left(0.5(\sqrt{\Delta_{i,j,t}\beta_\lambda(\Delta_{i,j,t}\beta_\lambda - 4)} + \Delta_{i,j,t}\beta_\lambda - 2) e^{\alpha_\lambda \beta_\lambda}) \right) \tag{13}$$

[15] Note that in our model we focus on bilateral interbank lending and the recourse to central bank facilities. Hence the optimal monitoring decisions are based on interbank lending only, and do not reflect any other bilateral exposure between banks.

for $\Delta_{i,j,t}\beta_\lambda(\Delta_{i,j,t}\beta_\lambda - 4) \geq 0$. The optimal search strategy hence shows that for positive expected return net of search cost the solutions satisfies Equation (13), with $s(\cdot)' \geq 0$. Thus search efforts increases in the expected surplus. Note that $\lambda(0) > 0$ and hence even without search efforts two banks will eventually get connected to each other and bargain about loan outcomes.

It is important to highlight that lender i's monitoring level with respect to borrower j depends on the expectation of being contacted for transaction by the latter. Similarly borrower j's search level with respect to lender i depends on the expected surplus she can obtain from borrowing from i. This interrelation between monitoring and counterparty selection, tied together by banks expectations about profitability, generates an amplification mechanism that is at the core of our model and explains several empirical regularities of interbank lending market.

3.5.2 Adaptive Expectations

The optimal monitoring and search levels in Equation (11) and (12) depend on expectations about bilateral credit availability and conditions. In an opaque over-the-counter market such as the interbank lending market, information about credit availability and conditions of other banks is not publicly available. We assume that banks form bank-specific *adaptive expectations* about credit conditions at other banks in the market. Following Chow (1989) and Chow (2011), the adaptive expectation of bank i concerning variable $x_{i,j,t}$, denoted by $x^*_{i,j,t} := \mathbb{E}_t x_{i,j,t+1}$, follows an exponentially weighted moving average (EWMA)

$$x^*_{i,j,t} = (1 - \lambda_x)x^*_{i,j,t-1} + \lambda_x x_{i,j,t}, \tag{14}$$

where all variables are in deviation from the mean steady-state values. Banks use this forecasting rule for variables that are always observed by bank i ($\tilde{\sigma}_{i,j,t+1}$ and $B_{i,j,t+1}$). The parameter $\lambda_x \in (0, 1)$ determines the weight of the new observations at time t relative to the previous expectation.

However, a crucial implication of the over-the-counter structure of the market is that a bank learns about credit condition (that is volumes $y_{i,j,t+1}$ and rates $r_{i,j,t+1}$) at *other* banks only once upon contact. We incorporate this feature of decentralized interbank markets by assuming that bank i uses the following forecasting rule,

$$x^*_{i,j,t} = (1 - \lambda_x)x^*_{i,j,t-1} + \lambda_x B_{i,j,t} x_{i,j,t}. \tag{15}$$

Recall that $B_{i,j,t}(s_{j,i,t-1}) = 1$ denotes an 'open' connection. Hence new information about a counterparty is added to the last forecast only if the banks had an established contact; otherwise the last forecast is not maintained but discounted by a factor $(1 - \lambda_x)$. Thus if bank i and j are not in contact for a many periods their expectations converge to the mean steady-state values. .

The adoption of adaptive expectations is justified in the first place by the fact that adaptive expectations are much easier to handle, see Mlambo (2012). Indeed, it is impossible to use the deterministic steady-state of the model as an approximation point for perturbation methods (see Appendix A). This renders rational expectation solution method computationally impractical. On the contrary, since adaptive expectations are solely dependent on past observations, the numerical nature of the equilibrium point does not present extra difficulties.

Second, in many settings, there exists a collection of very strong econometric evidence supporting the adaptive expectations hypothesis against the rational expectations hypothesis; see e.g. Chow (1989) and Chow (2011). More recently, Evans and Honkapohja (1993) and Evans and Honkapohja (2001) showed that adaptive expectations are not only reasonable, but in many ways the most rational forecast method when the true process is unknown. This seems argument seems specifically relevant for modeling decisions in a highly complex system such as an over-the-counter trading network.

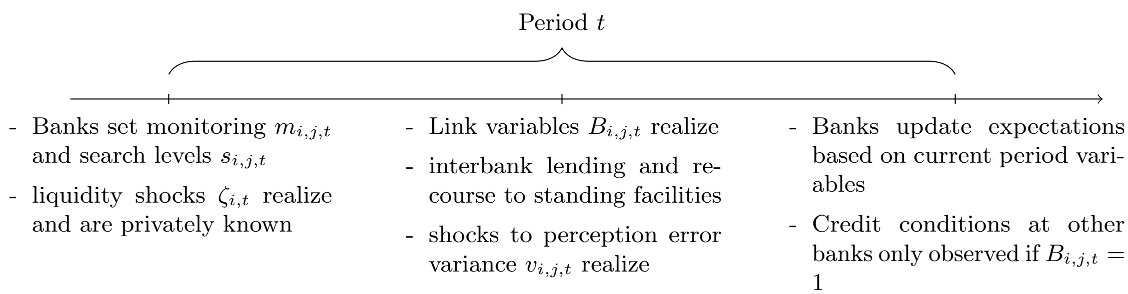


Figure 1: Time line illustrating the sequence of events in period t.

The formulation of the expectation mechanism completes the model description. Figure 1 summarizes the sequence of events within one period. From the structural model we obtain a reduced form representation that allows simulating from the parametric model under some fixed parameter vector; details on the reduced form representation and stability conditions are provided in Appendix B.

4 Parameter Estimation

We now turn to the estimation of the structural model parameters using loan level data from the Dutch overnight interbank lending market. Due to the complexity of the model (non-linearity and non-standard distributions) the likelihood function is not known even up to a constant, rendering maximum likelihood estimation or Bayesian methods impossible. We therefore resort to the simulation-based method of indirect inference that builds on an appropriate set of auxiliary statistics to estimate the structural model parameters.

4.1 Auxiliary Statistics and Indirect Inference

Following the principle of indirect inference introduced in Gourieroux et al. (1993), we estimate the vector of parameters $\boldsymbol{\theta}_T$ by minimizing the quadratic distance between the auxiliary statistics $\hat{\boldsymbol{\beta}}_T$ obtained from the observed data $X_1, ..., X_T$, and the average of the auxiliary statistics $\tilde{\boldsymbol{\beta}}_{TS}(\theta) := (1/S)\sum_{s=1}^{S} \tilde{\boldsymbol{\beta}}_{T,s}(\theta)$ obtained from S simulated data sets $\{\tilde{X}_{1,s}(\boldsymbol{\theta}), ..., \tilde{X}_{T,s}(\boldsymbol{\theta})\}_{s=1}^{S}$ generated under $\theta \in \Theta$. Formally the indirect inference estimator is thus given as

$$\hat{\boldsymbol{\theta}}_T := \arg\max_{\boldsymbol{\theta}\in\Theta} \Big[\hat{\boldsymbol{\beta}}_T - \frac{1}{S}\sum_{s=1}^{S}\tilde{\boldsymbol{\beta}}_{T,s}(\boldsymbol{\theta})\Big]' \mathbf{W}_T \Big[\hat{\boldsymbol{\beta}}_T - \frac{1}{S}\sum_{s=1}^{S}\tilde{\boldsymbol{\beta}}_{T,s}(\boldsymbol{\theta})\Big],$$

where Θ denotes the parameter space of $\boldsymbol{\theta}$ and $\mathbf{W}_T$ is a weighting matrix. Under appropriate regularity conditions this estimator is consistent and asymptotically normal. In particular, consistency holds as long as, for given $S \in \mathbb{N}$, the auxiliary statistics converge in probability to singleton limits $\hat{\boldsymbol{\beta}}_T \xrightarrow{p} \boldsymbol{\beta}(\boldsymbol{\theta}_0)$ and $\tilde{\boldsymbol{\beta}}_{T,s} \xrightarrow{p} \boldsymbol{\beta}(\boldsymbol{\theta})$ as $T \to \infty$ and the so-called *binding function* $\boldsymbol{\beta} : \Theta \to \mathcal{B}$ is injective. Convergence in probability is precisely ensured through the application of a law of large numbers for strictly stationary and ergodic data (see e.g. White (2001)). Similarly, asymptotic normality of the estimator is obtained if the auxiliary statistics $\hat{\boldsymbol{\beta}}_T$ and $\tilde{\boldsymbol{\beta}}_{T,s}$ are asymptotically normal, see Gourieroux et al. (1993) for details. By application of a central limit theorem (see e.g. White (2001)), the asymptotic normality of the auxiliary statistics can again be obtained by appealing to the strict stationarity and ergodicity of both observed and simulated data.

The injective nature of the binding function is the fundamental identification condition which ensures that the structural parameters are appropriately described by the auxiliary statistics. This condition cannot be verified algebraically since the binding function is analytically intractable. However, identification will be ensured as long as the set of auxiliary statistics adequately describes both observed and simulated data. We hence select auxiliary statistics that provide a rich characterization of the interbank market represented by the network of bilateral loans and associated volumes and interest rates. Specifically, we explore the auto-covariance structure, higher-order moments, and a number of network statistics as auxiliary statistics. The use of means, variances and auto-covariances is in line with the literature concerned with the estimation of structural models such as Dynamic Stochastic General Equilibrium models (see e.g. DeJong and Dave (2006) and Ruge-Murcia (2007)). The use of higher-order moments such as measures of skewness and kurtosis is justified by the nonlinearity of the model and the fact that some structural parameters might be well identified by such statistics.

In addition to these standard auxiliary statistics, we base the indirect inference estimator of the network model on auxiliary statistics that characterize specifically the topological structure of the interbank lending network. In particular, our model focuses on explaining relationship lending and the sparse core-periphery structure and we hence include statistics that measure these characteristics. Thereby, we follow the large empirical literature on the structure of interbank lending networks and use key network statistics that are common in empirical analysis (see e.g. Bech and Atalay (2010)).

Moreover, we don't include computationally expensive network statistics due to the large number of simulated networks in the estimation procedure.

First, we consider global network statistics. In particular, the *density* defined as the ratio of the actual to the potential number of links is a standard measure of the connectivity of a network. A low density characterizes a sparse network with few links. The *reciprocity* measures the fraction of reciprocal links in a directed network. For the interbank market this relates to the degree of mutual lending between banks. The *stability* of a sequence of networks refers to the fraction of links that do not change between two adjacent periods. Note that all three statistics are bounded between zero and one.

Second, we include bank (node) level network statistics. The (unweighted) *in-degree* of a bank is defined as the number of lenders it is borrowing funds from, the (unweighted) *out-degree* as the number of borrowers it is lending funds to. We summarize this bank level information using the mean and standard deviation of the (in-/out-) degree distribution as well as its skewness. The (local) *clustering coefficient* of a node quantifies how close its neighbors are to being a clique (complete graph). In the interbank network it measure how many of a bank's counterparties have mutual credit exposures to each other. We compute the clustering coefficients for directed networks as proposed by Fagiolo (2007) and consider the average clustering as auxiliary statistic.

Third, we focus on simple bilateral network statistics that measure the intensity of a bilateral trading relationship based on past lending activity during a rolling window. Similar to Furfine (1999) and Cocco et al. (2009), we compute the number of loans given from bank i to bank j during the last week and denote this variable by $l^{rw}_{i,j,t}$. We then compute a cross-sectional correlation between these relationship variables and loan outcomes at time t (decision to grant a loan and interest rate). The first variable $\text{Corr}(l_{i,j,t}, l^{rw}_{i,j,t-1})$ is hence a measure of the bilateral stability of lending relations; $\text{Corr}(r_{i,j,t}, l^{rw}_{i,j,t-1})$ proxies for effects of relationship lending on interest rates.

We compute all described network statistics for each lending network within the sequence of networks such that we obtain a sequence of network statistics associated with the sequence of networks. We then obtain the unconditional means, variance and/or auto-correlation of these sequences as auxiliary statistics and base the parameter estimations on the values of the auxiliary statistics only. In Appendix C we provide the formulae of the described network statistics. For further details on network statistics in economics, see Jackson (2008).

Our estimator is based on a quadratic objective function with a diagonal weighting matrix $\mathbf{W}_T$. We refrain from using an asymptotically efficient weighing matrix. First, the inverse of the covariance matrix is only optimal under an axiom of correct specification. In addition, even under correct specification the (asymptotically) optimal weighing matrix can lead to larger variance of the estimator in finite samples. Second, for economic theoretic reasons, there are a number of auxiliary statistics that we wish to approximate better than others. As such we adopt a matrix $\mathbf{W}_T$ corresponding to a an identity matrix, but the weight of the density and $\text{Corr}(l_{i,j,t}, l^{rw}_{i,j,t-1})$ is set to 10 and the weight of $\text{Corr}(r_{i,j,t}, l^{rw}_{i,j,t-1})$ is set to 50 as we want to match these characteristics particularly well.

In all estimations we use $S = 24$ simulated network paths each of length $T^* = 3000$ with 1000 periods burned to minimize dependence on initial values. Note that in the estimation some of the structural parameters are calibrated as these are not identified by the data. For example, it is clear that several combinations of β_σ, $\beta_{1,\phi}$ and $\beta_{2,\phi}$ imply the same distribution for the data, and hence, also for the auxiliary statistics. The same applies for ϵ and σ. Further, we fix the common default threshold ϵ and the common true variance of the financial distress σ^2 to obtain a upper bound on the true probability of default of 0.01. We calibrate the corridor width to the average observed value during our sample of 1.5% and set the discount rate to 1.75%. The scaling parameter of the logistic function that approximates the step function is set to 200.

4.2 Data Description

The original raw data that we use in the estimation procedure comprises the daily bilateral lending volumes and interest-rates practiced in the over-night unsecured lending market between all Dutch banks. In particular, our empirical analysis is based on a confidential transaction level dataset of interbank loans that has been compiled by central bank authorities based on payment records in the European large value payment system TARGET2. This panel of interbank loans has been inferred using a modified and improved version of the algorithm proposed by Furfine (1999) for the US fedwire system. The idea of Furfine-type algorithms is to match payment legs between pairs of banks and classifies those as interbank loans depending on the size of the initial payment and size and date of candidate repayments; for details on the data set and methodology see Heijmans et al. (2011) and Arciero et al. (2013).

Compared to interbank lending data derived from the US fedwire data and payment systems of other countries, our dataset has three major advantages. First, TARGET2 payments have a flag for transactions related to interbank credit payments, which restricts the universe of all payments to be searched by the algorithm. Second, information about the actual sender and receiver bank is available. Unlike settlement banks, sender and receiver banks are the ultimate economic agents involved in the contract. In particular the sender bank is exposed to inherent counterparty credit risk that is at the core of our model. Third, and most importantly, euro area interbank lending data derived from Furfine-type algorithms has been cross-validated with official Spanish and Italian interbank transaction level data yielding type I errors of less than 1%. That is less than 1% of all payments are incorrectly paired and classified as interbank loans (see Arciero et al. (2013) and de Frutos et al. (2014)).

Our interbank loan level dataset contains observations on daily bilateral volumes ($y_{i,j,t}$) and interest spreads ($r_{i,j,t}$) for the period between 18 February 2008 until 28 April 2011 ($T = 810$ trading days). From this data we construct the loan indicator $l_{i,j,t}$ that equals one if a loan from lender i to borrower j at day t is observed, and zero otherwise. For computational reasons we focus on overnight interbank lending between the $N = 50$ largest banks according to their overnight trading frequency (as both borrower and lender) throughout the entire sample. As a result, the data from which the auxiliary statistics are obtained consists of three $N \times N \times T$ ($50 \times 50 \times 810$) arrays with elements $l_{i,j,t}$, $y_{i,j,t}$ and $r_{i,j,t}$. The arrays for $y_{i,j,t}$ and $r_{i,j,t}$ contain missing values if and only if $l_{i,j,t} = 0$.[16]

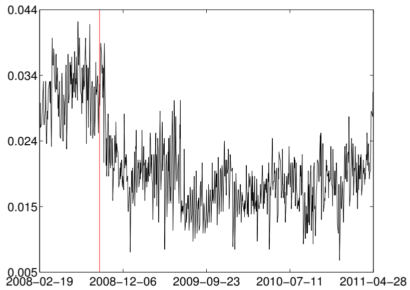
Table 1: Descriptive statistics. The table reports moment statistics for different sequences of network statistics and cross-sectional correlations that characterize the sequence of observed Dutch unsecured interbank lending networks. The statistics are computed on a sample of daily frequency from 18 February 2008 2008 to 28 April 2011.

Statistic	Mean	Std	Autocorr
Density	0.0212	0.0068	0.8174
Reciprocity	0.0819	0.0495	0.2573
Stability	0.9818	0.0065	0.8309
Mean out-/indegree	1.0380	0.3323	0.8174
Mean clustering	0.0308	0.0225	0.4149
Corr($r_{i,j,t}, l^{rw}_{i,j,t-1}$)	-0.0716	0.1573	0.4066
Corr($l_{i,j,t}, l^{rw}_{i,j,t-1}$)	0.6439	0.0755	0.4287
Mean log-volume	4.1173	0.2818	0.4926
Mean rate	0.2860	0.3741	0.9655

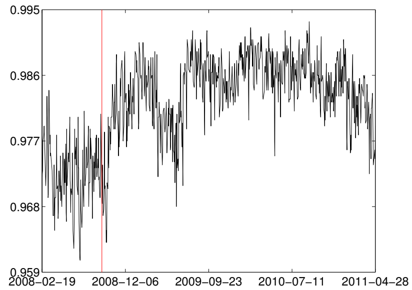
Table 1 shows key summary statistics of the data used in the analysis, more detailed summary statistics are provided in Appendix D. Note that: (i) the moments of bilateral volumes of granted loans are for values stated in (logarithm of) EUR millions; (ii) the moments of bilateral interest rates of granted loans are reported in percentage points per annum above the ECB deposit facility rate (lower bound of the interest rate corridor); (iii) the daily interbank network is very sparse with mean density 0.02 (on average 1.04 lenders and borrowers) and low clustering; (iv) the distribution of interest rates, volumes, degree centrality and clustering are highly skewed. It is also important to highlight here that the high auto-correlation of the density, the high stability of the network as well as the positive expected correlation between current period lending and past lending activity can be seen as evidence of 'trust relations' between banks, and shows that past trades affect future trading opportunities. Similarly, the negative expected correlation between past lending activity and present interest rates provides evidence of lower perceived risk that may be result from monitoring efforts as postulated by the proposed structural model.

Figure 2 graphs the time evolution of the daily network density, stability, average (log) volume, total volume, and mean and standard deviation of the daily spreads throughout the sample. From the plots we see that the network density and total trading volume declined after Lehman's failure on 15 September 2008, which is indicated by the vertical red line. In economic terms, the total trading volume declines from about EUR 20 billions to EUR 10 billions. At the same time the network stability and the daily cross-sectional standard deviation of interest rate spreads increases more than threefold. Moreover, the mean spreads of granted loans are close to the deposit facility as of October

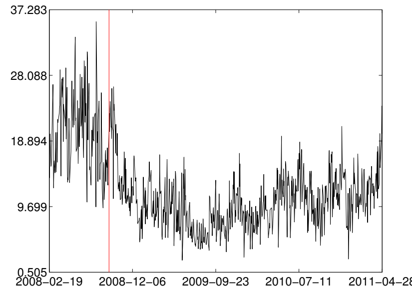
[16]The dataset contains only loan of at least 1 million euros volume as typically banks with shocks below that value do not go to the market. Therefore, Equation (7) for the volumes of granted loans changes accordingly.



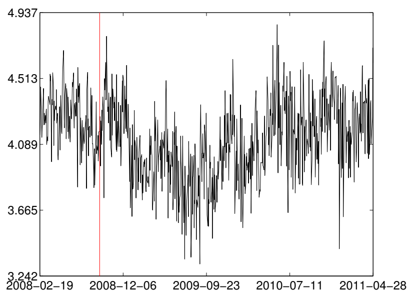
(a) Density



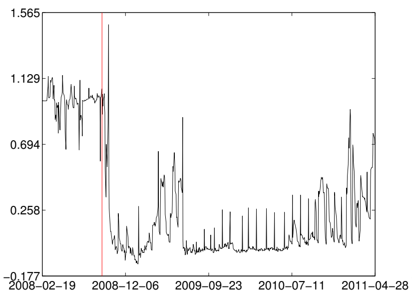
(b) Stability



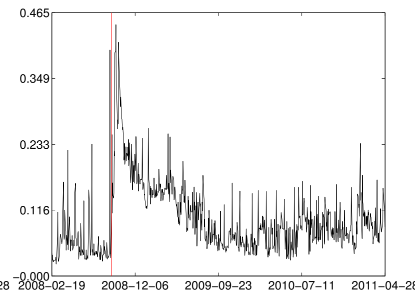
(c) Total volume



(d) Mean (log) volume



(e) Mean spread



(f) Stdev spread

Figure 2: Time series plots of daily network density, stability, total traded volume (in EUR billions), and mean volume (in log EUR millions), mean spread (to deposit rate), and standard deviation of granted loans from 18 February 2008 to 28 April 2011. Vertical red line corresponds to Lehman's failure on 15 September 2008.

2008. Further, the plots reveal that the data exhibits well documented end-of-maintenance period effects that we clean out in the construction of the auxiliary statistics by regressing each sequence of network statistics on end-of-maintenance period dummies before computing auxiliary statistics.

4.3 Estimation Results

We next turn to the estimation results of the interbank network model. Table 2 shows the point estimates, standard errors and 90% confidence intervals of the structural parameters using the auxiliary statistics reported in Table 4. Naturally, standard errors and confidence intervals are not provided for calibrated parameters. For comparison with the indirect inference estimates, we also present an alternative, calibrated parameter vector ($\boldsymbol{\theta_a}$) which equals $\hat{\boldsymbol{\theta}}_T$ but restricts the effects of monitoring to zero ($\beta_{\phi,1} = 0$).

The parameter estimates reported in Table 2 are interesting in several respects. First, the relatively large and significant intercept of the autoregressive log-variance process that can be seen as evidence for high levels of prevailing bank-to-bank uncertainty. Also the autoregressive parameter γ_σ is estimated to be 0.66 indicating some autocorrelation in bilateral credit risk uncertainty in the absence of new information. The estimate of the scaling δ_σ is positive and significant, indicating that shocks to credit risk uncertainty are important drivers of bank-to-bank uncertainty. Moreover, the estimated coefficient $\beta_{\phi,1}$ that determines the effect of peer monitoring on the added information is positive and statistically significant. Hence, we find evidence that monitoring is a significant factor in the reduction of prevailing bank-to-bank counterparty risk uncertainty. On the other hand, the estimated coefficient that determines the effect of the occurrence of a trade is close to zero and statistically insignificant. This suggests that the credit risk uncertainty is not merely mitigated by repeated transaction but depends on monitoring efforts.

Table 2: Estimated parameter values. The table reports the estimated structural parameters ($\hat{\boldsymbol{\theta}}_T$) and corresponding standard errors and 90% confidence bounds. The parameter $\boldsymbol{\theta}_a$ represents an calibrated model parametrization without monitoring ($\beta_{\phi,1} = 0$). For calibrated parameters no standard errors and confidence bounds are reported. The indirect inference estimator is based on $S = 24$ simulated network sequences of length $T^* = 3000$. Note also that $\sigma^*_\mu = \log(\sigma_\mu)$.

Structural parameters		Calibrated $\boldsymbol{\theta}_a$	Estimated $\hat{\boldsymbol{\theta}}_T$	St.Errors ste($\hat{\boldsymbol{\theta}}_T$)	90% Bounds LB	UB
Added information	α_ϕ	-1.5000	-1.5000	-	-	-
	$\beta_{\phi,1}$	0.0000	9.6631	0.0006	9.6619	9.6643
	$\beta_{\phi,2}$	0.0001	0.0001	0.0445	-0.0871	0.0873
Perception error variance	α_σ	1.2890	1.2890	0.0028	1.2835	1.2945
	β_σ	2.0000	2.0000	-	-	-
	γ_σ	0.6648	0.6648	0.0183	0.6289	0.7007
	δ_σ	0.3383	0.3383	0.0451	0.2499	0.4267
Search technology	α_λ	0.0001	0.0001	0.1159	-0.2271	0.2273
	β_λ	72.8331	72.8331	0.0006	72.8319	72.8343
Liquidity shocks	μ_μ	0.0000	0.0000	-	-	-
	σ^*_μ	1.9903	1.9903	0.0228	1.9456	2.0350
	μ_σ	1.9492	1.9492	0.0218	1.9065	1.9919
	σ_σ	1.9810	1.9810	0.0213	1.9393	2.0227
	ρ_ζ	-0.7826	-0.7826	0.0423	-0.6997	-0.8655
Expectations	λ^y	0.8472	0.8472	0.0443	0.7604	0.9340
	λ^B	0.9278	0.9278	0.0470	0.8357	1.0199
	λ^r	0.4008	0.4008	0.0466	0.3095	0.4921
	$\lambda^{\tilde{\sigma}}$	0.0318	0.0318	0.0414	-0.0493	0.1129
Bargaining lender	θ	0.6897	0.6896	0.0441	0.6032	0.7760
CB corridor width	r	1.5000	1.5000	-	-	-
Default threshold	ϵ	3.0000	3.0000	-	-	-
Financial distress	σ	0.1000	0.1000	-	-	-
Discount rate	r^d	1.7500	1.7500	-	-	-
Scale logistic	β_I	200.00	200.00	-	-	-

Second, the positive estimates for α_λ and β_λ shows that counterparty search is a crucial feature in the formation of interbank networks. In particular, the large and significant estimate for β_λ is 73 and suggests that links are not formed at random but are strongly influenced by banks' search towards preferred counterparties. With such large scaling the logistic function mimics a step function quite well. The significant role of endogenous counterparty selection also highlights the effect of expected profitability (expected volumes and interest rates) on the search decisions . In this respect, the positive point estimate of 0.85 for λ^y indicates quite persistent expectation about available bilateral loan volumes. Similarly, the estimated value of 0.93 for λ^B indicate a strong persistence in the expectations of being contacted by a specific borrower. These persistent expectation eventually contribute to the high persistence of bilateral trading relationship. The estimated value for λ^r is considerably lower (0.40) and suggests that new information about bilateral interest rates gets much more weight in the expectation formation process, as compared to expectations about volumes and contacts which are relatively more persistent. On the other hand, the changes in bank-to-bank credit risk uncertainty immediately feed into the expectations as the small estimate for $\lambda^{\tilde{\sigma}}$ indicates (value of 0.03).

Third, the estimated values of the hyper-parameters of the distribution that parameterizes banks' individual liquidity shock distributions point towards significant heterogeneity. The estimated log normal distribution implies that there are a few banks with very large liquidity shock variances that are very active players in the market. Moreover, the notion that some banks are structural liquidity providers or suppliers is supported by the positive estimate of the variance parameter of the mean. Note also the estimated negative correlation parameter, indicating that banks with a small liquidity shock variance typically have a positive mean. We discuss the role of bank heterogeneity in more detail in Section 5.

Table 3: Coefficients of linear policy rule for optimal monitoring

Variable	$\tilde{\sigma}_{i,j,t}$	$\mathbb{E}_t\tilde{\sigma}_{i,j,t+1}$	$\mathbb{E}_t B_{i,j,t+1}$	$\mathbb{E}_t y_{i,j,t+1}$
Coefficient	0.0024	-0.0043	0.0348	0.0019

In Table 3, we report the coefficients of the linear policy rule for the optimal monitoring levels as implied by the estimated parameters (monitoring expressed in deviations from steady-state values). It is particular noteworthy that the optimal monitoring level towards a particular bank depends positively on the expected probability of being approached by this bank to borrow funds during future trading sessions, $\mathbb{E}_t B_{i,j,t+1}$. Indeed, this positive coefficient and the significantly positive effect of search on link formation (endogenous counterparty selection) creates the interrelation between monitoring and search as the source of persistent trading relationships. Moreover, note that the current state of credit risk uncertainty positively affects monitoring during the current period period. Higher expected future uncertainty, however, reduces these efforts as expected profitability of interbank lending decline. The positive coefficient of the amount of granted loans shows that banks prefer to monitor those counterparties where they expect to trade larger volumes since the surplus that can be generated by reducing credit risk uncertainty is larger. Hence, monitoring reacts positively to increased expected profitability in the future, similar to banks optimal search decision.

The estimated policy rules for monitoring and search imply that shocks to interbank trading profitability lead to an endogenous multiplier effect. Figure 3 shows this basic amplification mechanism. As a response to a larger loan volume in $t = 5$ (liquidity shocks scaled by factor 5),

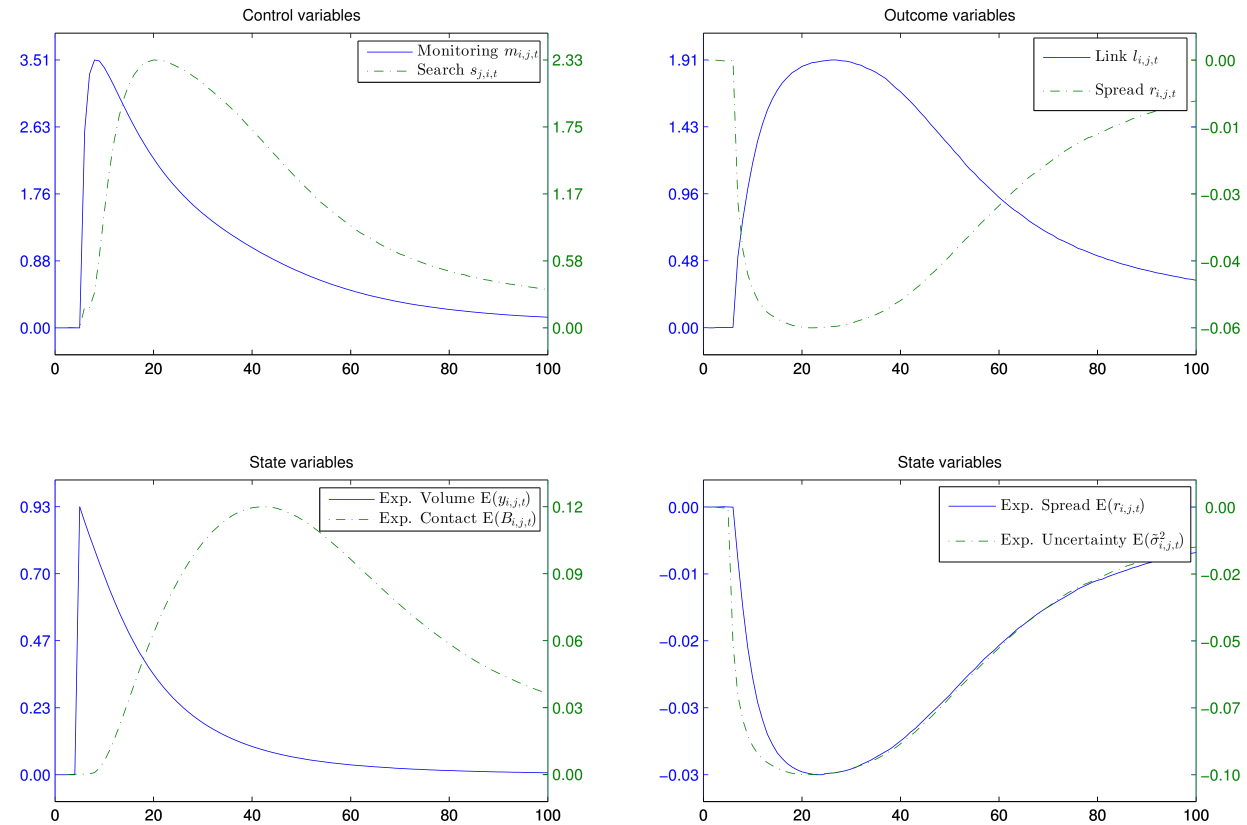


Figure 3: Amplification mechanism of peer monitoring and search. Mean responses to positive shock to bilateral volume in $t = 5$ as implied by the estimated model (scaling of factor 5). Series are expressed in percentage changes to initial value (solid line on left axis, dashed line on right axis). MC results based on 5000 networks each with $T = 100$.

expected profitability increases and banks increase monitoring and search efforts.[17] As a consequence, more loans are granted and interest rates decrease. This in turn feeds into banks' expectations about spreads and bilateral link probabilities, which further fosters monitoring and search. Note that the initial shock to the bilateral volume dies out relatively fast. On the other hand, peer monitoring levels stay above initial values for extended time periods. The low mean reversion is driven by an increased expected link probability that has positive effects on monitoring. As a consequence, the multiplier effect of monitoring and endogenous counterparty selection further drives up the link probability and reduces interest rates until 20 periods after the shock. Thus, the initial shock to interbank profitability is reinforced by the interrelation between control variables, outcomes, and state variables. This basic amplification mechanism is at the core of our model and can explain several features of the observed interbank network as we discuss next.

[17] In our example, we shock the bilateral loan volumes. A similar pattern emerges if we shock the links $l_{i,j,t}$ or the spreads $r_{i,j,t}$ (by $B_{i,j,t}$ and $u_{i,j,t}$ respectively).

5 Model Analysis

In this section, we use the estimated structural model to study the effects of key frictions on the network structure. In the line of our analysis, we focus on assessing the role of private information, gathered through peer monitoring and repeated interaction, in shaping the network of bilateral lending relations and associated interest rates and volumes. Moreover, we use the model to analyze the effect of changes in the central bank discount window on the interbank lending structure.

5.1 Comparison of Auxiliary Statistics

We first analyze the model fit and the observed and simulated values of the auxiliary statistics under the estimated structural parameter. We benchmark our estimated model against an alternative model parametrization $\boldsymbol{\theta}_a$, where the effects of monitoring on the perception error variance are restricted to be zero ($\beta_{\phi,1} = 0$). By setting off the monitoring channel but keeping all other things equal (in particular the parameters related to banks' liquidity shock distribution and search technology), we evaluate the role of peer monitoring on the network structure and associated bilateral credit conditions.

Table 4 shows how the estimated structural parameter vector $\hat{\boldsymbol{\theta}}_T$ produces an accurate description of the data when compared to the alternative calibrated parameter vector $\boldsymbol{\theta}_a$. First note the remarkable improvement in model fit compared to the calibrated example. This is brought about by the indirect inference estimation, as judged by (i) the value of the (log) criterion function that is about 54 times smaller for the estimated model, and (ii) the comparison between auxiliary statistics obtained from observed data, data simulated at the calibrated parameter, and data simulated at the estimated parameters. For instance, the Euclidean norm and the sup norm of the difference between observed and simulated auxiliary statistics are about 3.5 and 5 times larger under the calibration without monitoring, respectively.

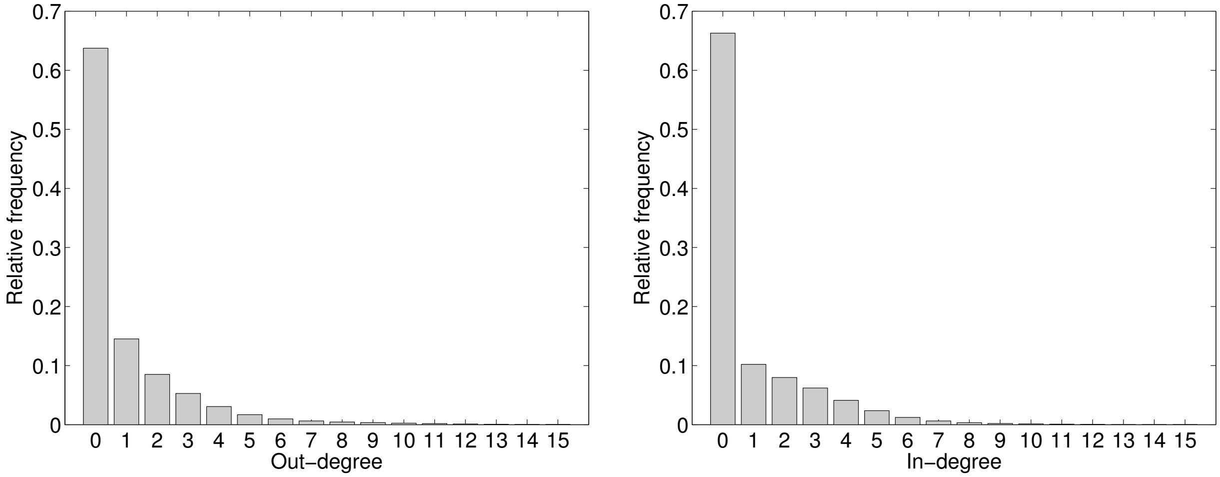
A closer look at individual auxiliary statistics reveals several interesting features of the estimated model. First, it is important to highlight the significant improvement in the fit of the density compared to the calibrated example. In fact, the estimated model matches the sparsity of the Dutch interbank network very well, with a density of about 0.02. Hence only a few banks pairs trade in the market on a daily basis. Likewise, the present structural model provides a very accurate description of the high stability of the network, with values of 0.98. Similarly, the average clustering coefficient improves considerably compared to the calibrated model and matches the data very well with a small value of 0.03. Moreover, the estimated model implies that about 6.3% of all links are reciprocal, compared to 8.2% in the observed data.

Second, a comparison of observed and simulated auxiliary statistics shows that the model is also

Table 4: Auxiliary network statistics. The table reports the values of the observed auxiliary statistics $\hat{\boldsymbol{\beta}}_T$ used in the indirect inference estimation along with the HAC robust standard errors. The simulated average of the auxiliary statistics $\tilde{\boldsymbol{\beta}}_{TS}$ for $S = 24$ paths is shown for the estimated parameter vector $\hat{\boldsymbol{\theta}}_T$ and the alternative calibration $\boldsymbol{\theta}_a$ (model without monitoring). The observed statistics are computed on a sample of daily frequency from 18 February 2008 to 28 April 2011 of size $T = 810$. Objective function is quadratic form with diagonal weight matrix, see Equation (4.1).

	Simulated		Observed	
	Calibrated	Estimated		
Auxiliary statistic	$\tilde{\boldsymbol{\beta}}_{TS}(\boldsymbol{\theta}_a)$	$\tilde{\boldsymbol{\beta}}_{TS}(\hat{\boldsymbol{\theta}}_T)$	$\hat{\boldsymbol{\beta}}_T$	ste($\hat{\boldsymbol{\beta}}_T$)
Density (mean)[a]	0.1121	0.0193	0.0212	0.0026
Reciprocity (mean)	0.0453	0.0627	0.0819	0.0029
Stability (mean)	0.8247	0.9795	0.9818	0.0025
Avg clustering (mean)	0.1097	0.0347	0.0308	0.0027
Avg degree (mean)	5.4948	0.9441	1.0380	0.1291
Std outdegree (mean)	3.2901	1.6547	1.8406	0.0918
Skew outdegree (mean)	0.4512	2.3649	2.8821	0.3537
Std indegree (mean)	4.7450	1.6950	1.6001	0.0995
Skew indegree (mean)	0.3300	2.2801	2.4030	0.3143
Corr($r_{i,j,t}, l^{rw}_{i,j,t-1}$) (mean)	0.0000	-0.1231	-0.0716	0.0113
Corr($l_{i,j,t}, l^{rw}_{i,j,t-1}$) (mean)	0.2345	0.6001	0.6439	0.0107
Avg log volume (mean)	2.8298	3.9422	4.1173	0.0516
Std log volume (mean)	1.0547	1.0865	1.6896	0.0200
Skew log volume (mean)	-0.1187	-0.1357	-0.3563	0.0317
Avg interest rates (mean)	1.0348	1.1353	0.2860	0.1331
Std interest rates (mean)	0.0000	0.1004	0.1066	0.0142
Skew interest rates (mean)	0.0251	1.6010	0.6978	0.5295
Corr(density,stability)	-0.4688	-0.3837	-0.7981	0.0275
Corr(density,rates)	0.0296	0.0896	0.7960	0.0229
Autocorr(density)	0.0034	0.2455	0.8174	0.0243
Autocorr(avg volume)	0.0014	0.0760	0.4926	0.0555
Autocorr(avg rate)	0.9991	0.2425	0.9655	0.0031
Objective function value	227.3328	4.2407		
Euclidean norm $\lVert\hat{\boldsymbol{\beta}}_T - \tilde{\boldsymbol{\beta}}_{TS}\rVert$	6.8563	2.0035		
Sup norm $\lVert\hat{\boldsymbol{\beta}}_T - \tilde{\boldsymbol{\beta}}_{TS}\rVert_\infty$	4.4568	0.9032		

[a] Not included in vector of auxiliary statistics as proportional to average degree.



(a) Out-degree distribution

(b) In-degree distribution

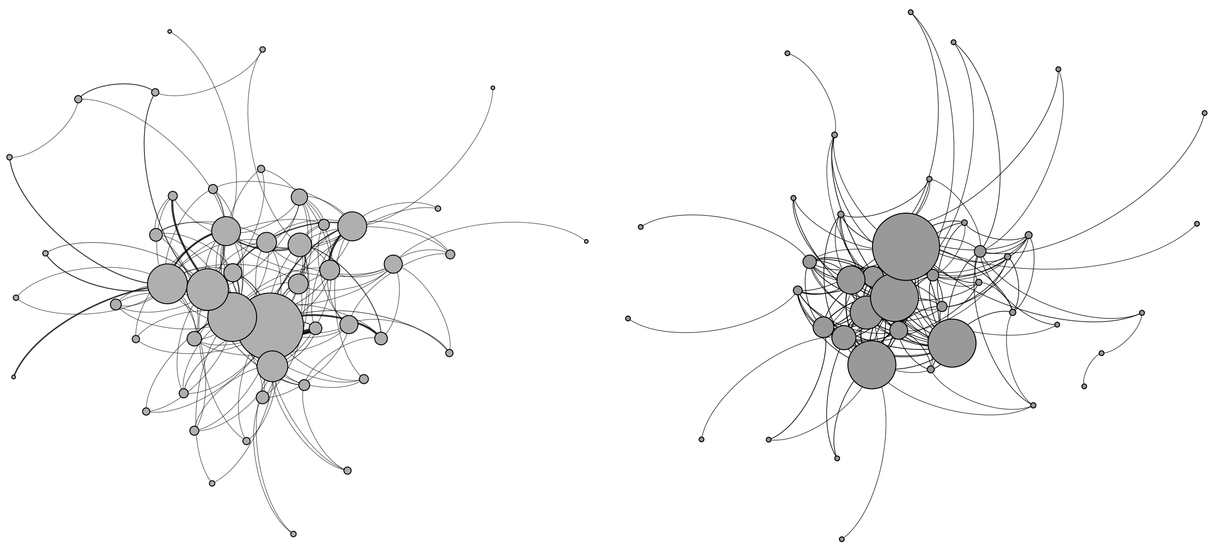
Figure 4: Marginal in-degree and out-degree distribution based on 5000 simulated network paths of size T = 25 under the estimated parameter vector $\hat{\boldsymbol{\theta}}_T$.

able to replicate well the first three moments of the observed degree distribution.[18] In particular, the estimated model generates a high positive skewness of both the in-degree and out-degree distribution (compare the simulated skewness of 2.4 and 2.3 with the observed one 2.8 and 2.4, respectively). Similarly, the standard deviation of both degree distributions are quite accurate with values of 1.7 and 1.7 as compared to the observed counterparts of 1.8 and 1.6. Figure 4 plots the simulated (marginal) in-degree and out-degree distribution under the estimated model parameters. The figure is the result of a MC analysis based on 5000 different networks each with T = 25. About 65% of all banks have zero trading partners on a daily basis (isolated vertexes), i.e. they do not lend or borrow in the market. Moreover, the majority (about 60%) of active banks has at most two borrowers and lenders. At the same time both degree distributions have a very long right tail indicating that there are few banks that borrow and lend from many other banks. Yet banks that have more than ten counterparties on a daily basis are very rare, with a relative frequency below 1%.

To illustrate the basic network topology, Figure 5 depicts the observed interbank network along with a network simulated from the estimated model. The graph shows the sparse and concentrated market structure with few banks in the center of the network that trade large volumes on either side of the market (scale of nodes relates to volume). The visualization also highlights the skewed degree distribution of the observed and simulated network. In particular, large banks in the core have multiple counterparties, while small banks typically have few trading partners and they are typically connected with banks in the center

Comparison of the estimated auxiliary statistics with those obtained from the calibrated model $\boldsymbol{\theta}_a$ without monitoring, shows that monitoring is an important factor in explaining the basic topology

[18]Note that the density is just a rescaled version of the average degree centrality, we did not include the density in the estimation but show it for convenience in Table 4.



(a) Observed network

(b) Simulated network (under $\hat{\boldsymbol{\theta}}_T$)

Figure 5: Interbank network during one week. Nodes are scaled according to total trading volume. Observed network corresponds to first week in April 2008. Simulated network under estimated parameter $\hat{\boldsymbol{\theta}}_T$ is randomly picked realization. Isolated nodes are not shown.

of the observed lending network. The calibrated model – in contrast to the estimated model with moitoring – fails to match specifically the skewed out- and in-degree distribution of the network with a simulated value of 0.45 and 0.33 (note that bank's liquidity shock distributions and all other parameters are held constant). Indeed, as we discuss in detail in the next section, the amplification mechanism of peer monitoring reinforces the tiered market structure.

Third, and key to our analysis, the estimated structural model is able to generate patterns of relationship lending where banks repeatedly interact with each other and trade at lower interest rates. In particular, the positive correlation of 0.60 between past and current bilateral lending activity, i.e. the measure of the stability of bilateral lending relationships, matches well the observed value of 0.64. Moreover, the model generates a negative correlation between interest rates and past trading of -0.12 (as compared to -0.07 for the observed data). As reported in Table 3, monitoring efforts depend positively on the expectation about being approached by a specific borrower. Once a contact between two banks is established, banks positively adjust their expectation and increase monitoring. This has dampening effects on the bilateral interest rate level and thereby further attracts the borrower leading to yet increased expectations about a contact. The role of bank-to-bank peer monitoring as the crucial driver behind the observed dynamic structure of the interbank market is also confirmed by comparing the fit of the auxiliary statistics simulated under the calibrated parameter with those of the estimated parameter. Clearly, under the calibrated example where there is no role for monitoring, bilateral stability of trading relations is low (0.23) and there is no effect of past trading on current prices as the effect of trading activity on reduction in uncertainty ($\beta_{\phi,2}$) is small and not significant.

Moreover, our model replicates rather well the mean and skewness of the distribution of (log) volumes of granted loans. Also the standard deviation points towards heterogeneity in bilateral loan amounts, thought the simulated value is not as large as the observed value. Note that the distribution of the potential bilateral volumes depends on the bank specific liquidity shocks through Equation (7). However, the decision to lend is endogenous and hence the distribution of granted loans also depends on other model parameters. Note also that the model does a poorer job in explaining the observed average interest rate level, while it captures very well the cross-sectional standard deviation of spreads that is in our model related to heterogeneous counterparty risk perceptions. Further, the skewness of the cross-sectional interest rate distribution has the right sign but is twice as large as the observed value.[19]

Finally, we find that our estimated model is able to generate quite some autocorrelation in the density (0.25) and the average interest rate of granted loans (0.24), in contrast to the average volume of granted loans. Clearly, the estimated values are not as high as the observed values (0.81 and 0.97, respectively). However, note that there are no common factors in the model and all shocks are iid. The only persistent processes are at the bank-to-bank level: credit risk uncertainty and the bank-to-bank specific expectations. Hence, the generated autocorrelation in these aggregate figures results from the same banks trading with each other in adjacent periods. Similarly, the model also generates a negative correlation between the density and the stability and a positive correlation between density and average spreads. Thus when there are less loans granted, the average spread of these loans decreases. In our model, this happens because when counterparty risk uncertainty is high, only bank pairs with low uncertainty (and hence low spreads) continue to trade.

5.2 Comparative Statics of Network Structures

We further evaluate the role of peer monitoring, credit risk uncertainty and counterparty search on the model outcomes. While in the previous section we compared two model parametrizations, we now vary the structural parameters and analyze responses in the auxiliary statistics. Figure 6 graphs how the mean density, reciprocity, skewness of out-degree and in-degree distribution, mean monitoring and mean search respond to changes in structural parameters by +/- 10% from their estimated values. Specifically, we focus on varying the coefficient of monitoring ($\beta_{\phi,1}$) in Equation (3), the autoregressive coefficient of the log perception error variance (γ_σ) in Equation (1) and the parameter that determines the location of the logistic link probability function (α_λ) in Equation (5), while holding constant all other parameters at the estimated values.

In the left panel we see that an increase in the persistence of the log perception error variance leads to a lower network density and a higher fraction of reciprocal lending relationship. Moreover, for the plotted range of values of γ_σ, both the in- and out-degree skewness exhibits a hump shaped

[19] Recall that the model abstracts from any bank heterogeneity beyond differences in liquidity shocks, in particular differences in balance sheet strength or heterogeneous outside options. Moreover, in the current model there is no room of excess liquidity that might have effects on the level of interest rates.

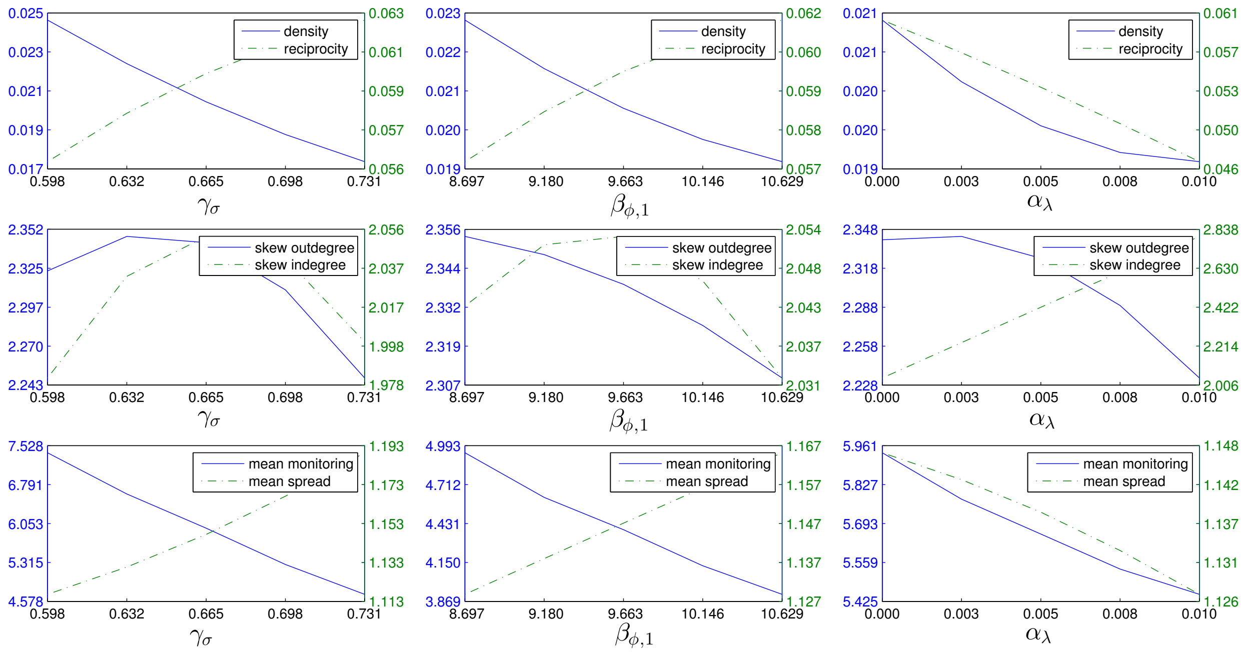


Figure 6: Comparative statics of network statistics. Simulated mean of network statistics as a function of key structural parameters related to credit risk uncertainty (γ_σ), efficacy of peer monitoring ($\beta_{\phi,1}$), and search frictions (α_λ). Parameters range from +/-10% around estimated values, holding fix all other parameters. Each graph is based on 5000 MC repetitions each with T=500. Left (right) axes correspond to solid (dashed) lines.

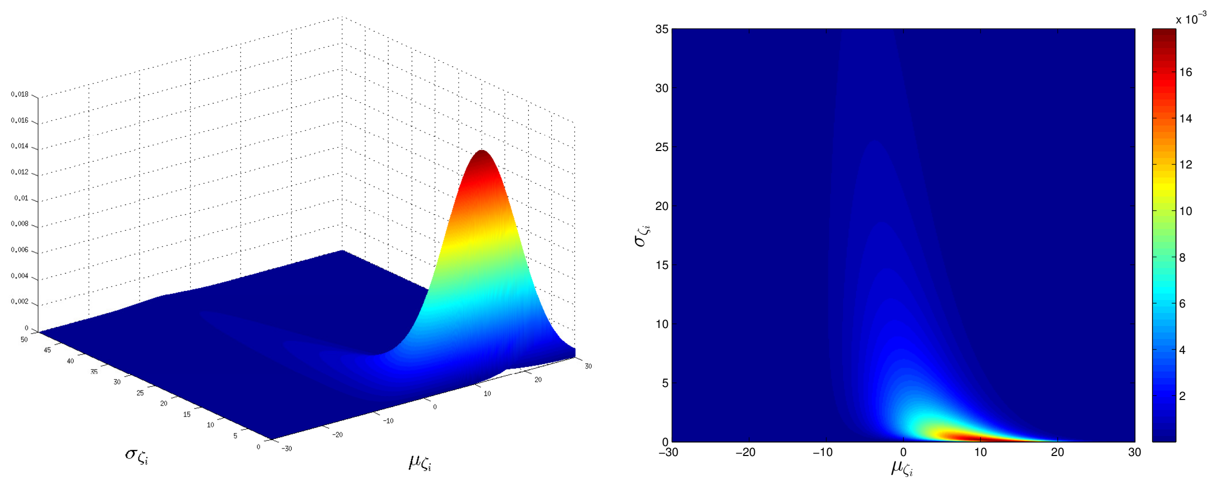
form. For a low persistence in credit risk uncertainty, an initial increase in γ_σ leads to higher skewness of the degree distributions, in particular the in-degree becomes more asymmetrically distributed. Economically, as the persistence of credit risk uncertainty increases some banks lose trading partners – potentially not accessing the market at all – while few highly connected banks can still maintain sufficiently many lending relations (these money center banks are intensively monitored as they are frequent borrowers in the market demanding large volumes). As the uncertainty increases further, however, lender banks will also occasionally refrain from providing credit to money center banks and the skewness decreases again. Moreover, a more persistent uncertainty leads to higher spreads of granted loans and decreases monitoring efforts due to lower profitability.

The network shows a qualitatively similar response to an local increase in the marginal effect of monitoring on the added information, specifically the density decreases and lending becomes more reciprocal (center panel). At the same time the average spread of granted loans increases and banks on average reduce peer monitoring efforts (bottom plot). The decline in monitoring occurs because $\frac{\partial \pi_{i,t}}{\partial m_{i,j,t} \partial \beta_{\phi,1}}|_{\theta=\hat{\theta}} < 0$ for sufficiently large $m_{i,j,t}$, in particular at the expansion point of the FOCs. Intuitively, banks' steady state monitoring levels are such that uncertainty is already relatively low and an increase in $\beta_{\phi,1}$ further reduces the marginal benefit from monitoring. To maintain the equality between the (constant) marginal cost and benefit a reduction in monitoring is necessary. The results confirm our findings of the previous section where we compare the estimated model with a model without monitoring ($\beta_{\phi,1} = 0$).

The right panel reveals that if banks need to invest more to maintain the same link probability, less trade occurs and lending becomes less reciprocal because some banks will not find it profitable to maintain some of their trading relationships. However, as the large increase in the in-degree skewness suggests, the reduction in lending partners at the borrower level is again asymmetrically distributed. In particular, as the cost of link formation increases, borrowing becomes more concentrated towards few highly connected core banks. At the same time, the reduction in out-degree skewness reflects that highly connected lenders lose some of their borrowers that don't find it profitable anymore to incur the search cost, thereby reducing the asymmetry of the degree distribution. Moreover, while the average monitoring expenditures decrease as a reaction to higher cost of linking, the mean spread of granted loans decreases because those bank pairs that continue trading have lower uncertainty about their counterparts.

5.3 Bank Heterogeneity and Lending Relationships

Heterogeneous liquidity shock distributions are the only source of bank heterogeneity in our model. Yet they are important in determining the exogenous volumes of granted loans. As banks monitoring and search depend on expected loan volumes, they determine the distribution of the monitoring multiplier effects that is crucial in matching the basic network structure of the interbank market, such as the high skewness of the degree distribution as described in the previous subsection. In our model the distribution of liquidity shocks in the banking system is characterized by the probabilistic structure described in Section 3.



(a) Density (b) Contour plot

Figure 7: Joint distribution of mean (μ_{ζ_i}) and standard deviation (σ_{ζ_i}) of banks' liquidity shock distributions and contour plots as implied by the estimated model parameters.

Figure 7 plots the joint distribution of the bank specific mean μ_{ζ_i} and standard deviation σ_{ζ_i} of the liquidity shocks as implied by the estimated structural parameters $\hat{\mu}_\mu = 0$, $\hat{\sigma}_\mu = 1.99$, $\hat{\mu}_\sigma = 1.94$,

$\hat{\sigma}_\sigma = 1.98$ and $\hat{\rho}_\zeta = -0.78$. First, note that most probability mass is located around $\mu_{\zeta_i} = 0$ and at small values of σ_{ζ_i}. Hence, the median bank has small liquidity shocks that are about zero on average. Second, the distribution of μ_{ζ_i} is more dispersed for low values of σ_{ζ_i}. Thus for banks with a small variance parameter of the liquidity shock distribution (small banks) there is higher heterogeneity with respect to their mean parameter μ_{ζ_i}. Third, the contour plot reveals a banana shaped form of the distribution. In particular, small banks with very small-scaled liquidity shocks typically tend to have a liquidity surplus, while banks with very large-scaled shocks typically have a negative mean, indicating a liquidity deficit. This relation is driven by the correlation parameter ρ_ζ that we estimate to be -0.78. Finally, the long tail in the dimension of σ_{ζ_i} shows that few banks feature very large liquidity shock variances.

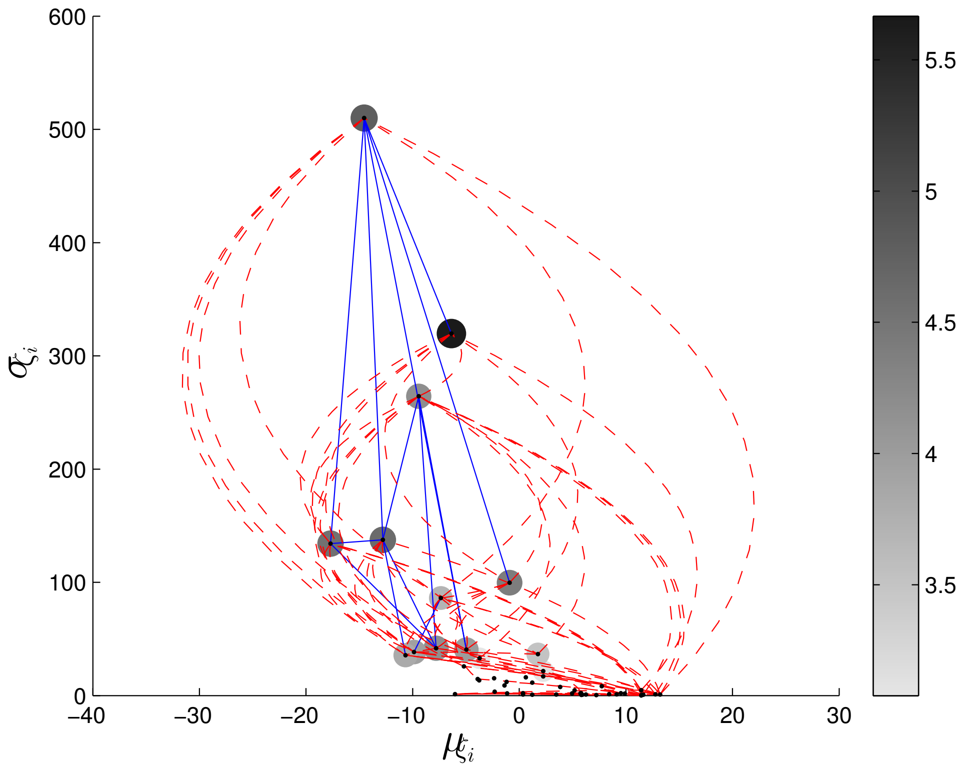


Figure 8: Simulated interbank network (5 trading days). Banks position in μ_ζ-σ_ζ plane given by mean and standard deviation parameter $(\mu_{\zeta_i}, \sigma_{\zeta_i})$. Node shading relates to average loan volume per bank. For each node incoming links are shown as dashed red lines coming from the right, outgoing links leave nodes from the left (counterclockwise). Solid blue lines represent reciprocal links.

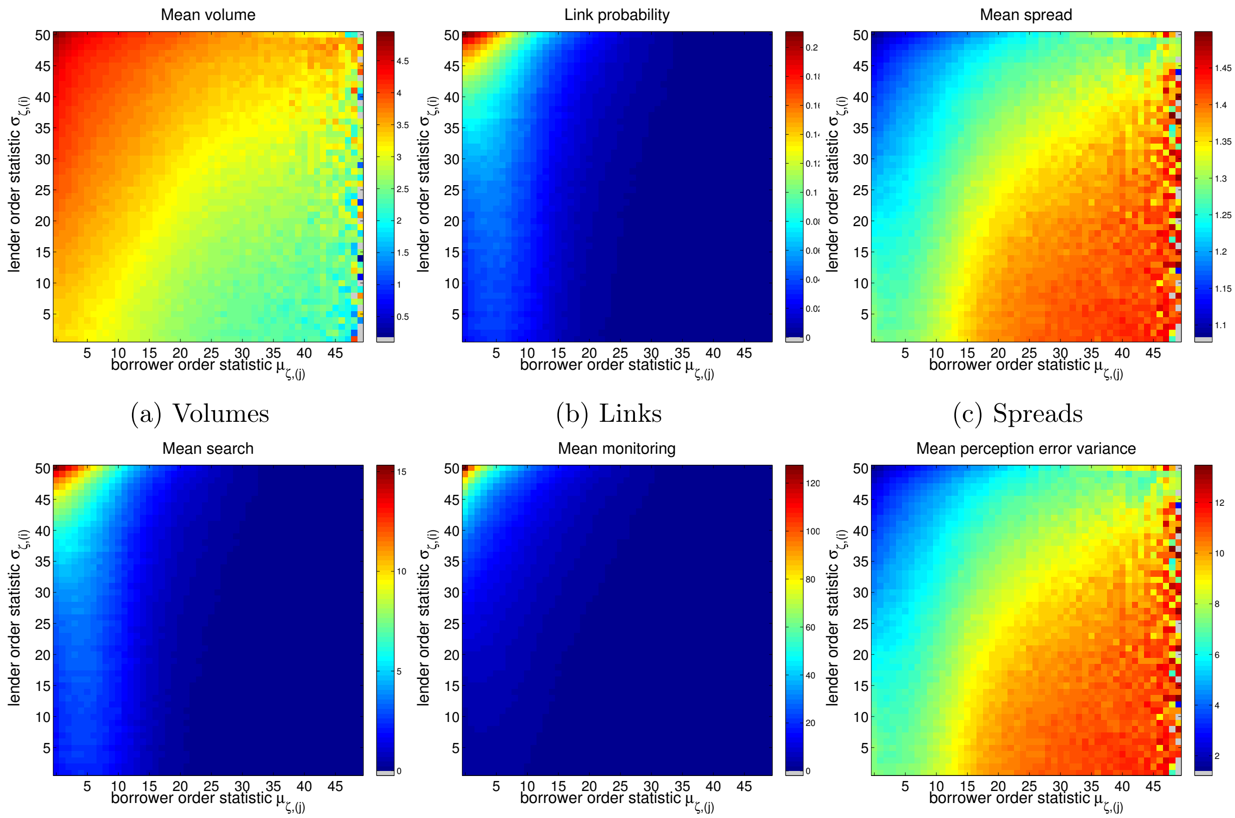
The estimated bank heterogeneity has important consequences for pairwise credit availability and conditions as well as for search and monitoring expenditures. In Figure 8 we graph the interbank activity during one week (five days) for 50 randomly drawn liquidity shock parameters (associated with 50 banks). Each bank is indicated by a black dot and its position in the μ_ζ-σ_ζ plane is given by the values of the bank specific mean and standard deviation parameters $(\mu_{\zeta_i}, \sigma_{\zeta_i})$. The figure reveals that small banks (small liquidity shock variance) are typically providing liquidity to the market, in particular to big banks (those that on average have a positive demand for liquidity) or small banks with complementary liquidity shocks.[20] In the market intermediation emerges as big banks (money center banks) act as lenders and borrowers at the same time. For small banks it is most efficient to trade with big banks that have large liquidity shocks than with banks with small ones. Moreover, big banks form a tightly inter-connected core where each member of the core has reciprocal lending relationships (solid blue lines) with other core banks, see the core-periphery analysis by Craig and von Peter (2014) and van Lelyveld and in 't Veld (2012). Clearly, big banks trade on average larger loan volumes than small banks as a result of their larger-scaled liquidity shocks.

We next present more rigorous Monte Carlo evidence to analyze the role of bank heterogeneity as the fundamental source of persistent trading opportunities. For this purpose, we simulate 5000 network paths and sort for each draw the lender banks according to their variance parameter σ_{ζ_i} and the borrower banks according to their mean parameter μ_{ζ_i} in increasing order. Hence, we compute the order statistics of both parameters. We then simulate for each draw $T = 25$ periods and compute the mean link probability, mean volume and spreads of granted loans as well as mean search and monitoring efforts between the lender order statistics and the borrower order statistics of all possible bank pairs.

Figure 9 shows the results of the MC analysis. Panel (a) depicts the mean granted loan volumes for different bank pairs. In particular, we see that banks with a structural liquidity deficit (on the left of the horizontal axis) are borrowing larger loan amounts than banks with a structural liquidity surplus (on the right of the horizontal axis). Both type of banks borrow larger volumes from big banks with a large variance parameters (on the top of the vertical axis). Due to the negative correlation parameter ρ_ζ, banks with low order statistic $\mu_{\zeta_{(i)}}$ are typically big banks and thus borrowing volumes with other big banks (with large $\sigma_{\zeta_{(i)}}$) are high. Similarly, mean traded volumes are low for banks with a structural liquidity surplus and lender banks with a small scaled variance parameter see the blue region in Panel (a).

The distribution of (exogenous) loan volumes affects monitoring decisions that eventually affect the prices at which bank pairs trade liquidity (see Panel (c) and (e)). Those bank pairs that can exchange large loan amounts (either because they have large variance or because they have complementary shocks) monitor more and trade at lower spreads (see the banana shaped contour plots). Again we see that the very large banks have high monitoring efforts and trade at very low rates with each other (up to 40 bps lower than high spread pairs). Hence, these core banks are not only highly interconnected but also credit risk uncertainty among these banks is very low (Panel (f)). Due to the interrelation with monitoring and search low interest regions in the graphs correspond to bank pairs where search levels are high and this leads to high link probabilities (see Panel (b)). Moreover, borrowers with a structural liquidity deficit obtain larger volumes at lower prices when

[20] This results is in line with similar empirical findings by Furfine (1999) and Bräuning and Fecht (2012) amongst others that small banks are net lenders.



(a) Volumes (b) Links (c) Spreads

(d) Search (e) Monitoring (f) Uncertainty

Figure 9: Bank heterogeneity and trading relationships. The order statistics of lender variance parameters $\sigma_{\zeta_{(i)}}$ is depict on vertical axis, order statistics of borrower mean parameter $\mu_{\zeta_{(i)}}$ is depict on horizontal axis, i.e. lender banks are ordered by variance parameter σ_{ζ_i} such that $\sigma_{\zeta_{50}} > \sigma_{\zeta_{49}} > ... > \sigma_{\zeta_1}$ and borrower banks ordered by mean parameter such that $\mu_{\zeta_{50}} > \mu_{\zeta_{49}} > ... > \mu_{\zeta_1}$. Results based on 10000 MC repetitions, each of length T=100.

borrowing from large banks than from small banks. This further highlight the role of intermediation in the model. Intermediaries have less credit risk uncertainty about their borrowers due to higher monitoring intensities, and in turn borrowers have lower credit risk uncertainty about intermediary banks because lenders channel monitoring towards those banks. Hence, the network's tiered structure that results from differences in liquidity shocks is reinforced by the presence of credit risk uncertainty and peer monitoring.

5.4 Dynamic Responses to Credit Risk Uncertainty Shocks

In this section, we analyze the effect of shocks to the perception error variance on the dynamics of the estimated network model. To account for the uncertainty about the precise latent liquidity shock distributions, we perform a simulation study by first drawing the bank properties (as described by the parameters μ_{ζ_i} and σ_{ζ_i}) and then calculating a set of key network statistics for T = 25 time periods. This procedure is then repeated in a Monte Carlo setting with 5000 replications. In all simulated structures we impose a large positive shock to the perception error variance in period $t = 4$ (affecting the perception error variance in $t = 5$) to investigate how our key network statistics react to increases in credit risk uncertainty.

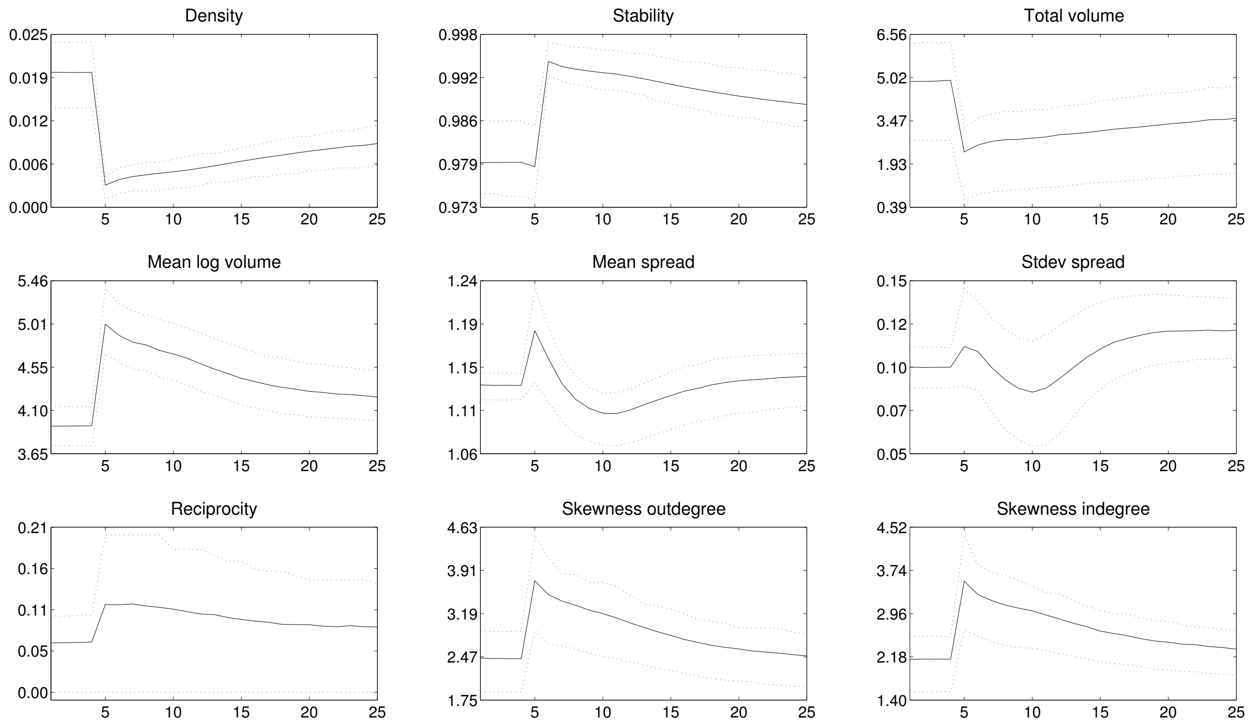


Figure 10: Simulated impulse responses to a common ten standard deviations shock in credit risk uncertainty in t = 4. Results are based on 5000 MC repetitions. The solid line is the mean impulse response, the dotted lines refer to the interquartile range across all network structures. Total volume is in billions and mean volume is the mean log volume (in millions) of granted loans.

The solid lines in Figure 10 depict the mean responses across all network structures to an extreme ten standard deviation shock in credit risk uncertainty, i.e. we impose $u_{i,j,4} = 10 \; \forall i, j$. The interquartile range (dotted lines) in this figure reflects essentially the uncertainty about the exact network structure as described by the unobserved liquidity shock distributions. For instance, the interquartile range of the mean network density is between 0.014 and 0.023 and the mean is about 0.019 depending on the precise network structure.[21] In the top panel, we see that at the time of the shock to the credit risk uncertainty, the network density drops by more than 75%. Both density and total volume remain at low levels and twenty trading days after the shock they are still at 50% of their pre-crisis value. Moreover, the log of total transaction volume plummets by more than 50% as a result of less trading activity. At the same time we observe an increase in the average (log) volume of granted loans compared to pre-shock levels and an increase in the network stability one period after the shock. Similarly, both in-degree and out-degree distribution become more positively skewed

[21] For any fix structure of liquidity shocks, the interquartile range is much tighter around the mean response.

and the reciprocity increases more than twofold.[22] Hence, the network shrinks and trading becomes more concentrated among highly interconnected core banks.

These changes are driven by the fact that some bank pairs that are active before the shock, stop trading as the lenders' risk assessments deteriorate, implied interest rate spreads explode and are for some pairs too high compared to the outside option. These loans are substituted by increased recourse to the central bank's standing facilities (not shown), which moves inversely to the density and total transaction volume. In fact, the increased average loan volume shows that in period t=5 relatively more bank-pairs are trading that exchange larger volumes (due to their size and/or complementarity of liquidity shocks). As discussed in the previous section, these are bank pairs where monitoring is particularly profitable and bank-to-bank uncertainty is low, making interbank lending still more attractive than the outside option even after the shock. Yet, also those trades that do occur are associated with increased spreads due to higher uncertainty; the average spread of granted volumes increases by about 6 bps right after the shock. Thus the compositional effects do not immediately outweigh the uncertainty-induced increases in interest rates. However, about two periods after the shock, the average spread of trades that do occur are back to initial levels and a further decrease can be observed until about period 10 when the average rate of traded loans is below the level before the shock. A similar pattern can be observed for the cross-sectional standard deviation of interest rates. While it increases to 0.11 as the shock hits, it decreases to about 0.09 in period 10 when mean spreads are lowest and then increases to levels higher than before the shock for an extended period of time.

Figure 11 depicts the impulse responses for the banks' expectations and control variables as crucial drivers behind the discussed changes in observable network statistics. Again the solid line refers to the mean and the dotted lines to the inter-quartile range representing the uncertainty about the latent network structure. The top left panel shows how mean credit risk uncertainty peaks at the time of the shock in period t=5. Clearly, the increase in mean credit risk uncertainty translates into an increase in the mean expectations about future credit risk uncertainty that show a similar behavior yet at lower values. As a consequence of the higher (expected) uncertainty (that is directly translated into higher bilateral equilibrium rates) after the shock the expected profitability of interbank borrowing decreases as the spread that can be earned compared to discount window borrowing declines. This leads borrower banks to invest less in counterparty search, further bringing down interbank trade. Note that the impaired funding conditions due to higher credit risk uncertainty only feed gradually into borrower's expectations about interbank profitability, as banks only update their expectations once they got in contact with a lender. Therefore, mean search effort declines gradually until it reaches a minimum in period 10. Moreover, these search effort is picked up by lenders' expectations about future contacting probabilities that gradually decline from period 5 onwards until the end of the plotted sample (though the decrease in the mean expectation

[22] The lower bound remains at zero because for some network structures interbank lending breaks down completely leading to zero reciprocity.

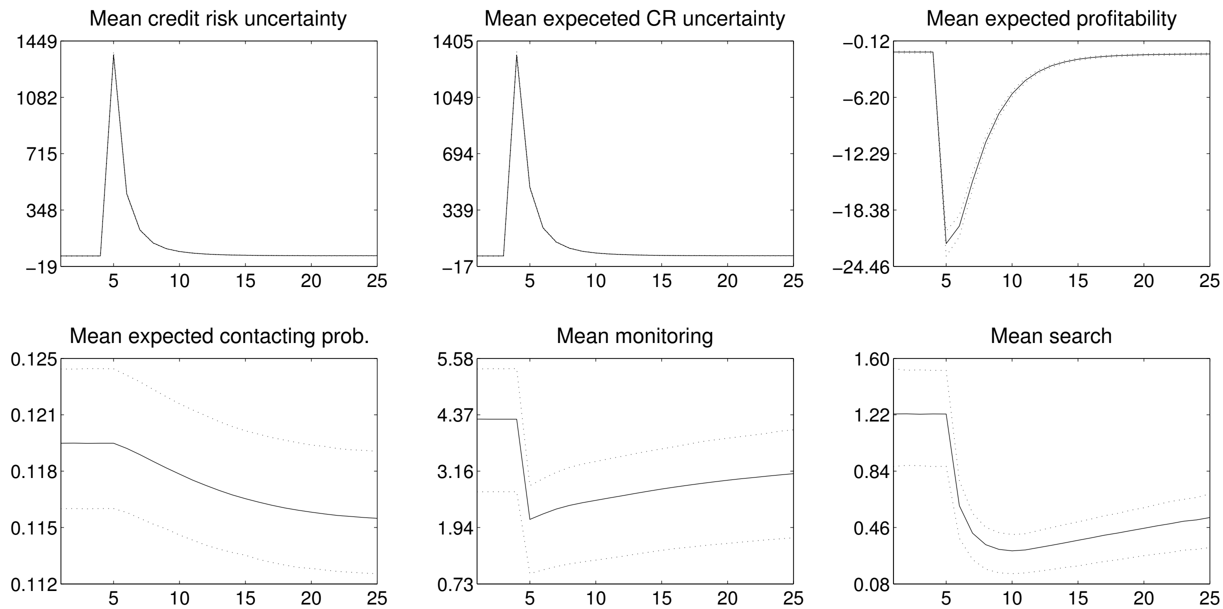


Figure 11: Simulated impulse responses to a common ten standard deviations shock in credit risk uncertainty in t = 4. Results are based on 5000 MC repetitions. The solid line is the mean impulse response, the dotted lines refer to the interquartile range across all network structures. Expectations are in deviations from steady state values. Monitoring and search expenditures are in thousands of euros.

is arguably small).

Moreover, as a response to the increased perception error variance banks adjust their monitoring expenditures from about 4.5 thousand euros on average (per bank-pair) downwards to two thousand euros. Several channels drive this decease that contributes to the prolonged period of interbank trading inactivity and prevents a fast recovery of the market. From the estimated linear policy rules we see that banks increase monitoring as a response to higher credit risk uncertainty, however at the same time adjustments to future expected uncertainty makes them decrease peer monitoring. Because the estimated EMWA forecast parameter is low, these expectations follow closely the actual credit risk uncertainty that has quite persistent dynamics. In sum, the negative effect of future uncertainty dominates such that the overall mean effect of this large ten standard deviations shock is negative. In addition, the gradual decrease in expected future contacting probability due to lower search efforts further dampens bank's monitoring expenditures and prevents a faster recovery of the market. Note that we plot here the mean values of bank-to-bank specific expectations and control variables. Of course, other moments change as well in response to the shock. In particular, the distribution of monitoring and search efforts becomes more skewed.

In Figure 12, we analyze heterogeneous responses to a shock in credit risk uncertainty. To this end, for each MC repetition we group banks into three categories based on total trading volume during the 100 periods preceding the shock at t = 4. Group 1 includes the largest 5 banks in terms

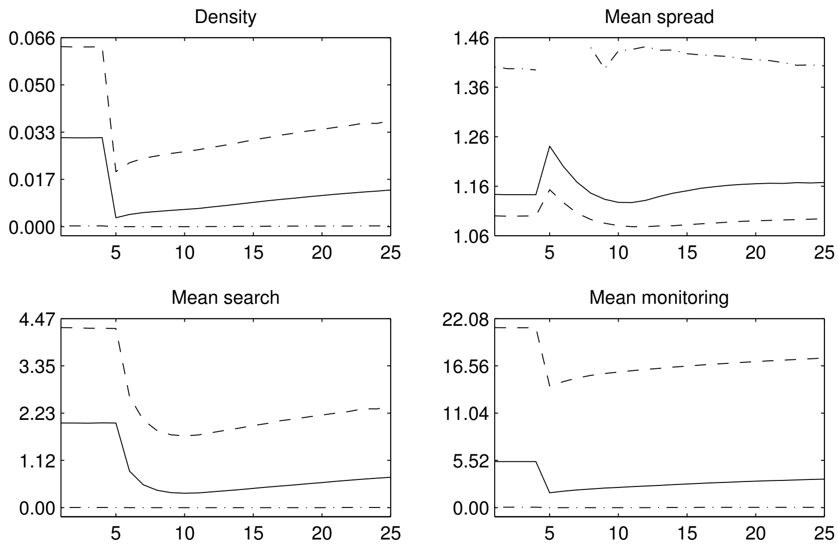


Figure 12: Heterogeneous impulse responses of top 5 (dashed line), 6-25 (solid line), 26-50 banks (dashed-dotted line) in terms on total volume in 100 periods preceding the common ten standard deviations shock to credit risk uncertainty in t = 5. Simulation results based on 5000 MC repetitions.

of total trading volume (1th-10th percentile), group 2 includes the 6th to 25th (11th-50th percentile) and group 3 includes the remaining banks (51th-100th percentile). In the top left panel, we see that the large banks have on average a much higher link probability than the medium sized or small ones. In fact, the latter group has a very low participation rate close to zero. With respect to the dynamics after the shock, we observe a drop in trading activity for big and medium sized banks and a subsequent increase with a similar pattern as for the aggregate figures. For the small banks that are mostly inactive anyway, there is no effect of the shock.

In the top right panel we plot the mean interest rate spreads for the three groups. Large banks trade at spreads of about 1.10 % while medium sized banks pay about 1.16 %. For small banks the mean spread of granted loans is by about 25 basis points higher with average values of 1.40 %. Interestingly, we see a heterogeneous reaction to the shock. The spreads large and medium banks pay increase as a response to higher uncertainty, for large banks however the increase is only about 5 bps whereas for medium banks it is about 10 bps. On the other hand, small banks completely stop borrowing in the market for three periods once the shock hits. Because the spread those banks pay is already very high, a slight increase in uncertainty will lead to bilateral rates that are outside the corridor and banks will not settle a deal.

5.5 Monetary Policy Analysis: The Interest Corridor Width Multiplier

The central bank's interest rate corridor is a key parameter of the model as it determines the price of the outside options to interbank lending. We next analyze how changes in the width the corridor affect the interbank lending network and associated credit conditions.

Figure 13 shows how the width of the ECB corridor produces significant changes in the structure of the interbank lending network that are driven by changes in banks' monitoring and search efforts. Again, the uncertainty captured by the interquartile range captures to a large extent the uncertainty about the precise latent distribution of liquidity shocks in the banking system. The most striking feature in Figure 13 is that an increase of roughly 100% in the width of the ECB's interest rate corridor (from 1 to 2 percentage points) produces a more than threefold increase in the mean network density (average number of daily trades), from roughly 1% density to over 3%. Furthermore, at a corridor width of 2%, the lower bound of the interquartile range across all network structures is larger than the upper bound on the interquartile range across network structures at a corridor width of 1%. This analysis shows that these effects are highly significant and that the ECB corridor width plays an important role in the intensity of interbank activity.

A second important feature of Figure 13 is that, since the model is nonlinear, the multiplier value is not constant over the range of ECB corridor widths. In particular, Figure 13 shows that the multiplier value is decreasing with the corridor width. Indeed, a 10 basis points increase of the bound has a much larger relative effect in density for low corridor widths compared to large ones. For instance, increasing the bounds from 1% to 1.25% leads to an (relative) increase in density by about 45%, while an increase from 1.75% to 2% leads to an (relative) increase of about 28%. The presence of this multiplier, as well as its nonlinearity, are both explained by the role that monitoring and search efforts play in the interbank market. Just like in the usual Keynesian multiplier, the effects of a change in the width of the ECB bounds can also be decomposed into *(i)* an immediate short-run effect, and *(ii)* a long-run effect that is created by the multiplier that results from feedback loops between the effect of monitoring and search on loan outcomes and expectations about credit conditions.

Consider a decrease in the width of the ECB corridor. In the advent of such a shock, the interbank market immediately shrinks as a fraction of loans are no longer profitable given the new tighter ECB bounds. In this immediate mechanical effect, part of the interbank market is simply substituted by lending and borrowing with the central bank authority which now plays a more important role. This immediate short-run effect is however only a fraction of the total long-run effect. Indeed, given that the possibilities of trade are now smaller, expected future profit is diminished and the incentive to monitor and search partners is reduced. This reduction in monitoring and search (depicted in Figure 13) will further reduce the mean density and mean traded volumes in the interbank market. These, in turn force banks to revise the expected profitability of monitoring and search efforts, pulling these variables further down. This 'negative spiral' that defines the multiplier will bring the market to a new level of operation that can be orders of magnitude lower than that observed values prior to the bounds.

Similarly, an increase in the size of the corridor leads to a wider market participation again fostered by banks' increased monitoring levels. Moreover, from Figure 13 we see that with a wider

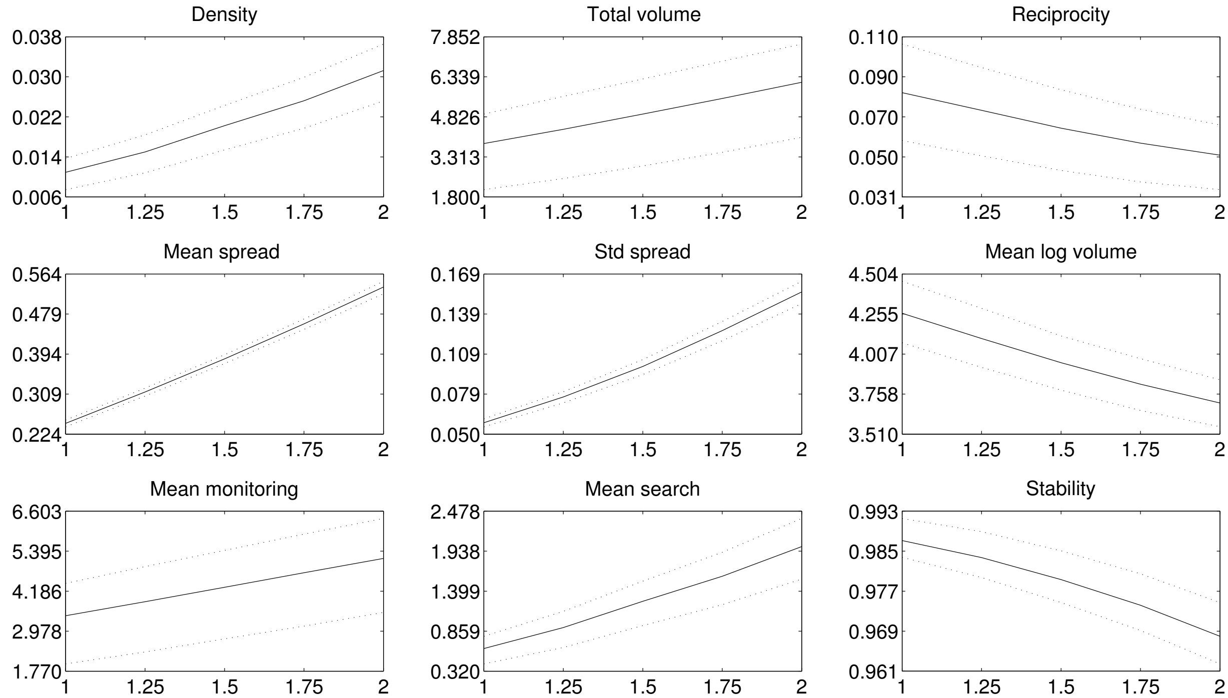


Figure 13: Effect of changes in ECB corridor width. Simulated mean and inter-quartile range of key network statistics and mean monitoring and mean search per bank over alternative ECB interest corridor width. Total volume is in billion euros. MC results based on 5000 networks each with $T = 25$.

interest rate corridor both the mean spread of granted loans (to the center of the corridor) as well as the cross-sectional standard deviation increases, while the average (log) volume traded decreases. The changes in these market outcomes are driven by bank pairs that did not trade under the narrower interest rate corridor but preferred to deposit funds at the central bank. After the increase of the corridor trading becomes profitable for those bank pairs, though interest rates are still high driving up the average rate and standard deviation of granted loans. Similarly, the reciprocity of the trading network and the stability decreases because with a wider corridor, trading becomes more attractive for banks that only occasionally access the market.

Hence, if the ECB wishes to get tighter control over the traded rates by narrowing the corridor, it has to expect further adverse effects on interbank lending activity triggered by a reduction of monitoring and search. On the other hand, if the ECB wants to foster an active decentralized interbank lending market as a means to explore benefits from peer monitoring, it is essential to consider policies that increase the difference between the standing facilities for depositing and lending funds. Only then is the interbank market profitable enough to encourage intense peer monitoring and search among banks. In both cases the multiplier effect should be taken into account when considering policy changes.

6 Conclusion

In this paper we propose and estimate a structural micro-founded network model for the unsecured interbank lending market where banks can lend and borrow funds in an over-the counter market to smooth liquidity shocks or resort to the central bank standing facilities. Banks choose which counterparties to approach for bilateral Nash bargaining about interest rates and set monitoring efforts to mitigate asymmetric information problems about counterparty risk. We estimate the structural model parameters using network statistics of the Dutch unsecured overnight interbank lending market from February 2008 to April 2011.

Our estimated model shows that prevailing bank-to-bank uncertainty and peer monitoring in interaction with counterpart search generates an amplification mechanism that can generate key characteristics of interbank markets. First, banks form long-term lending relationships that are associated with improved credit conditions. Second, the lending network exhibits a sparse core-periphery structure. Moreover, our dynamic analysis shows that shocks to credit risk uncertainty can bring down lending activity for extended periods of time.

We use the estimated model to discuss monetary policy implications. In particular, we show that in order to foster trading activity in unsecured interbank markets and exploit benefits from peer monitoring, an effective policy measure is to widen the bounds of the interest rate corridor. The effects of a wider corridor result from both a direct effect and a non-linear, indirect multiplier effect triggered by increased monitoring and search activity among banks.

For future research, we believe that our framework could be used to study several interesting extensions. First, in this paper we do not study the effects of liquidity hoarding and excess liquidity on market participation and the bilateral bargaining problem and monitoring decisions.[23] Second, this paper leaves open the question of the optimal corridor size, which requires assumptions on the central bank's preferences. Third, an interesting analysis could ask how the failure of an interbank relationship lender that disposes of private information tightens credit conditions for its respective borrowers, thereby giving room to contagion from the asset side.

[23] In this paper, we do for instance not address the effects of the LTROs as of end 2011.

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A Solution of FOCs

From the first-order condition for the optimal search expenditure in Equation (10) we get

$$
\begin{aligned}
&\frac{\partial}{\partial s_{i,j,t}}\mathbb{E}_t\Big[(r-r_{j,i,t})y_{j,i,t}l_{j,i,t}\Big]=1\\
\Leftrightarrow\ &\frac{\partial}{\partial s_{i,j,t}}\mathbb{E}_t\Big[(r-r_{j,i,t})y_{j,i,t}I_{j,i,t}B_{j,i,t}\Big]=1\\
\Leftrightarrow\ &\mathbb{E}_t\Big[(r-r_{j,i,t})y_{j,i,t}I_{j,i,t}\Big]\frac{\beta_\lambda\exp(-\beta_\lambda(s_{i,j,t}-\alpha_\lambda))}{(1+\exp(-\beta_\lambda(s_{i,j,t}-\alpha_\lambda)))^2}=1
\end{aligned}
$$

where the first step uses the definition, the second step uses independence of $B_{j,i,t}$. The above equation can be solved analytically for $s_{i,j,t}$ leading to Equation (13).

The first-order condition for monitoring in Equations (9) is

$$
1=\frac{1}{1+r^d}\frac{\partial\xi_{i,j,t}}{\partial m_{i,j,t}}\mathbb{E}_t\left(\frac{\frac{\partial\xi_{i,j,t+1}}{\partial\tilde{\sigma}^2_{i,j,t+1}}}{\frac{\partial\xi_{i,j,t+1}}{\partial m_{i,j,t+1}}}-\frac{\partial\pi_{i,j,t+1}}{\partial\tilde{\sigma}^2_{i,j,t+1}}\right).
$$

Using the product rule, we get $\frac{\partial\pi_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}=\frac{\partial\bar{R}_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}y_{i,j,t}l_{i,j,t}+\bar{R}_{i,j,t}y_{i,j,t}B_{i,j,t}\frac{\partial I_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}$, which we can further unfold using the following partial derivatives

$$
\begin{aligned}
&\frac{\partial\phi_{i,j,t}}{\partial m_{i,j,t}}=\beta_\phi,\quad \frac{\partial P_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}=\frac{\epsilon^2}{(\sigma^2+\tilde{\sigma}^2_{i,j,t}+\epsilon^2)^2},\quad \frac{\partial r_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}=0.5/\epsilon^2\\
&\frac{\partial\xi_{i,j,t}}{\partial\phi_{i,j,t}}=\exp(\alpha_\sigma+\gamma_\sigma\log\tilde{\sigma}^2_{i,j,t}+\beta_\sigma\phi_{i,j,t}+\delta_\sigma u_{i,j,t})\beta_\sigma,\\
&\frac{\partial\xi_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}=\exp(\alpha_\sigma+\gamma_\sigma\log\tilde{\sigma}^2_{i,j,t}+\beta_\sigma\phi_{i,j,t}+\delta_\sigma u_{i,j,t})/\tilde{\sigma}^2_{i,j,t},\\
&\frac{\partial\tilde{R}_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}=-\frac{\partial P_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}+\frac{\partial 1-P_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}r_{i,j,t}+(1-P_{i,j,t})\frac{\partial r_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}\\
&\frac{\partial I_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}=\frac{\beta_I\exp(-\beta_I(r-r_{i,j,t}))}{1+\exp(-\beta_I(r-r_{i,j,t}))}(-\frac{\partial r_{i,j,t}}{\partial\tilde{\sigma}^2_{i,j,t}}),
\end{aligned}
$$

Equation (9) is highly non-linear and does not have an analytical solution. We therefore follow the standard practice to compute an approximate solution based on a Taylor expansion. To this end, write the Euler equation more compactly as

$$\mathbb{E}_t f(m_{i,j,t}, \tilde{\sigma}^2_{i,j,t}, \tilde{\sigma}^2_{i,j,t+1}, B_{i,j,t+1}, y_{i,j,t+1}) = 0.$$

The local Taylor approximation of f requires an expansion point. The usual steady state (resulting from the absence of any shocks to the system) proves inappropriate in our setting, as steady state volumes would be zero and, as a consequence, the steady-state corresponds to a critical point where all derivatives of f are zero. We therefore linearize the function f around the stable point $(\tilde{m}_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j}, \tilde{\lambda}_{i,j}, \tilde{y}_{i,j})$. This expansion point is obtained as the steady state of the system when $y_{i,j,t+1}$ is fixed at the expected loan volumes for two banks characterized by a liquidity shock distribution with mean parameter $\mathbb{E}(\mu_{\zeta_i}) = \mu_\mu$ and variance parameter $\mathbb{E}(\sigma^2_{\zeta_i}) = \exp(\mu_\sigma + \sigma^2_\sigma/2)$ (two 'average' banks).[24] As a result the expansion point is the same for each bank pair (i, j).[25]

In the following expansion we write $h_x := \frac{\partial h(x,y)}{\partial x}$ and use $\hat{x} := x - \tilde{x}$ to denote deviation from the expansion point. Applying the first-order Taylor expansion gives

$$f \approx \tilde{f} + f_{m_{i,j,t}} \hat{m}_{i,j,t} + f_{\tilde{\sigma}^2_{i,j,t}} \hat{\tilde{\sigma}}^2_{i,j,t} + f_{\tilde{\sigma}^2_{i,j,t+1}} \hat{\tilde{\sigma}}^2_{i,j,t+1} + f_{B_{i,j,t+1}} \hat{B}_{i,j,t+1} + f_{y_{i,j,t+1}} \hat{y}_{i,j,t+1}$$

where $\tilde{f} := f(\tilde{m}_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j}, \tilde{\lambda}_{i,j}, \tilde{y}_{i,j})$ and all derivatives are evaluated at the expansion point. Note that $\tilde{f} = 0$ by construction.

We then obtain the approximate Euler equation for monitoring as

$$\mathbb{E}_t \left[f_{m_{i,j,t}} \hat{m}_{i,j,t} + f_{\sigma^2_{i,j,t}} \hat{\tilde{\sigma}}^2_{i,j,t} + f_{\tilde{\sigma}^2_{i,j,t+1}} \hat{\tilde{\sigma}}^2_{i,j,t+1} + f_{B_{i,j,t+1}} \hat{B}_{i,j,t+1} + f_{y_{i,j,t+1}} \hat{y}_{i,j,t+1} \right] = 0$$

which we rearrange to get the linear policy function

$$m_{i,j,t} = a_m + b_m \tilde{\sigma}^2_{i,j,t} + c_m \mathbb{E}_t \tilde{\sigma}_{i,j,t+1} + d_m \mathbb{E}_t B_{i,j,t+1} + e_m \mathbb{E}_t y_{i,j,t+1},$$

that constitutes an approximate solution to the problem. Note that the intercept and the coefficients of the linear policy function are functions of the structural parameters.

B Reduced Form, Stationarity and Ergodicity

Substituting the adaptive expectation mechanism in (14) and (15) into the Euler equation for monitoring in (11) and the optimal search strategy in (12) allows us to re-write the full system in

[24] Due to the normality assumption for the liquidity shocks we can compute $\tilde{y}_{i,j} := \mathbb{E}(y_{i,j,t})$ analytically. Given $\tilde{y}_{i,j}$ we solve for the steady state values of $\tilde{m}_{i,j}, \tilde{\sigma}^2_{i,j}, \tilde{\lambda}_{i,j}$ under the absence of shocks to $\tilde{\sigma}^2_{i,j}$.

[25] Computationally it is infeasible to compute $N(N-1)$ different expansion points depending on banks' liquidity distribution.

reduced form. The reduced form can be written as a nonlinear Markov autoregressive process,

$$\mathbf{X}_t = \mathbf{G}_{\boldsymbol{\theta}}(\mathbf{X}_{t-1}, \mathbf{e}_t)$$

where $\mathbf{G}_{\boldsymbol{\theta}}$ is a parametric vector function that depends on the structural model parameter $\boldsymbol{\theta}$, and $\mathbf{X}_t$ is the vector of all state-variables and control variables (observed or unobserved), and $\mathbf{e}_t$ is the vector of shocks driving the system. These shocks are the bank-specific liquidity shocks $\{\zeta^j_{i,t}\}$, the bank-to-bank-specific shocks to the perception error variance $\{u_{i,j,t}\}$, and the shocks that determine if a link between any two banks is open and trade is possible $\{B_{i,j,t}\}$. Obtaining the reduced form representation is crucial as it allows us to simulate network paths for both state and control variables under a given structural parameter vector. Furthermore, this model formulation allows us to describe conditions for the strict stationarity and ergodicity of the model that are essential for the estimation theory that is outlined in Section 4.

In particular, following Bougerol (1993), we note that under appropriate regularity conditions, the process $\{\mathbf{X}_t\}$ is strictly stationary and ergodic (SE).

LEMMA 1. *For every $\boldsymbol{\theta} \in \Theta$, let $\{\mathbf{e}_t\}_{t\in\mathbb{Z}}$ be an SE sequence and assume there exists a (nonrandom) $\mathbf{x}$ such that $\mathbb{E}\log^+\|\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{e}_t) - \mathbf{x}\| < \infty$ and suppose that the following contraction condition holds*

$$\mathbb{E}\ln \sup_{\mathbf{x}'\neq\mathbf{x}''} \frac{\|\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}', \mathbf{e}_t) - \mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}'', \mathbf{e}_t)\|}{\|\mathbf{x}' - \mathbf{x}''\|} < 0 \tag{16}$$

Then the process $\{\mathbf{X}_t(\mathbf{x}_1)\}_{t\in\mathbb{N}}$, initialized at $\mathbf{x}_1$ and defined as

$$\mathbf{X}_1 = \mathbf{x}_1 \quad , \quad \mathbf{X}_t = \mathbf{G}_{\boldsymbol{\theta}}(\mathbf{X}_{t-1}, \mathbf{e}_t) \ \forall \ t \in \mathbb{N}$$

converges e.a.s. to a unique SE solution $\{\mathbf{X}_t\}_{t\in\mathbb{Z}}$ for every $\mathbf{x}_1$, i.e. $\|\mathbf{X}_t(\mathbf{x}_1) - \mathbf{X}_t\| \overset{e.a.s.}{\to} 0$ as $t \to \infty$.[26]

The condition that $\mathbb{E}\log^+\|\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{e}_t) - \mathbf{x}\| < \infty$ can be easily verified for any given distribution for the innovations $\mathbf{e}_t$ and any given shape function $\mathbf{G}_{\boldsymbol{\theta}}$. The contraction condition in (16) is however much harder to verify analytically.

Fortunately, the contraction condition can be re-written as

$$\mathbb{E}\log\sup_{\mathbf{x}} \|\nabla\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{e}_t)\| < 0 \tag{17}$$

where $\nabla\mathbf{G}_{\boldsymbol{\theta}}$ denotes the Jacobian of $\mathbf{G}_{\boldsymbol{\theta}}$ and $\|\cdot\|$ is a norm. By verifying numerically that this inequality holds at every step $\boldsymbol{\theta} \in \Theta$ of the estimation algorithm, one can ensure that the simulation-based estimation procedure has appropriate stochastic properties.

The contraction condition of Bougerol (1993) in (17) states essentially that the maximal Lyapunov exponent must be negative uniformly in $\mathbf{x}$.

[26] A stochastic sequence $\{\xi_t\}$ is said to satisfy $\|\xi_t\| \overset{e.a.s.}{\to} 0$ if $\exists\ \gamma > 1$ such that $\gamma^t\|x_t\| \overset{a.s.}{\to} 0$.

Definition 1. *The maximal Lyapunov exponent is given by* $\lim_{t\to\infty} \frac{1}{t} \log \max_i \Lambda_{i,t} = \mathbb{E} \log \max_i \Lambda_{i,t}$ *where* $\Lambda_{i,t}$*'s are eigenvalues of the Jacobian matrix* $\nabla \mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}_t, \mathbf{e}_t)$.

A negative Lyapunov exponent ensures the stability of the network paths. Table 5 uses the Jacobian of the structural dynamic system $\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{e}_t)$ to report numerical calculations of the maximal Lyapunov exponent of our dynamic stochastic network model at the parameters $\boldsymbol{\theta}_0$ and $\hat{\boldsymbol{\theta}}_T$ described in Table 2 of Section 4.3 below. These points in the parameter space corresponds to the starting point for the estimation procedure described in Section 4 and the final estimated point.

Table 5: Lyapunov stability of the dynamic stochastic network model

Parameter vector	$\boldsymbol{\theta}_0$	$\hat{\boldsymbol{\theta}}_T$
Lyapunov exponent	-0.6451	-0.2462

Despite the higher degree of persistence at $\hat{\boldsymbol{\theta}}_T$ compared to $\boldsymbol{\theta}_0$ (higher Lyapunov exponent), the contraction condition is satisfied in both cases as the maximal Lyapunov exponent is negative. This ensures that both $\boldsymbol{\theta}_0$ and $\hat{\boldsymbol{\theta}}_T$ generate stable network paths.

C Network Auxiliary Statistics

In this section, we provide formulæ for the non-standard auxiliary statistics that characterize specifically the (dynamic) structure of the interbank lending network. First, the global network statistics that relate to the sparsity, reciprocity and stability are given as

$$\mathit{density}_t = \frac{1}{N(N-1)} \sum_{i,j} l_{i,j,t}, \quad \mathit{reciprocity}_t = \frac{\sum_{i,j} l_{i,j,t} l_{j,i,t}}{\sum_{i,j} l_{i,j,t}},$$
$$\mathit{stability}_t = \frac{\sum_{i,j} \left(l_{i,j,t} l_{i,j,t-1} + (1 - l_{i,j,t})(1 - l_{i,j,t-1}) \right)}{N(N-1)}.$$

Further, we maintain information about the *degree distribution*. In the interbank market, the degree centrality of a bank counts the number of different trading partners. For directed networks the out- and in-degree of node i are given by

$$d_{i,t}^{out} = \sum_j l_{i,j,t} \quad \text{and} \quad d_{i,t}^{in} = \sum_j l_{j,i,t}.$$

Instead of considering all $2N$ variables individually, we consider the mean, variance and skewness of the out-degree and in-degree distribution. Note that the mean of degree distribution is proportional to the density. In the estimation procedure we include therefore only the average degree.

The (local) clustering coefficient of node i in a binary unweighted network is given by

$$c_{i,t} = \frac{1/2 \sum_j \sum_h (l_{i,j,t} + l_{j,i,t})(l_{i,h,t} + l_{h,i,t})(l_{j,h,t} + l_{h,j,t})}{d_{i,t}^{tot}(d_{i,t}^{tot} - 1) - 2d_{i,t}^{\leftrightarrow}},$$

where $d_{i,t}^{tot} = d_{i,t}^{in} + d_{i,t}^{out}$ is the total degree and $d_{i,t}^{\leftrightarrow} = \sum_{j \neq i} l_{i,j,t} l_{j,i,t}$, see Fagiolo (2007) for details. We consider the average clustering coefficient which is the mean of the local clustering coefficients.

Second, we compute simple bilateral local network statistics that measure the intensity of a bilateral trading relationship based on a rolling window of size $T_{rw} = 5$ (one week). As a simple measure of bilateral relationships, we compute the number of loans given from bank i to bank j during periods $t' = \{t - T_{rw} + 1, ..., t\}$ and denote this variable by

$$l_{i,j,t}^{rw} = \sum_{t'} l_{i,j,t'},$$

where the sum runs over $t' = \{t - T_{rw} + 1, ..., t\}$. We then consider for each t the correlation between current access and past trading intensity, and between current interest spreads (for granted) loans and past trading intensity,

$$\text{Corr}(l_{i,j,t}, l_{i,j,t}^{rw}) \text{ and } \text{Corr}(r_{i,j,t}, l_{i,j,t}^{rw}).$$

All described network statistics are computed for the network of interbank lending at each time period t such that we obtain a sequence of network statistics. We then obtain the unconditional means, variance and/or auto-correlation of these sequences as auxiliary statistics and base the parameter estimations on the values of the auxiliary statistics only.

D Summary Statistics

Table 6: Descriptive Statistics of Dutch Interbank Network: The table shows moment statistics for different sequences of network statistics and cross-sectional correlations that characterize the sequence of observed Dutch unsecured interbank lending networks. The statistics are computed on a sample of daily frequency from 18 February 2008 to 28 April 2011.

Statistic	Mean	Std	Autocorr	Skew	Kurtosis
Density	0.0212	0.0068	0.8174	0.8667	3.1983
Reciprocity	0.0819	0.0495	0.2573	0.2903	2.8022
Stability	0.9818	0.0065	0.8309	-0.8590	3.0503
Mean-out-/indegree	1.0380	0.3323	0.8174	0.8667	3.1983
Std-outdegree	1.8406	0.4418	0.6882	0.0553	2.4326
Skew-outdegree	2.8821	1.0346	0.7035	0.6074	2.4572
Mean-indegree	1.0380	0.3323	0.8174	0.8667	3.1983
Std-indegree	1.6001	0.4140	0.6880	0.6997	3.4529
Skew-indegree	2.4030	0.8787	0.6576	0.6714	2.7434
Mean-clustering	0.0308	0.0225	0.4149	0.7900	3.2473
Std-clustering	0.0880	0.0490	0.3587	0.1561	2.7280
Skew-clustering	3.7367	1.5454	0.1213	-0.2213	3.1281
Avg log-volume	4.1173	0.2818	0.4926	-0.2820	2.8220
Std log-volume	1.6896	0.1685	0.3623	0.1541	3.4546
Skew log-volume	-0.3563	0.2818	0.2970	-0.0669	3.2151
Avg rate	0.2860	0.3741	0.9655	1.1044	2.6965
Std rate	0.1066	0.0632	0.7865	1.6668	6.8848
Skew rate	0.6978	1.6399	0.5492	0.6832	2.9469
Corr($r_{i,j,t}$,$l^{rw}_{i,j,t}$)	-0.0716	0.1573	0.4066	0.0817	2.8539
Corr($l_{i,j,t}$,$l^{rw}_{i,j,t}$)	0.6439	0.0755	0.4287	-0.7653	4.2833